# Coupling Constants of $D^{*} D_{s} K$ and $D_{s}^{*} D K$ Processes 

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#### Abstract

We calculate the coupling constants of $D^{*} D_{s} K$ and $D_{s}^{*} D K$ vertices using the QCD sum rules technique. We compare our results with results obtained in the limit of $S U(4)$ symmetry and we found that the symmetry is broken at the order of $40 \%$.


Keywords: Coupling constants; Form Factors; QCD Sum Rule

The knowledge of coupling constants in hadronic vertices is crucial to estimate cross sections when hadronic degrees of freedom are used. The kaon is one of the commovers light mesons that can annihilate the charmonium in a nuclear medium, given as result D and $D_{s}$ mesons. Therefore, the absorption of charmonium by kaons in a nuclear medium can be used to study the $J / \psi$ suppression in heavy-ion collisions, which is one of the signatures of the formation of the quark gluon plasma (QGP) [1]. The processes of absorption of $J / \Psi$ by kaons can be visualized in the Figure 1.

To evaluate theoretically the cross section for these processes, one can use the approach based on effective $\operatorname{SU}(4)$ Lagrangians [2, 4]. The effective Lagrangians that describe the processes represented in Fig. 1 are:

$$
\begin{align*}
& \mathcal{L}_{D s D^{*} K}=i g_{D s D^{*} K} D^{* \mu}\left(\bar{D}_{s} \partial_{\mu} K-\left(\partial_{\mu} \bar{D}_{s}\right) K\right)+H . c .  \tag{1}\\
& \mathcal{L}_{D s^{*} D K}=i g_{D s^{*} D K} D_{s}^{* \mu}\left(\bar{D} \partial_{\mu} \bar{K}-\left(\partial_{\mu} \bar{D}\right) \bar{K}\right)+H . c . . \tag{2}
\end{align*}
$$

In this formalism it is necessary to know the form factors and coupling constants in the hadronic vertices to obtain the cross section. In ref. [2] it was shown that the use of appropriated form factors can lead to a change in the value of the cross section by a factor two. Also, the values of the coupling constants used when $D$ mesons are involved are evaluated using $\operatorname{SU}(4)$ exact symmetry, which means that the coupling constants are evaluated using the same values for the masses of the quarks $u, d, s$ and $c$. In this case, the values of the coupling constants


FIG. 1: Annihilation of $J / \Psi$ by kaons given $D s, D^{*}, D_{s}^{*}$ and $D$ mesons production.
for the two vertex of the right side in the processes, in Fig. 1, are identical:

$$
\begin{equation*}
g_{D s^{*} D K}=g_{D s D^{*} K}=\frac{g}{2 \sqrt{2}} \tag{3}
\end{equation*}
$$

In this work we study the $D^{*} D_{s} K$ and $D_{s}^{*} D K$ vertices using the QCD Sum Rules technique [5], to evaluate the form factors and to estimate the coupling constants.

We have been working on the problem of computing coupling constants for others processes and have a consistent method for this [6-14]. Following the QCDSR formalism described in our previous works [6-14], we write the three-point correlation function associated with the $D^{*} D_{s} K$ vertex, which is given by

$$
\begin{align*}
\Gamma_{\mu}^{(K)}\left(p, p^{\prime}\right)= & \int d^{4} x d^{4} y e^{i p^{\prime} \cdot x} e^{-i\left(p^{\prime}-p\right) \cdot y} \\
& \langle 0| T\left\{j_{\mu}^{D^{*}}(x) j^{K^{\dagger}}(y) j^{D_{s}^{\dagger}}(0)\right\}|0\rangle \tag{4}
\end{align*}
$$

for $K$ meson off-shell, where the interpolating currents are $j_{\mu}^{D^{*}}=\bar{c} \gamma_{\mu} d, j^{K}=i \bar{s} \gamma_{5} d$ and $j^{D_{s}}=i \bar{c} \gamma_{5} s$, and

$$
\begin{align*}
\Gamma_{\mu v}^{\left(D_{s}\right)}\left(p, p^{\prime}\right)= & \int d^{4} x d^{4} y e^{i p^{\prime} \cdot x} e^{-i\left(p^{\prime}-p\right) \cdot y} \\
& \langle 0| T\left\{j_{\mu}^{K}(x) j^{D_{s}^{\dagger}}(y) j_{v}^{D^{\dagger \dagger}}(0)\right\}|0\rangle \tag{5}
\end{align*}
$$

for $D_{s}$ meson off-shell, with the interpolating currents $j_{\mu}^{K}=$ $\bar{u} \gamma_{\mu} \gamma_{5} s, j^{D_{s}}=i \bar{c} \gamma_{5} s, j_{\mu}^{D^{*}}=\bar{u} \gamma_{\mu} c$, with $u, d, s$ and $c$ being the up, down, strange and charm quark fields respectively. In both cases, each one of these currents have the same quantum numbers as the corresponding mesons.

