

Magnetic Resonance Force Microscopy: Recent Results

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Force detection of magnetic resonance has been demonstrated experimentally and used for imaging in EPR. This paper will review the basic principles of magnetic resonance force microscopy (MRFM) and will report some recent results in NMR imaging and the operation of a low-temperature MRFM.

I. Introduction

Since the discovery of electron paramagnetic resonance^[1,2] and of nuclear magnetic resonance^[3,4], EPR and NMR have become two of the most important spectroscopic tools in many areas, especially physics, chemistry, biophysics, biochemistry and materials science. Very detailed information at the molecular level can be obtained from ESR and NMR spectra, while magnetic resonance imaging (MRI)^[5,6] is now routinely used by radiologists for clinical diagnosis. The reason that ESR and, in particular NMR, are so informative lies in their extremely high spectral resolution. A disadvantage of magnetic resonance, and especially NMR, is the low sensitivity of these methods due to the relatively low transition energies (4×10^{-5} eV for NMR and 10^{-2} eV for ESR) even at the highest polarizing fields currently available. As a result, samples containing at least 10^{15} nuclear spins or 10^9 electron spins are required for most NMR and ESR experiments, respectively. This limitation leads to a requirement for samples that are much larger than those used for studies by optical spectroscopy and restricts the investigation of surface and interface phenomena to materials with extremely high internal surface area. Therefore, the suggestion by Çidles⁷ that it may be possible to detect a single nuclear spin was spectacular. Sidles proposed coupling the spin via magnetic force to a mechanical os-

cillator holding a field gradient source, a method that is very different from the inductive techniques used in conventional magnetic resonance. Reviews of the first theoretical and experimental work in magnetic resonance force microscopy (MRFM) have been written summarizing the progress made to date in demonstrating remarkable sensitivity for ESR and NMR^[8,9].

A sketch of an idealized atomic-scale MRFM is shown in Fig. 1. The sketch shows that the MRFM is a magnetic force microscope^[10] modified to do magnetic resonance experiments. The magnetic spins in the sample experience a force in the large gradient generated by a sharp magnetic tip attached to the cantilever. The force is given by

$$\mathbf{F} = (\mathbf{m} \cdot \nabla)\mathbf{B} , \quad (1)$$

where \mathbf{m} is the magnetic moment due to electron or nuclear spins and \mathbf{B} is an inhomogeneous magnetic field. At resonance, the alternating magnetic field generated by the proximate coil causes a change in the force, which is detected with an optical fiber interferometer^[11]. The gradient from the magnetic tip not only determines the strength of the force signal but, as in conventional magnetic resonance imaging, determines the spatial resolution of the MRFM. Nano-fabrication and chemical etching methods have demonstrated that it is possible to produce tips with a radius of curvature of 300\AA ,

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[12] which would generate magnetic field gradients as large as 100 gauss/Å (10^8 T/m). Therefore, even if all the spins shown in Fig. 1 were identical, it should be possible, using such high gradients, to excite one spin at a time, permitting direct atom-by-atom imaging of molecules. MRFM thus has the potential for atomic-scale microscopy with elemental specificity. Although such a capability may be difficult to achieve, the experiments reported here show that micron-scale ESR^[13,14] and NMR^[15] images have already been obtained using MRFM. Good agreement between the estimated and observed spin sensitivity in the MRFM results reported to date suggest that the detection of a single electron spin is a realistic possibility⁸. In this paper, basic principles of MRFM are reviewed, and some recent results in NMR detection and imaging are reported.

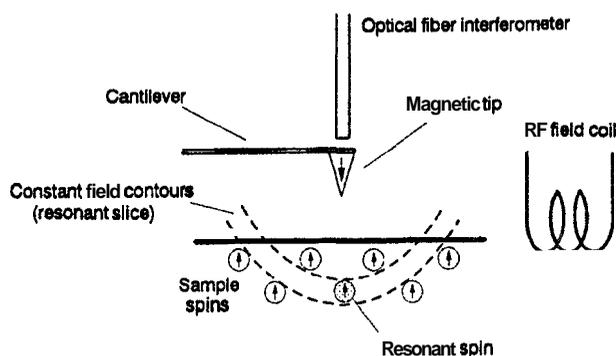


Figure 1. Sketch of a proposed magnetic resonance force microscope with atomic resolution. The gradient of the field produced by the nanoscale-size magnetic tip is strong enough to isolate a single electron or nuclear spin. The rf coil is used to excite the magnetic resonance.

