Revista Brasileira de Ensino de Física, v. 30, n. 3, 3303 (2008)
www.sbfisica.org.br

Sliding doors

(Portas corrediças)

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Recebido em 14/1/2008; Aceito em 31/1/2008; Publicado em 2/8/2008

Looking closely to the mechanism which make sliding doors move, one sees that the motion of the commonly used objects can be understood by solving a problem related to the motion of blocks and pulleys. The system is by itself interesting to be modelled as a problem in Newtonian mechanics, and the solution of the equation of the motion can lead us to estimate the time it takes for a sliding door to open or close.

Keywords: newtonian mechanics, physics education, problem solving.

Examinando em detalhes o mecanismo que produz o movimento das portas corrediças, observa-se que o movimento desses objetos usualmente usados podem ser entendidos resolvendo um problema modelado por blocos e polias. O sistema é per si interessante para ser modelado como um problema na mecânica newtoniana e a solução da equação do movimento pode nos levar a estimar o tempo de abertura e fechamento de uma porta corrediça.

Palavras-chave: mecânica newtoniana, ensino de física, solução de problemas.

1. Introducing the problem

It is a common experience to go through a sliding door when walking in a store and to observe its rather smooth motion. By looking closely to the mechanism responsible for the motion of the door, we notice that it can be schematized by a rather simple system, made of two pulleys and two material points. In order to adopt a simple model for the motion of sliding doors, let us consider a rather common way of mounting them, as shown in Fig. 1.

In Fig. 1 we notice that the two pulleys of radius r, free to rotate about a vertical axis, are connected by a string of negligible mass. Two doors, of mass M_D, follow the horizontal motion of points A and B of this string by means of connecting rods. In order for the string to transmit more easily horizontal motion to the doors in the two opposite senses, for a given angular velocity $\omega(t)$ of the pulleys, the weight of both doors is counterbalanced by the normal reactions given by a horizontal guide, on which a rather low friction force is present. In the case we neglected friction in the guides and the moment of inertia of the pulleys with respect to the quantity $M_D r^2$, the relation between the applied moment $M(t)$ and the acceleration $a(t)$ of the right door would be given by the rather simple relation

$$M(t) = 2rM_D a(t). \qquad (1)$$

However, still neglecting the mass of the pulleys, in the case viscous friction between the horizontal guides and the door supports is considered, the above relation is modified as follows

$$M(t) = 2rM_D a(t) + 2\beta r V(t), \qquad (2)$$

where β is the coefficient of viscous friction and $V(t)$ is the velocity of the right door. We shall derive the above dynamical equation and shall see, in what follows, how to deal with the friction term, which may be useful to calibrate the time of closure (or opening) of the sliding doors.

2. Finding the dynamical equation

In the present section we shall find a schematic representation of the system in Fig. 1, describing the apparatus by which sliding doors are set in motion. Successively, by means of elementary mechanics, we shall derive the equation of the motion of the system.

We start by schematizing the system in Fig. 1 taking the two sliding doors as point particles of mass M_D attached to a weightless string connected to two cylindrical pulleys of negligible mass and radius r, as shown

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in Fig. 2. When the moment $M(t)$ is applied to one of the pulleys, let us say the right one, the string is made to circulate and the attached masses follow the horizontal motion of the rectilinear portions of the string itself, their abscissa being, respectively, $x(t)$ and $-x(t)$ for the right and left door as in Fig. 2. We further assume that the two point particles move on a plane horizontal surface in such a way that the viscous friction force on the right particle is $\mathbf{f}_t = -\beta V(t)\,\hat{\mathbf{x}}$, $\hat{\mathbf{x}}$ being the unit vector in the horizontal direction, while on the left particle, although the analytic expression of the viscous force is formally the same, it has opposite direction, since the velocity of the left particle is opposite to the velocity of the right particle.