Using the above currents to evaluate the correlation functions (4) and (5), the theoretical or QCD side is obtained. The framework to calculate the correlators in the QCD side is the Wilson operator product expansion (OPE). The Cutkosky's rule allows us to obtain the double discontinuity of the correlation function for each one of the Dirac structures appearing in the correlation function. Then we use spectral representation over the virtualities $p^{2}$ and $p^{\prime 2}$, holding $Q^{2}=-q^{2}$ fixed. The amplitudes receive contributions from all terms in the OPE. The leading contribution comes from the perturbative term.

| $m_{q}$ | $m_{s}$ | $m_{c}$ | $m_{K}$ | $m_{D_{s}}$ | $m_{D_{s}^{*}}$ | $m_{D}$ | $m_{D^{*}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 0.13 | 1.2 | 0.498 | 1.97 | 2.11 | 1.87 | 2.01 |

TABLE I: Masses of quarks and mesons used in the calculation of the QCD sum rule. All quantities are in GeV .

| $f_{K}[16]$ | $f_{D_{s}}[17]$ | $f_{D^{*}}[18]$ | $f_{D}[20]$ | $f_{D_{s}^{*}}[19]$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.160 | 0.280 | 0.240 | 0.200 | 0.330 |

TABLE II: Decay constant used in the calculation of the QCD sum rule. All quantities are in GeV .

The phenomenological side of the sum rule, which is written in terms of the mesonic degrees of freedom, is parametrized in terms of the form factors, meson decay constants and meson masses. We introduce the meson decay constants $f_{K}$, $f_{D_{s}}$ and $f_{D^{*}}$, which are defined by the following matrix elements

$$
\begin{gather*}
\langle 0| j^{K}|K\rangle=\frac{m_{K}^{2} f_{K}}{m_{s}+m_{q}},  \tag{6}\\
\langle 0| j^{D_{s}}\left|D_{s}\right\rangle=\frac{m_{D_{s}}^{2}}{m_{c}+m_{s}} f_{D_{s}} \tag{7}
\end{gather*}
$$

and

$$
\begin{equation*}
\langle 0| j_{v}^{D^{*}}\left|D^{*}\right\rangle=m_{D^{*}} f_{D^{*}} \varepsilon_{v}^{*} \tag{8}
\end{equation*}
$$

where $\varepsilon_{v}$ is the polarization vector of the $D^{*}$ meson. The QCD sum rule is obtained by matching both representations, using the duality principle. The matching is improved by performing a double Borel transform on both sides. The perturbative contribution for both Eqs. (4) and (5) is given in details in ref.[14]. We chose one structure that appear in both sides and that has a good stability, which guarantees a good match between the two sides of the sum rule. The structures that obey these two points are $p_{\mu}^{\prime}$, in the case $K$ off-shell, and $p_{\mu}^{\prime} p_{v}^{\prime}$ in the case $D_{s}$ off-shell.

The Borel transformation [15] in the variables $P^{2}=-p^{2} \rightarrow$ $M^{2}$ and $P^{\prime 2}=-p^{\prime 2} \rightarrow M^{\prime 2}$ allows to get the final form of the sum rule, which allow us to obtain the form factors $g_{D^{*} D_{s} K}^{(M)}\left(Q^{2}\right)$ where $M$ stands for the off-shell meson.

We use Borel masses satisfying the constraint $M^{2} / M^{\prime 2}=$ $m_{\text {in }}^{2} / m_{\text {out }}^{2}$, where $m_{\text {in }}$ and $m_{\text {out }}$ are the masses of the incoming and outgoing meson respectively. The values of the parameters used in the calculation of the vertices are depicted in Table I and in Table II

The continuum thresholds $s_{0}$ and $u_{0}$ are important parameters to control the pole contribution and can be expressed in terms of the increments $\Delta_{s}$ and $\Delta_{u}$ (see ref. [14]). Using $\Delta_{s}=\Delta_{u}=0.5 \mathrm{GeV}$ for the continuum thresholds and fixing $Q^{2}=1 \mathrm{GeV}^{2}$, we found a good stability of the form factor $g_{D^{*} D_{s} K}^{(K)}$, as a function of the Borel mass $M^{2}$, in the interval $3<M^{2}<5 \mathrm{GeV}^{2}$. In the case of the form factor $g_{D^{*} D_{s} K}^{\left(D_{s}\right)}$ the interval for stability of the sum rule is $2<M^{2}<5 \mathrm{GeV}^{2}$.