II. Basic principles of the MRFM

The basic detection principle of MRFM is to couple the motion of spins to the motion of a mechanical cantilever and to monitor the position of the cantilever. A sketch of the probe that was used to do the first force-detected NMR experiment^[16] is shown in Fig. 2^[17]. It should be noted that the sample, rather than the magnetic tip, is mounted on the cantilever. This configuration was necessary due to the large size of the magnetic tip used as a gradient source in these experiments^[18]. If the force on the spins in the sample can be modulated at the cantilever frequency, f_c , then an enhancement of the amplitude of deflection of the cantilever can be realized:

$$A = (Q/k)F \quad (2)$$

where A is the deflection amplitude of the cantilever, Q is the quality factor of the cantilever oscillator and k the spring constant of the cantilever. F is amplitude of the oscillatory force on the spins in the gradient generated by the magnetic tip. From an analysis of the noise (see section V), a major goal in the design and construction of cantilevers is achieving a high Q and low force constant. This is most effectively realized by fabricating small, and, in particular thin cantilevers to be used in a modest vacuum (10^{-4} Torr)^[16,19]. For very small, light cantilevers with low force constant, typical cantilever frequencies are ≤ 100 kHz^[19]. These frequencies are orders of magnitude lower than the Larmor frequencies used nowadays in magnetic resonance (as high as 750 MHz for protons and 250 GHz for electrons).

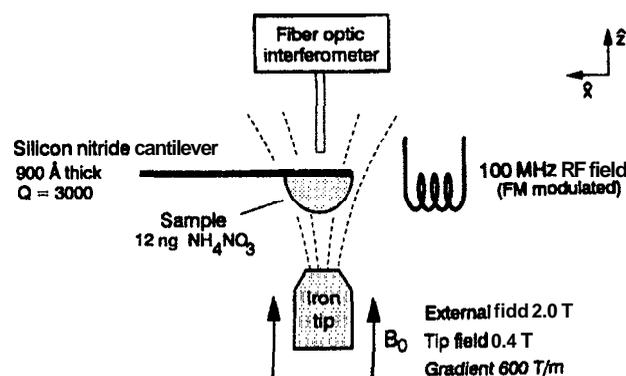


Figure 2. Sketch of the force detection apparatus used to detect NMR. In this configuration, the sample is mounted on the cantilever. The tip which generates the field gradient can be translated to sweep the field through the resonance condition. The position of the cantilever is monitored with a fiber optic interferometer.

The spin dynamics relevant to MRFM and conventional magnetic resonance experiments are thus very different. The spin system in a sample in a magnetic field oriented along the z axis is characterized by a magnetic moment $m = M_z V$, where M_z is the Curie spin magnetization and V is the sample volume. In the MRFM experiments performed to date, M_z is forced to vary slowly at frequencies $\sim f_c$, in contrast to conventional magnetic resonance, where the transverse magnetization $M_{x(y)}$ rotating at the Larmor frequency is detected. Since M_z is the projection of the spin magnetization, \mathbf{M}_0 , along the z -axis, a change in either the

magnitude or direction of \mathbf{M}_0 will result in a corresponding change in M_z and in the force on the spins of the sample mounted on the cantilever. The vibration of the cantilever can be excited by modulating the magnitude or direction of \mathbf{M}_0 at f_c , resulting in a time-varying force. The change in amplitude of the cantilever vibration is then registered by the fiber optic interferometer.

III. Summary of first results

1. Force Detection of ESR^[20]

Cyclic saturation^[21] was used to change the magnitude of \mathbf{M}_0 for force detection of ESR. This was accomplished by modulating the dc field through resonance, resulting in cyclic saturation of \mathbf{M}_0 , and a corresponding reduction in M_z . A calculation of this effect is shown in Fig. 3(top); since M_z is reduced twice each modulation cycle, a second-harmonic response [Fig. 3(bottom)] can be detected. Detection at the second harmonic circumvented interference due to spurious feed-through at the fundamental frequency. The first experimental result in MRFM^[20], the ESR spectrum of 30 nanograms of DPPH, is shown in Fig. 4(a); for comparison, a theoretical calculation of the expected force-detected signal is shown in Fig. 4(b). The only free parameter in this calculation is the strength of the rf field used to saturate the electron spins. A requirement for this experiment to work is that the electron spin-lattice relaxation time (T_{1e}) be short compared with the cantilever period. Since f_c was 4 kHz, and T_{1e} in DPPH is 60 nanoseconds, this condition was easily fulfilled. Sidles and co-workers have proposed and demonstrated the utility of anharmonic modulation for ESR force detection of DPPH, and have shown that this method suppresses noise and feed-through^[22].