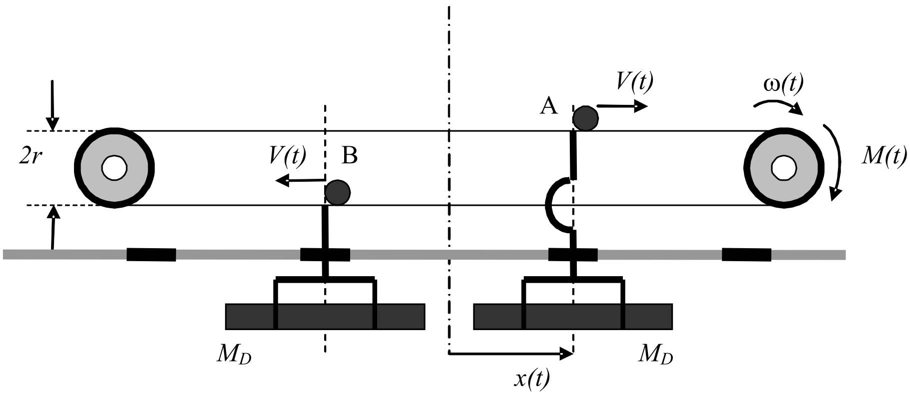

Figura 1 - A representation of the mechanisms for opening and closing sliding doors. An electric motor provides a moment $M(t)$ to the system of two pulleys connected by a string to which the doors are connected in such a way that points A and B on the string go in opposite directions. Points A and B, on their turn, are connected, by means of rods, to the right and left siding door, respectively, which follow the horizontal motion of these two points. The abscissa of point A is $x(t)$ and corresponds to the abscissa of the middle point of the right sliding door whose mass is M_D. The velocity of point A is $V(t) = r\,\omega(t)$, where r is the radius of the pulley and $\omega(t)$ its angular frequency.

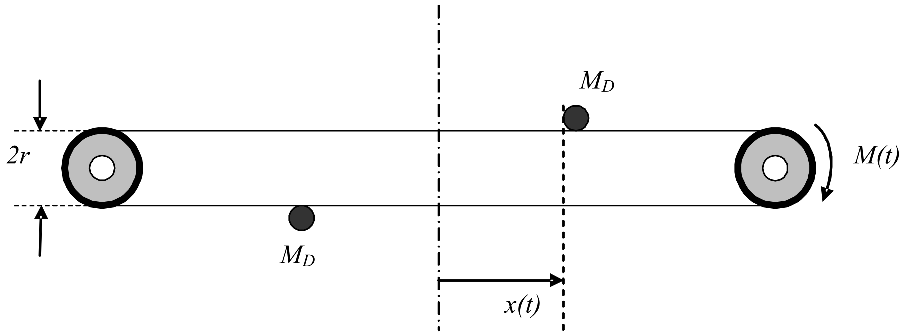

Figura 2 - A schematic version of the system in Fig. 1. The two doors are represented by point particles of mass M_D attached to a weightless string connecting two ideal pulleys. A friction force acts on both particles: its sense depends on the sign of the velocity vector of each door and, thus, on the sign of the angular velocity $\omega(t)$, taken as positive when the right door proceeds toward the right.

By applying Newton's second law to the point particles and by equating to zero all moments on the right pulley, referring to Fig. 3 we have

$$\begin{aligned} &M(t) + T_1 r - T_2 r = 0 \\ &T_2 - T_3 - f_t = M_D a \\ &T_3 - T_1 - f_t = M_D a. \end{aligned} \tag{3}$$

By dividing by r both members of the first equality in Eq. (3) and by summing all three equations, we get

$$\frac{M(t)}{r} - 2f_t = 2M_D a. \tag{4}$$

In this way, we can finally write

$$\frac{dV}{dt} + \frac{\beta}{M_D} V(t) = \frac{M(t)}{2M_D r}, \tag{5}$$

which is equivalent to Eq. (2). Notice that, for $\beta = 0$, Eq. (5) reduces to Eq. (1). Furthermore, one can easily argue that the above equation is similar to the dynamical equation for a particle moving in a viscous fluid [1]. In Eq. [5], however, we take the forcing term to depend on time. Indeed, by considering $M(t)$ as a decreasing linear function, *i.e.*, setting $M(t) = M_0 - At$, where M_0 is the applied moment at $t = 0$ and A is the absolute value of the slope of $M(t)$, we write