FIG. 2: $g_{D^{*} D_{s} K}^{(K)}$ (squares) and $g_{D^{*} D_{s} K}^{\left(D_{s}\right)}$ (triangles) form factors as a function of $Q^{2}$ from the QCDSR calculation of this work. The solid (dotted) line corresponds to the monopole (exponential) parametrization of the QCDSR results for each case.

Fixing $\Delta_{s}=\Delta_{u}=0.5 \mathrm{GeV}$ and $M^{2}=3 \mathrm{GeV}^{2}$, we evaluate the momentum dependence of both form factors. The results are shown in Fig. 2, where the squares corresponds to the $g_{D^{*} D_{s} K}^{(K)}\left(Q^{2}\right)$ form factor in the interval where the sum rule is valid. The triangles are the result of the sum rule for the $g_{D^{*} D_{s} K}^{\left(D_{s}\right)}\left(Q^{2}\right)$ form factor.
In the case that the $K$ meson is off-shell, our numerical results can be parametrized by an exponential function (dotted line in Fig. 2):

$$
\begin{equation*}
g_{D^{*} D_{s} K}^{(K)}\left(Q^{2}\right)=2.83 e^{-\frac{Q^{2}}{4.19}} \rightarrow g_{D^{*} D_{s} K}^{(K)}=3.01 \tag{9}
\end{equation*}
$$

where the coupling constant, $g_{D^{*} D_{S} K}^{(K)}$ is given by the value of the form factor at $Q^{2}=-m_{K}^{2}$.

When the $D_{s}$ meson is off-shell, our sum rule results can be parametrized by a monopole form (solid line in Fig. 2):

$$
\begin{equation*}
g_{D^{*} D_{s} K}^{\left(D_{s}\right)}\left(Q^{2}\right)=\frac{9.01}{Q^{2}+6.86} \rightarrow g_{D^{*} D_{s} K}^{\left(D_{s}\right)}=3.02 \tag{10}
\end{equation*}
$$

where $g_{D^{*} D_{s} K}^{\left(D_{s}\right)}$ is the coupling constant given by the value of the form factor at $Q^{2}=-m_{D_{s}}^{2}$.

Comparing the results in Eqs.(9) and (10) we see that the method used to extrapolate the QCDSR results in both cases, $K$ and $D_{s}$ off-shell, allows us to extract values for the coupling constant which are in very good agreement with each other.

In order to study the dependence of this results with the continuum threshold, we vary $\Delta_{s}=\Delta_{u}$ in the interval $0.4 \leq$ $\Delta_{s}=\Delta_{u} \leq 0.6 \mathrm{GeV}$. This procedure give us uncertainties in such a way that the final results for the couplings in each case are:

$$
g_{D^{*} D_{s} K}^{(K)}=3.02 \pm 0.15
$$

and

$$
g_{D^{*} D_{s} K}^{\left(D_{s}\right)}=3.03 \pm 0.14
$$

Now we study the $D_{s}^{*} D K$ vertex. The treatment is similar to the previous case. For details of the calculation see reference [14]. The correlation functions are

$$
\begin{align*}
\Gamma_{\mu}^{(K)}\left(p, p^{\prime}\right)= & \int d^{4} x d^{4} y e^{i p^{\prime} \cdot x} e^{-i\left(p^{\prime}-p\right) \cdot y} \\
& \langle 0| T\left\{j_{\mu}^{D_{s}^{*}}(x) j^{K^{\dagger}}(y) j^{D^{\dagger}}(0)\right\}|0\rangle \tag{11}
\end{align*}
$$

for $K$ meson off-shell, where the interpolating currents are $j_{\mu}^{D_{s}^{*}}=\bar{c} \gamma_{\mu} s, j^{K}=i \bar{u} \gamma_{5} s$ and $j^{D}=i \bar{c} \gamma_{5} u$, and

$$
\begin{align*}
\Gamma_{\mu \nu}^{(D)}\left(p, p^{\prime}\right)= & \int d^{4} x d^{4} y e^{i p^{\prime} \cdot x} e^{-i\left(p^{\prime}-p\right) \cdot y} \\
& \langle 0| T\left\{j_{\mu}^{K}(x) j^{D^{\dagger}}(y) j_{v}^{D_{s}^{* \dagger}}(0)\right\}|0\rangle \tag{12}
\end{align*}
$$