2. Force-detected ESR Imaging^[13,14]

The detection technique described above was used in imaging two micron-sized particles of DPPH by mounting the gradient source on a movable x-y-z piezoelectric stage and gathering two-dimensional (x-y) data. This was done for the axial direction (z) defined by the axis of the gradient source, a conical piece of NdFeB^[13]. The axial and lateral gradients were 4.3 G/ μm and 0.94 G/ μm , respectively. As the tip is scanned, the mag-

nitude of the spin magnetization in the DPPH particles will become cyclically modulated when the field at the particle has the resonance value B_0 and the corresponding change in force will be detected using the method described above. The spatial variation of the field generated by a conical tip is shown in Fig. 5(a), and the surface of constant field strength is approximately a paraboloid. For the point sample shown in Fig. 5(a), a scan of the tip in the x-y plane at $z = z_2$ will generate a circular force "map" indicating where the magnetic resonance condition is met on the surface $B_z(x, y, z) = B_0 = \text{const}$. Typical force maps demonstrating this behavior for different distances from the tip are shown in Fig. 5(b). A photomicrograph of the two DPPH particles glued to the silicon nitride cantilever^[23] is shown in Fig. 6(a), and a force map generated as described above is shown in Fig. 6(b). An ESR image of the two DPPH particles reconstructed from the force map of Fig. 6(b) using conventional image reconstruction techniques is shown in Fig. 6(c)^[14]. Comparison of Figs. 6(a) and (c) shows that the ESR image reproduces the shape and dimensions of the DPPH particles very well. The spatial resolution in the axial and lateral directions was 1.2 μm and 5.3 μm , respectively.

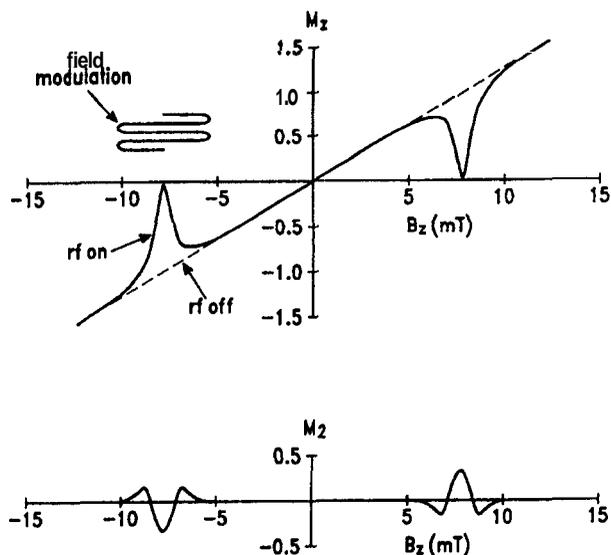


Figure 3. Top: calculation of the behavior of the longitudinal spin magnetization, M_z , as a function of external magnetic field with (solid curve) and without (dashed curve) rf irradiation. M_z is suppressed at magnetic resonance; bottom: second harmonic response of the change in M_z produced by modulating the dc field.

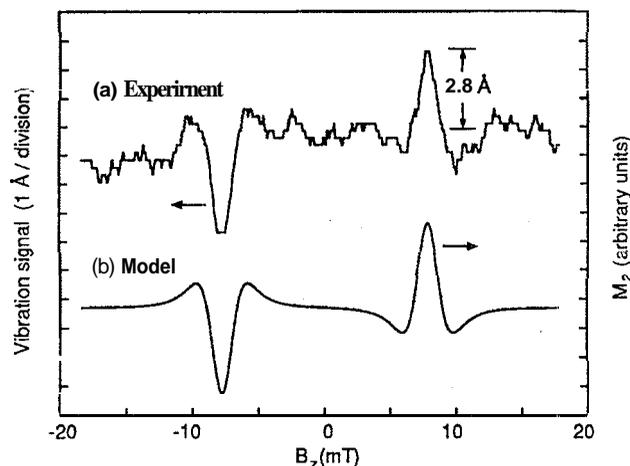


Figure 4. (a) amplitude of cantilever vibration measured at the second harmonic of the modulation frequency plotted as a function of the polarizing field for a peak modulation amplitude of 1 mT and rf excitation at 220 MHz. Vibration peaks occur at field values corresponding to electron spin resonance in diphenylpicrylhydrazyl (DPPH). (b) calculation of the second harmonic response using known experimental parameters with the exception of the strength of the rf field, $B_1 = 3.4$ mT, which was chosen to give the best fit with the data in (a).