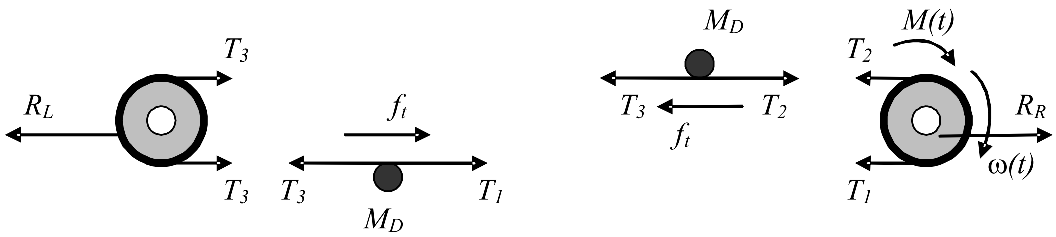

Figura 3 - Force diagram for the various parts of the schematic system in Fig. 2. Notice that the moment is applied in the same sense as the angular velocity $\omega(t)$ in such a way that the right door moves toward the right and, conversely, the left door moves toward the left. The reaction forces at the right and left hinges, are R_R and R_L, respectively.

$$\frac{M(t)}{2M_D r} = \frac{M_0 - At}{2M_D r} = m_0 - \alpha t, \tag{6}$$

so that a solution to Eq. (5) can be found in a similar way as in Ref. [1]. We assume that the applied moment acts up to time $t = \tau$, at which the doors reach their full aperture d with zero velocity, or

$$\begin{aligned} x(\tau) - x(0) &= \frac{d}{2}, \\ V(\tau) &= 0. \end{aligned} \tag{7}$$

Furthermore, in order to define the initial conditions for the differential equation (5), we take

$$V(0) = 0. \tag{8}$$

3. Solving the dynamical equation

We are now ready to solve the dynamical equation (5). Indeed, by substituting Eq. (6) into Eq. (5) we first write

$$\frac{dV}{dt} + \frac{\beta}{M_D} V(t) = m_0 - \alpha t, \tag{9}$$

so that the general solution for $V(t)$ can be found [2]

$$V(t) = e^{-\frac{\beta}{M_D}t}\left[c + \int_0^t e^{\frac{\beta}{M_D}\xi}(m_0 - \alpha\xi)\, d\xi\right], \tag{10}$$

where the constant c can be evaluated by imposing the initial condition (8), giving $c = 0$. By now evaluating the integral on the right hand side of Eq. (10), we have

$$V(t) = \frac{M_D}{\beta}\left[\left(m_0 + \alpha\frac{M_D}{\beta}\right)\left(1 - e^{-\frac{\beta}{M_D}t}\right) - \alpha t\right]. \tag{11}$$

Let us now impose the condition $V(\tau) = 0$ in Eq. (7), so that

$$\left(m_0 + \alpha\frac{M_D}{\beta}\right)\left(1 - e^{-\frac{\beta}{M_D}\tau}\right) = \alpha\tau. \tag{12}$$

We now derive the position $x(t)$ of the point particle with respect to its initial position $x(0)$, by further integrating Eq. (11) with respect to time, so that

$$x(t) - x(0) = \frac{M_D}{\beta}\left[\left(m_0 + \alpha\frac{M_D}{\beta}\right)t - \frac{M_D}{\beta}\left(m_0 + \alpha\frac{M_D}{\beta}\right)\left(1 - e^{-\frac{\beta}{M_D}t}\right) - \frac{1}{2}\alpha t^2\right]. \tag{13}$$

4. Calculating characteristic properties of the system

In the previous section we have completely solved the dynamical problem of opening (or closing) sliding doors, assuming that the friction force on them had a viscous character. In the present section we shall calculate the time in which this process is accomplished.

Let us first evaluate the expression for the position of the door at $t = \tau$ and set

$$\frac{d}{2} = x(\tau) - x(0) = \frac{M_D}{\beta}\left[\left(m_0 + \alpha\frac{M_D}{\beta}\right)\tau + \frac{M_D}{\beta}\left(m_0 + \alpha\frac{M_D}{\beta}\right)\left(1 - e^{-\frac{\beta}{M_D}\tau}\right) - \frac{1}{2}\alpha\tau^2\right]. \tag{14}$$

Because of Eq. (12), we can rewrite Eq. (14) as follows

$$\frac{d}{2} = \frac{M_D}{\beta}\left(m_0\tau - \frac{1}{2}\alpha\tau^2\right). \tag{15}$$