for $D$ meson off-shell, with the interpolating currents $j_{\mu}^{K}=$ $\bar{u} \gamma_{\mu} \gamma_{5} s, j_{v}^{D_{s}^{*}}=\bar{c} \gamma_{v} s$, and $j^{D}=i \bar{u} \gamma_{5} c$. We introduce the decay constants $f_{D}$ and $f_{D_{s}^{*}}$, which are defined by the following matrix elements:

$$
\begin{align*}
\langle 0| j^{D}|D\rangle & =\frac{m_{D}^{2}}{m_{c}+m_{q}} f_{D}  \tag{13}\\
\langle 0| j_{v}^{D_{s}^{*}}\left|D_{s}^{*}\right\rangle & =m_{D_{s}^{*}} f_{D_{s}^{*}} \varepsilon_{v}^{*} \tag{14}
\end{align*}
$$

where $\varepsilon_{v}$ is the polarization vector of the $D_{s}^{*}$ meson.
In Fig. 3 the squares correspond to the $g_{D_{s}^{*} D K}^{(K)}\left(Q^{2}\right)$ form factor in the interval where the sum rule is valid. The triangles are the result of the sum rule for the $g_{D_{s}^{*} D K}^{(D)}\left(Q^{2}\right)$ form factor.

In the case when the $K$ meson is off-shell, our numerical results can be parametrized by an exponential function (dashed curve in Fig. 3) and the coupling constant is extracted as the value of the form factor at $Q^{2}=-m_{K}^{2}$ :

$$
\begin{equation*}
g_{D_{s}^{*} D K}^{(K)}\left(Q^{2}\right)=2.69 e^{-\frac{Q^{2}}{4.39}} \rightarrow g_{D_{s}^{*} D K}^{(K)}=2.87 \tag{15}
\end{equation*}
$$

When the $D$ meson is off-shell, the sum rule results are represented by the triangles in Fig. 3, and they can be parametrized by a monopole form (solid line in the figure). The coupling constant is the value of the form factor at $Q^{2}=-m_{D}^{2}$ :

$$
\begin{equation*}
g_{D_{s}^{*} D K}^{(D)}\left(Q^{2}\right)=\frac{7.78}{Q^{2}+6.34} \rightarrow g_{D_{s}^{*} D K}^{(D)}=2.72 \tag{16}
\end{equation*}
$$

Studying the dependence of our results with the continuum threshold, for $\Delta_{s, u}$ varying in the interval $0.4 \leq \Delta_{s, u} \leq$ 0.6 GeV , we obtain the following values, with errors, for the couplings in each case:

$$
g_{D_{s}^{*} D K}^{(K)}=2.87 \pm 0.19
$$

and

$$
g_{D_{s}^{*} D K}^{(D)}=2.72 \pm 0.31
$$

Concluding, we have studied the form factors and coupling constants of $D^{*} D_{s} K$ and $D_{s}^{*} D K$ vertices in a QCD sum rule


FIG. 3: $g_{D_{s}^{*} D K}^{(K)}$ (squares) and $g_{D_{s}^{*} D K}^{(D)}$ (triangles) form factors as a function of $Q^{2}$ from the QCDSR calculation of this work. The dashed (solid) line corresponds to the exponential (monopole) parametrization of the QCDSR results for each case.
calculation. For each case we have considered two particles off-shell, the lightest and one of the heavy ones: the $K$ and $D_{s}$ mesons for the $D^{*} D_{s} K$ vertex, and the $K$ and $D$ mesons for the $D_{s}^{*} D K$ vertex. In the two situations, the off-shell particles give compatible results for the coupling constant in each vertex. The results are :

$$
\begin{equation*}
g_{D^{*} D_{s} K}=3.02 \pm 0.14 \tag{17}
\end{equation*}
$$

and

$$
\begin{equation*}
g_{D_{s}^{*} D K}=2.84 \pm 0.31 \tag{18}
\end{equation*}
$$

We can compare our result with the prediction of the exact $\mathrm{SU}(4)$ symmetry [4], which would give the following relation among these numbers [4]: $g_{D^{*} D_{s} K}=g_{D_{s}^{*} D K}=5$. Eqs. (17) and (18) shows that the coupling constants in the vertices $D^{*} D_{s} K$ and $D_{s}^{*} D K$ are consistent with each other, but that they are smaller than the value given by the $\mathrm{SU}(4)$ symmetry in the ref. [4]. Therefore, we conclude that the $\mathrm{SU}(4)$ symmetry is broken by approximately $40 \%$ in the calculation performed here. This is expected because the coupling constant obtained by the exact $\mathrm{SU}(4)$ symmetry uses the same mass for the quarks $u, d, s$ and $c$. In this case there is not experimental value to compare our results. However, we believe that our results are very robust since they were obtained using two different extrapolations in each vertex and we have obtained compatible results from both extrapolations.

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