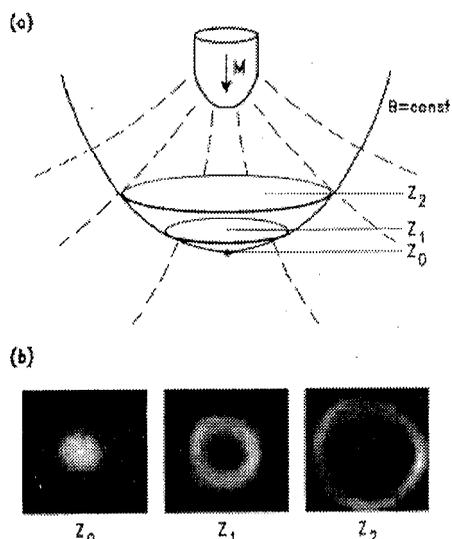
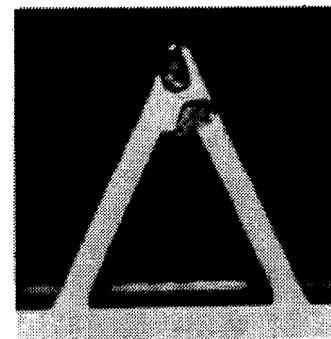


Figure 5. (a) dashed lines show the magnetic field lines from the tip used for EPR imaging. The surface of constant field strength is a paraboloid and the circles indicate where the magnetic resonance condition is met on the surface $B = B_0 = \text{const.}$ as the tip is scanned in the x - y plane at a fixed sample distance, z , from the tip. (b) force maps measured for a small DPPH particle scanned in the x - y plane at three different values of z : $z_0 \simeq 200 \mu\text{m}$, $z_1 = z_0 - 3 \mu\text{m}$, $z_2 = z_0 - 8 \mu\text{m}$. The gray scale represents cantilever vibration amplitude, with white representing maximum amplitude. The image size is $200 \times 200 \mu\text{m}^2$.

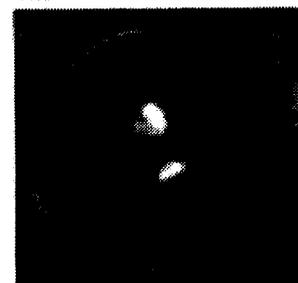
(a) Sample



(b) Force map



(c) Reconstructed image



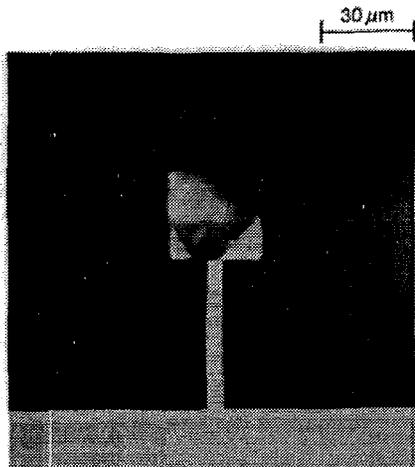
50 μm

Figure 6. (a) optical micrograph showing two DPPH particles attached to a silicon nitride cantilever; (b) magnetic resonance force map (*vide infra*) of the sample, consisting of a 128×128 pixel array acquired during a 90-minute scan; (c) reconstructed electron spin density image obtained by deconvolving the data in (b); the large rings slightly visible in the image are artifacts of the deconvolution procedure.

3. Force Detection of NMR^[16]

Force detection of NMR presented a major challenge since (1) the magnetic moment of nuclei is $\sim 10^3$ times smaller than the electron spin moment and (2) the shortest spin lattice relaxation times for nuclei are a few hundred μs so that the use of the cyclic saturation technique described above for ESR is precluded. In order to overcome these difficulties, very sensitive cantilevers were fabricated^[19], a high external field was used to increase the nuclear spin polarization and cyclic adiabatic reversal^[24] was used to change the direction of the nuclear magnetic moments at f_e ^[16,25]. A photo-

900Å Thick Silicon Nitride Cantilever



- $k = 10^{-3}$ N/m
- $Q = 3000$ in vacuum
- 5×10^{-16} N/ $\sqrt{\text{Hz}}$ force sensitivity at 300 K.