We can now solve the algebraic Eq. (15) for τ, obtaining the following solutions

$$\tau_\pm = \frac{m_0}{\alpha}\left[1 \pm \sqrt{1 - \frac{\alpha\beta d}{m_0^2 M_D}}\right]. \tag{16}$$

The largest solution is the one sought, since it gives a finite value of τ for very small β. In this case, indeed, which we shall consider to be real (a door with much too friction would be too difficult to open!) we can make a third order expansion of the exponential function in Eq. (12) and write

$$\left(m_0+\alpha\frac{M_D}{\beta}\right)\left(\frac{\beta}{M_D}\tau-\frac{1}{2}\frac{\beta^2}{M_D^2}\tau^2+\frac{1}{6}\frac{\beta^3}{M_D^3}\tau^3+O\left(\frac{\beta\tau}{M_D}\right)^4\right)=\alpha\tau, \quad (17)$$

where we shall retain only terms in which β appears to second order or less. In this way

$$\frac{\beta}{M_D}\tau\left[m_0\left(1-\frac{1}{2}\frac{\beta}{M_D}\tau\right)+\alpha\tau\left(\frac{1}{2}+\frac{1}{6}\frac{\beta}{M_D}\tau\right)\right]=0. \quad (18)$$

By setting to zero the term in parenthesis and by solving, to first order in $\beta\tau/M_D$, the resulting algebraic equation, we obtain

$$\tau_C=\frac{2m_0}{\alpha}\left[1-\frac{m_0\beta}{3\alpha M_D}\right]. \quad (19)$$

We may notice that this closure time τ_C is compatible with the solution τ_+ in Eq. (16) if the first order expansion of the latter, namely

$$\tau_+\approx\frac{2m_0}{\alpha}\left[1-\frac{\alpha\beta d}{4m_0^2M_D}\right], \quad (20)$$

coincides with τ_C, as expressed in Eq. (20). In this way, by simply comparing Eq. (19) and Eq. (20), we have

$$\alpha^2=\frac{4m_0^3}{3d}. \quad (21)$$

Finally, then, by substituting the value of α in Eq. (19), we obtain

$$\tau_C=\sqrt{\frac{3d}{m_0}}\left[1-\frac{\beta}{2M_D}\sqrt{\frac{d}{3m_0}}\right]. \quad (22)$$

From the above analysis we may notice that the absolute value α of the slope of the applied moment is expressed, in Eq. (21), in terms of the sole parameters m_0 and d, due to the fact that the speed of the two doors is taken as null when the latter reach their final positions. As a consequence, the closure time τ_C is seen to depend on the parameters of the problem as in Eq. (22).

In order to better understand the role played by the parameter α in determining change in τ_C, let us take $\beta/M_D = 0.1$ s^{-1} and $d = 2.4$ m to be fixed parameters. Let us then consider two different cases: the first for $m_0 = 6.0$ m/s^2, the second for $m_0 = 12.0$ m/s^2. In the second case the value of m_0 is simply doubled with respect to the first. In Fig. 4a and Fig. 4b the displacement and the velocity of the right door, as function of time, are shown for $m_0 = 12.0$ m/s^2, respectively.

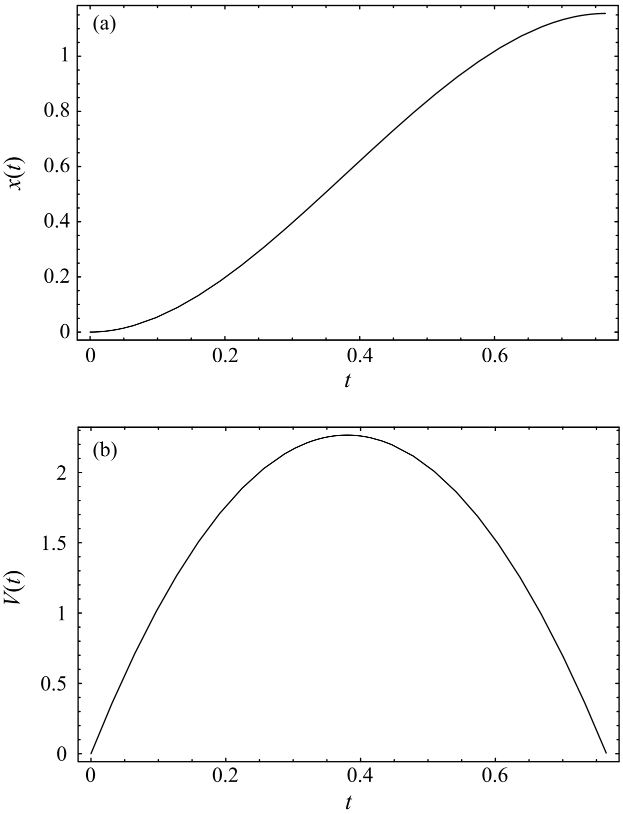

Figura 4 - Displacement in meters (a) and velocity in m/s (b) of the right door as a function of time, expressed in seconds, for $\beta/M_D = 0.1$ s^{-1}, $d = 2.$ m and for $m_0 = 12.0$ m/s^2. The calculated closure time is $\tau_C = 0.764$ s.

In Fig. 5, on the other hand, the applied moment M, divided by $2M_Dr$, is shown as a function of time for the two cases. From Fig. 4a we notice the smooth S-shaped motion of the right door, starting from rest and ending its run with zero velocity; therefore, the velocity of this door, shown in Fig. 4b, initially rises, until it attains a maximum value, and then decreases, until it reaches again the t-axis at $t = \tau_C$. The different dynamical behavior of the door for two different values of $m_0 = M(0)/2M_Dr$ can be resumed in Fig. 5, where the appropriate coefficient α, which determines the downward slope of the curve, is calculated as in Eq. (21). From Eq. (21) it is clear that, for higher values of the initially applied moment, the parameter α must be larger, so that, by Eq. (22), it results that the closure time τ_C must be lower. Therefore, the curve obtained for $m_0 = 12.0$ m/s^2 is truncated at time $t = \tau_C = 0.764$ s, while the curve obtained for $m_0 = 6.0$ m/s^2, for which the parameter α is lower, has a higher value of τ_C (1.075 s).

In order to see in more details for what range of m_0 the closure time diminishes as m_0 increases, we take the derivative of τ_C with respect to m_0, assuming β/M_D and d constant, obtaining

$$\frac{d\tau_C}{dm_0}=-\frac{1}{2m_0^2}\left(\sqrt{3m_0d}-\frac{\beta d}{M_D}\right). \quad (23)$$

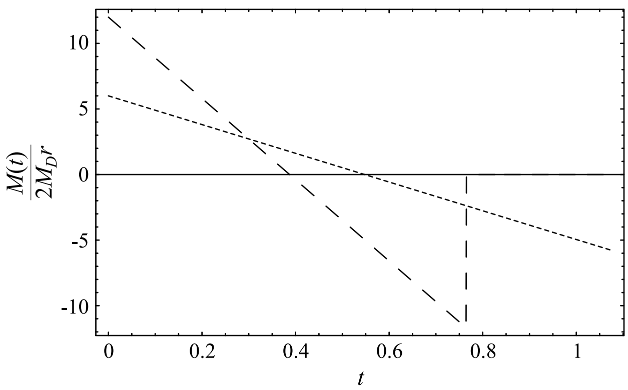

Figura 5 - Two different curves of the applied moment M, divided by $2M_D r$, as a function of time, expressed in seconds, as defined in the text. Both curves are plotted for $\beta/M_D = 0.1$ s^{-1} and $d = 2.4$ m. The dotted curve is for $m_0 = 6.0$ m/s^2, the dashed one for $m_0 = 12.0$ m/s^2. Notice that the closure time for the second curve is shorter since the parameter α is larger in this case (see Eq. (21) in the text).

In this way, we obtain a negative value of $d\tau_C/dm_0$, if the following relation is satisfied

$$m_0 > \frac{1}{3}\left(\frac{\beta}{M_D}\right)^2 d. \tag{24}$$

Given now that we only consider the case $\beta\tau_C/M_D << 1$, and being the desired closure times of the order of one second, we see that the inequality in Eq. (24) can well be always satisfied for the characteristic parameters of the present problem.