Figure 7. Optical micrograph of the 900 Å-thick silicon nitride cantilever, with a 12-nanogram grain of ammonium nitrate mounted on the paddle-shaped end.

micrograph of a 900Å -thick cantilever holding a 12-nanogram sample of ammonium nitrate is shown in Fig. 7^[16]. The loaded cantilever, fabricated from silicon nitride^[19], resonates at 1.5 kHz, with $Q = 3000$ and $k = 10^{-3}$ N/m. The achieved force sensitivity was 5×10^{-16} N in a 1 Hz bandwidth at 300K (see eq. 3 in section V). The rf frequency modulation sequence used to generate cyclic adiabatic reversal of the protons is shown in Fig. 8. After allowing sufficient time for the protons to polarize in the external field (~ 4 s), the rf field is turned on far off resonance (to avoid saturation of the protons), and the frequency is then swept adiabatically through resonance and modulated back and forth across the resonance value at a frequency $f_c = 1.5$ kHz. This forces the proton magnetization to change its direction with respect to the external field direction (z) at f_c , and produces enhancement of the cantilever deflection given by eq. 2. The change in proton magnetization generates an alternating force which drives the vibration of the cantilever, monitored by the interferometer (Fig. 2). Force detection of the proton NMR in ammonium nitrate is shown in Fig. 9, in which the cantilever vibration amplitude is plotted against time while the rf srquence shown in Fig. 8(a) is applied.

The single-shot sensitivity at ambient temperature in a field of 2.35T (100 MHz proton NMR) is 1.6×10^{13} spins. This sensitivity is about three orders of magnitude higher than conventional solid state NMR methods.

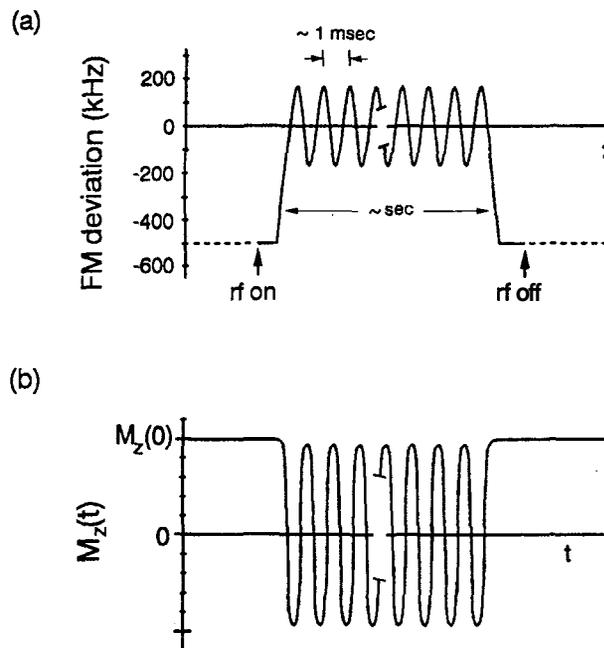


Figure 8. (a) frequency modulation sequence used for cyclic adiabatic inversion showing the rf frequency deviation $\Omega(t)/2\pi$; (b) time dependence of the longitudinal magnetization (M_z) resulting from the sequence in (a).

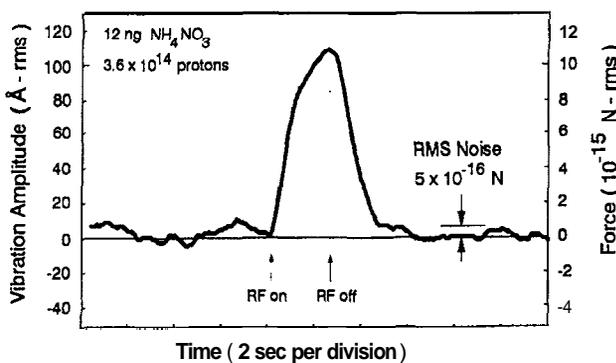


Figure 9. Force detected NMR signal of the protons in a 12 nanogram sample of ammonium nitrate. The sequence shown in Fig. 8(a) was turned on and off as indicated and the cantilever vibration amplitude measured at the fundamental cantilever frequency during this single-shot experiment (no averaging). The slow rise and fall times of the response are primarily a result of the 1 s time constant of the lock-in amplifier. Measurement conditions: rf field strength (B_1) = 12 G, $\Omega/2\pi = 170$ kHz and $Q_{\text{eff}} = 1000^{15}$.

The force detection method has also been used to measure the proton T_1 and to observe transient

nutations of the proton magnetization^[26]. A one-dimensional image of the ammonium nitrate sample was obtained and a resolution of $\sim 2.6\mu\text{m}$ estimated^[16]. The following section describes three-dimensional imaging of the protons in an ammonium nitrate crystal.

IV. Three-dimensional proton imaging^[15]

Three dimensional proton imaging was carried out using the NMR force detection method described above^[15]. The gradient source, a ferromagnetic tip (0.25 nm diameter iron wire, 4 nm long), was mounted on piezoelectric translators for motion in the x and y directions as shown in Fig. 10. The field at the sample is the sum of the homogeneous field from the superconducting magnet and the tip field. The field from the tip decreases as the distance from the tip increases. Therefore, by varying the rf frequency, ω , protons at different depths of the sample along a direction (z) defined by the axis of the cylinder, can be excited. In order to acquire the data for a 3-dimensional image, the tip is kept at a constant distance from the sample and scanned in the x - y plane and the rf frequency is scanned over a range $\Delta\omega$. Thus, an $x - y - \omega$ lattice of $31 \times 31 \times 50$ points was generated. The x and y coordinates range over 100 nm, while ω was changed from 96.3 MHz to 101.2 MHz. A range of $\Delta\omega$ corresponds to a distance $\Delta z = \Delta\omega(\gamma G_z)^{-1}$, where G_z is the gradient in the z direction. Since the z -gradient was 22 gauss/ μm with the tip positioned 150 μm from the sample, the ω scan was equivalent to a z -scan of 52 nm. At each lattice point, the response of the cantilever was measured while the excitation sequence shown in Fig. 8(a) was applied. Fig. 11 shows $x - \omega$ force maps for four of the 31 different y positions. The paraboloidal shape of the tip field profile is clearly reflected by the force maps. The force maps reflect the amplitude of the first harmonic response of the cantilever to the time-varying force imposed by modulation of the magnetization of the protons that are excited. The image of the sample consists of spatial variation of the proton density which can be deconvolved from the force maps with a knowledge of the spatial variation of the external magnetic field and the field gradient from the iron tip. The deconvolution was performed using a Wiener filter^[14].

1. Results and Spatial Resolution

A surface of constant proton density in an ammonium nitrate crystal obtained using this method is shown in Fig. 12. The flat part of the side view, parallel to the plane of the cantilever, represents the surface of the crystal facing the cantilever. The overall shape and dimensions of the surface shown in Fig. 12 closely reproduce an optical image of the sample. The spatial resolution of this method can be determined experimentally from relevant features of the proton spin density reconstructed from force maps. The resolution achieved in the axial direction was $3.6\mu\text{m}$ and $17.5\mu\text{m}$ in the lateral direction, yielding a volume resolution of $(10\mu\text{m})^3$. Volume resolution of $(10\mu\text{m})^3$ is three orders of magnitude greater than proton imaging in solids using conventional methods^[27], and at the limit of volume resolution calculated theoretically for solution proton imaging^[28]. Higher spatial resolution and sensitivity is expected of MRFM with the use of larger field gradients and more sensitive cantilevers.

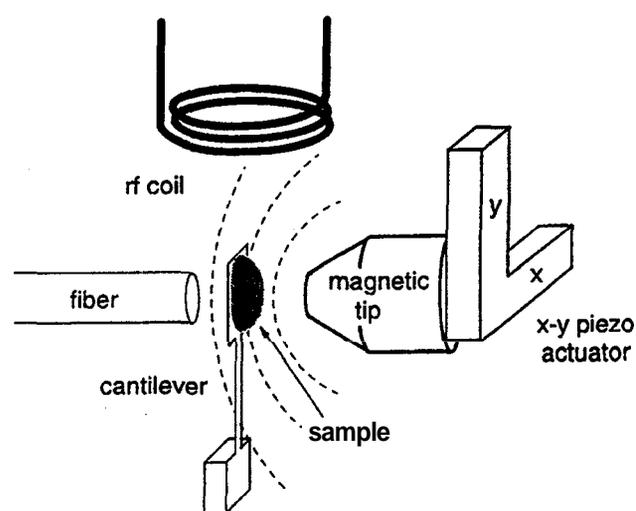


Figure 10. Schematic view of the NMR force microscope. The dashed lines represent contours of constant magnetic field.

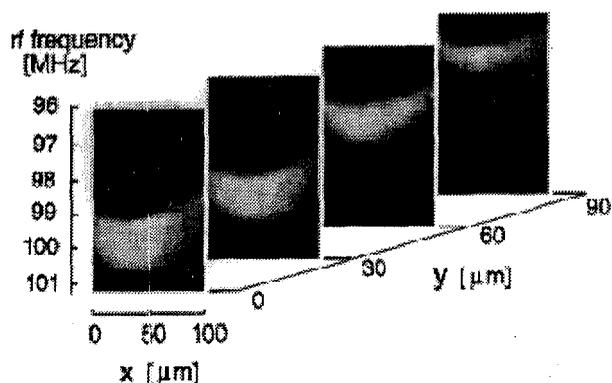


Figure 11. Three-dimensional force map of the sample obtained by scanning the rf frequency for each x-y position of the magnetic tip, moved by piezoelectric actuators over a 100 μm range. The conversion factor is 10.5 $\mu\text{m}/\text{MHz}$.