As a final remark, let us consider the instantaneous power $P(t)$ provided by the electric motor to the sliding doors. By definition, we have

$$P\,(t) = M\,(t)\,\omega\,(t) = M\,(t)\,\frac{V\,(t)}{r}. \tag{25}$$

In Fig. 6 we show the time dependence of the instantaneous power provided to the doors by the electrical motor. In particular, in Fig. 6a the value of $P\,(t)/2M_D$ *vs.* t is shown, which gives the instantaneous power per each kilogram of mass of the moving system necessary to get the desired motion for three different values of the normalized initially applied moment m_0. Notice that negative values of $P\,(t)$ means that the motor is applying an inverse torque to the sliding doors to make them decelerate. In Fig. 6b, on the other hand, the squared value of the quantity $P\,(t)/2M_D$ is shown for the same three values of the parameter m_0. The time average value of the square root of the latter quantity is indicative of the effective power dissipated by the motor (r.m.s. value). Notice, in this respect, that, as it is possible to argue from Fig. 6b simply by inspection, a lesser amount of energy is needed in decelerating the sliding doors rather than in accelerating them, for the given choice of parameters.

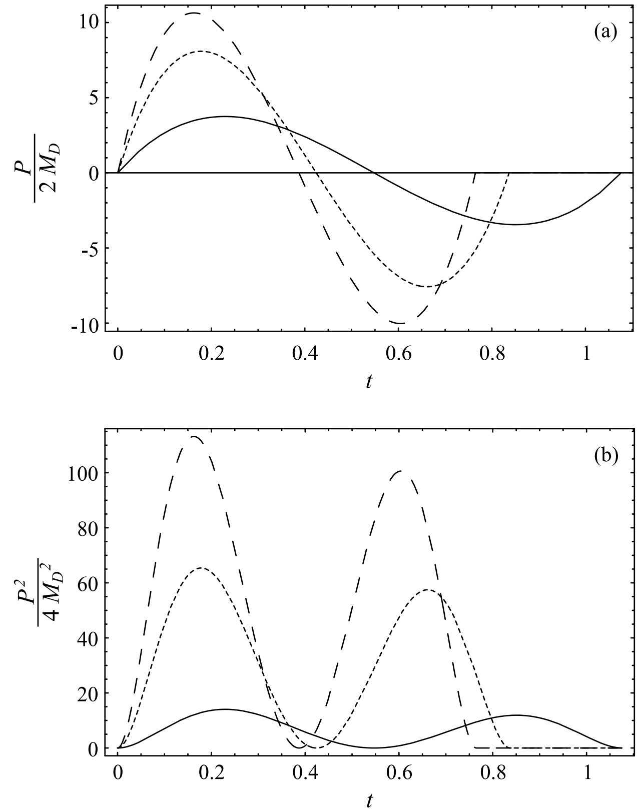

Figura 6 - Instantaneous power P, provided by the electrical motor to the sliding doors, divided by $2M_D$ (a) and its square value (b) for $\beta/M_D = 0.1$ s^{-1}, $d = 2.4$ m, and $m_0 = 6.0$ m/s^2 (full line), $m_0 = 10.0$ m/s^2 (dotted line), $m_0 = 12.0$ m/s^2 (dashed line).

5. Conclusions

We have considered the problem of the closure of two sliding doors. The rather simple way of realizing the motion of both doors has led us to consider a simplified model for describing the dynamics of the system. By the adopted scheme, the system consists of two point particles attached to a string, which runs over two ideal pulleys. The problem is rather interesting, since it makes use of the commonly adopted models for initiating the student to the study of the dynamics of point particles, and yet it is related to a frequently observed phenomenon: The closing or opening of a sliding door.

The characteristic properties of these systems are analyzed in details, by assuming that a viscous friction force is present between the guides and the doors supports. The resulting first-order differential equation, describing the motion of both doors, is solved and the closure time τ_C is found by imposing smooth closing, *i.e.*, by taking the final velocity of the door be zero at $t = \tau_C$.

Finally, because of what has been found, the present work can be useful not only as an application of basic physics to commonly observed phenomena, but also for engineering studies relating to the development of sliding doors with optimum closure or opening times.

Acknowledgments

The author is gratefully indebted to N. Di Brizzi, C. Formisano and F. Romeo for helpful discussions on the subject. The author also wishes to thank S. Ganci for advices.

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