V. Low Temperature MRFM Detection of ^{19}F NMR²⁹

Force sensitivity in MRFM is limited only by thermal cantilever noise. The minimum detectable force is^[30,31]

$$F_{\min} = (2kk_B T \Delta\nu / Q\pi f_c)^{1/2}, \quad (3)$$

where k_B is the Boltzmann constant, T is the temperature and $\Delta\nu$ is the detection bandwidth (Hz). It is clear from eq. 3 that low-temperature operation should increase the sensitivity of the MRFM. An additional advantage is that the magnetic moment due to spin polarization, which is inversely proportional to temperature, will also increase, resulting in a larger force signal (eq. 1).

1. Experimental details

The MRFM was suspended in a vacuum can to provide vibration isolation and cooling was achieved by heat-sinking the cantilever mount to the bottom of the can via copper braid. The can was immersed in the liquid helium bath of a cold-bore superconducting magnet used to generate an external magnetic field. The field gradient source, a cylindrical iron tip 1.5 mm diameter \times 3 mm long, generated a gradient of 6 gauss/ μm and an additional field of 0.41T at a distance of 600 μm to 700 μm from the sample. The cantilever, commercially fabricated from single crystal silicon, was 470 μm long by 45 μm wide by 1.5 μm thick, with $L = 0.07 \text{ N m}^{-1}$, $f_c = 9.8 \text{ kHz}$ (without sample) and $Q = 4 \times 10^4$ at room

temperature in the vacuum of 10^{-5} Torr. The effect of temperature on the thermal noise is shown in Fig. 13. The vibration amplitude of the cantilever measured at the resonance frequency was 1.1 \AA -rms at room temperature (Fig. 13a) and decreased to 0.26 \AA -rms at 6K (Fig. 13b); the cantilever Q also increased to 2×10^5 at this temperature. The minimum detectable force at 6K was estimated to be $8 \times 10^{-17} \text{ N}/(\text{Hz})^{1/2}$, about twice the expected theoretical value. The discrepancy may be due to some environmental noise resulting from incomplete vibration isolation.

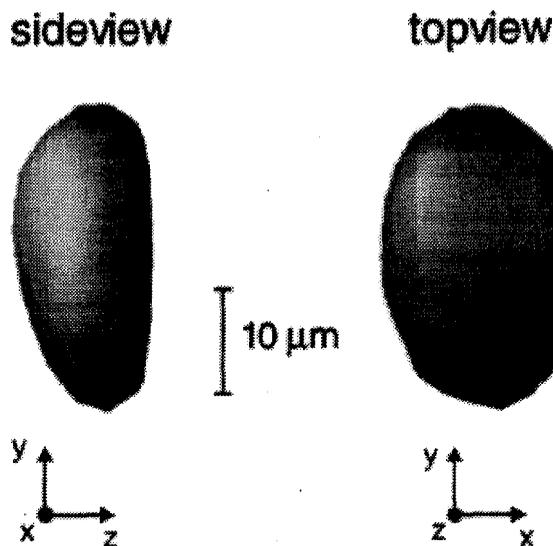


Figure 12. Reconstructed proton spin density of the ammonium nitrate sample, showing a constant density surface. The reconstruction was obtained from the force map in Fig. 11 by a deconvolution technique. The flat part in the side view represents the surface of the sample facing the cantilever.

2. Results

The first low-temperature force-detected magnetic resonance spectrum, the ^{19}F NMR spectrum in calcium fluoride (CaF_2) doped (1%) with Nd^{+3} ions, is shown in Fig. 14. The experiment was carried out at 14K in a dc field of 2.55T with an rf field strength of 19 gauss at 102.8 MHz, a (loaded) cantilever frequency of 1.6 kHz and a peak modulation amplitude of 280 kHz, equivalent to 70 gauss for ^{19}F . A special excitation sequence designed to circumvent the rapid decay of the ^{19}F magnetization during the cyclic adiabatic reversal sequence was used^[26]. The ^{19}F lineshape can be interpreted as a one-dimensional spatial map of the spin density projected onto the z axis, which is the symmetry axis of

the cylindrical iron tip. Since the field gradient produces different polarizing fields in different z positions of the sample, successive x-y slices of the sample come into resonance as the dc field is scanned, generating a magnetic force proportional to the number of ^{19}F spins within the slice. The observed ^{19}F linewidth of 500 gauss in the gradient of 6 gauss/ μm corresponds to a sample thickness (z direction) of $\sim 80\mu\text{m}$.

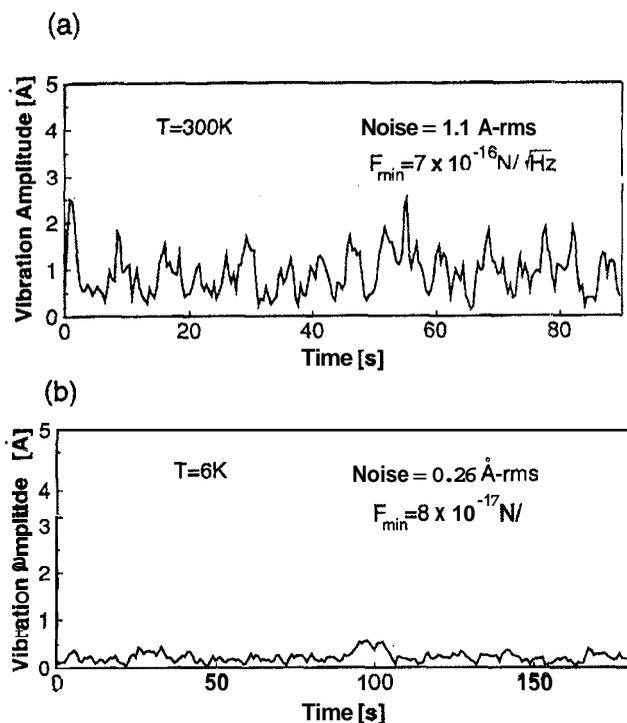


Figure 13. Vibration noise of the cantilever optically monitored at the resonant frequency (9.8 kHz) at (a) 300K and (b) 6K.

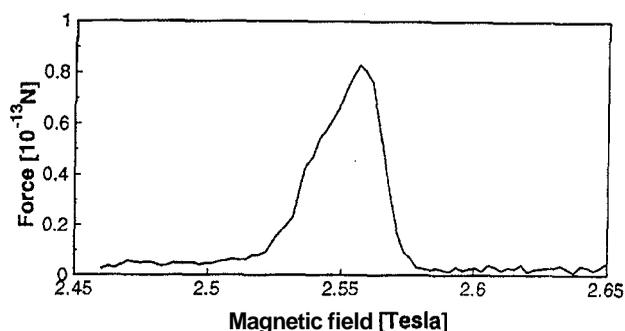


Figure 14. The NMR force signal of ^{19}F in 1% Nd-doped CaF_2 measured at 14 K as a function of polarizing field.

VI. Summary and Conclusions

The relatively brief history of MRFM has been summarized here with the following conclusions: MRFM

has been demonstrated experimentally for (1) the detection of both ESR and NMR with high sensitivity; (2) ESR imaging with resolution of $\sim 1\mu\text{m}$; (3) NMR imaging with volume resolution of $(10\mu\text{m})^3$; (4) MRFM at low temperatures, with nearly the expected increase in sensitivity as thermal cantilever noise is decreased. The ultimate goal of this work, as stated in the Introduction, is to develop a microscope that can detect the NMR of individual atoms with three-dimensional sub-Ångstrom spatial resolution. Apart from intrinsic value as a scientific tool, an atomic MRFM would have a broad range of applications in molecular biology, materials and polymer science, surface science and nanotechnology. In order to achieve this goal, considerable innovation will be required in a number of areas and in particular in developing improved sensors to detect extremely small forces and new methods for manipulating nuclear spins. Theoretical treatments of force signal and noise and of magnetic resonance, which have described closely the results obtained to date and provide a reliable basis for predicting the results of future experiments, encourage a sustained effort to develop MRFM with atomic resolution.

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18. A large (millimeter-size) magnetic tip was used in order to ensure that the resonant volume was large enough, i.e. contained a sufficient number of spins, to generate a measurable signal. In the NMR experiment on ammonium nitrate, for example, the force noise was such that the minimum detectable magnetic moment in a field of 2.5T at 300K corresponded to 1.6×10^{13} protons, a sample $1 \mu\text{m}$ thick by $20 \mu\text{m}$ square^[16]. This required an iron tip far too large to attach to the $50 \mu\text{m}$ -long cantilever. In general, as the tip size and the spacing between tip and sample are scaled to smaller dimensions, the gradient increases in inverse proportion to the scale factor. This has the beneficial effect of increasing the force per spin (or force per unit volume). However, the volume of resonant spins decreases faster as the dimensions are scaled down, resulting in a net reduction of force for most sample geometries. These comments obviously do not apply to the case of single-spin MRFM, for which the gradient may be increased without concern for the size of the resonant volume.
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