Cosmological Evolution in Ten Dimensions Driven by Viscous Dissipation, Monopole Condensation and a Non-Zero Cosmological Constant

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The evolution of the cosmological scale factors is studied in a ten dimensional universe with the normal three space adjoined to a six dimensional space with positive, negative or zero curvature and the matter content admitting a viscous fluid and a monopole configuration in a background of a cosmological constant.

I. Introduction

Early universe cosmology has become a fashionable and fertile field of investigation ever since the inception of inflation to explain the cosmological puzzles of flatness, horizon and absence of monopoles from tlie present universe[1]. Old inflation[1], new inflation[2], chaotic inflation[3], and extended inflation[4] all offer us a mechanism by which the initial state of the false vacuum can expand the universe at a rapid rate to resolve the horizon problem and then by either fine tuning or the intermediary of a Brans-Dicke scalar, new inflation, chaotic inflation and extended inflation can offer us a mechanism by which the true vacuum can percolate. The central problem of all inflationary cosrnology centers on the mechanism by which galaxies were initially seeded and large scale structure evolved. Galaxy formation is primarily concerned with two questions, what were the seeds of large scale structure (density perturbatiom or topological defects) and what kind of matter (hot or cold dark matter) was active in nucleating around these seeds to produce the generic origin of large scale structure[5]. One of the most beautiful features of inflation is that it makes use of the scalar sector of particle theory (Higgs sector) with the Higgs potential providing the driving force for inflation[6]. After tlie original proposal for inflation in the early eighties, higher dimensions attracted the interest of many theorists because of the attractive features of supei-gravity theories and super-string theory which sought to give us a generic reason for a G.U.T. group and a low energy standard model along with a primitive origin of Einstein gravity and higher curvature corrections[7-9]. Low energy manifestations of the super-string would be extra neutra1 gauge bosons in the range (100 - 400 GeV)[10] along with an extra scalar gravitational field (dilaton) that may provide the scalar field necessary in extended inflation[9]. To model the ten dimensional cosmology that emerges from the super-string, we follow the observation of Myung and Cho who suggested that once the field theory limit is reached the conversion of massive string modes to massless modes can be modeled by a dissipative fluid with an bulk viscosity term representing the conversion process and resulting entropy production process[11]. Actually bulk viscosity proportional to the curvature squared was shown by Gurovich and Starobinsky[12] to represent vacuum polarization in a background gravitational field and we have shown that bulk viscosity proportional to the curvature squared leads to inflation in any number of dimensions for a flat space topology[13]. If we also represent the vacuum effects of quantum fields by a cosmological constant Gleiser[14] and Accetta[15] have pointed out that if compactification begins tlie vacuum effects of quantum fields will drive the universe to a deSitter expansiori in all dimensions, however if a monopole configuration is present, it will stabilize tlie compactification process. Admitting the presence of all tliese competitive phenomena we study a ten dimensional cosmology with the matter represented by a viscous fluid in the presence of a monopole configuration, we also consider a background cosmological constant to represent tlie vacuum effects of quantum fields and allow it to be positive, zero or negative. In a previous note[16] we have studied a similar cosmological scenario only in that study we did not consider a background cosmological constant and did not allow for negative and zero curvature in the six dimensional space. The equations for R_3, R_6 (three and six dimensional scale factors) are difficult to solve but we are able to study the small time evolution away from tlie initial state by using a power series expansion about tlie initial state.

II. Ten **dimensional** cosmology **amitting viscous fluid,** monopole condensation **and a background** cosmological constant

We begin our analysis by writing the metric for a ten dimensional space with topology of a closed, open or flat three dimensional liomogeneous isotropic space adjoined to a six dimensional isotropic and homogeneous six space of positive, negative or zero curvature. Tlie form of the metric is

$$g_{\mu\nu} = \begin{pmatrix} -1 & & \\ & R_3^2\tilde{g}_{ij} & \\ & & R_6^2\tilde{g}_{mn} \end{pmatrix} \tag{1}$$

Here $\mu,\nu = 0,..9$; $i,j = 1,2,3$; $m,n = 4,..9$.

For the Ricci component corresponding to Eq. (2.1) we have[17]

$$R_{00} = 3\frac{\ddot{R}_3}{R_3} + 6\frac{\ddot{R}_6}{R_6}$$

$$R_{ij} = -\left[\frac{2K_3}{R_2^3} + \frac{\ddot{R}_3}{R_3} + 2\left(\frac{\ddot{R}_3}{R_3}\right)^2 + 6\frac{\dot{R}_6\dot{R}_3}{R_6R_3}\right] g_{ij} \quad (2)$$

$$R_{mn} = -\left[\frac{5K_6}{R_2^6} + \frac{\ddot{R}_6}{R_6} + 5\left(\frac{\dot{R}_6}{R_6}\right)^2 + 3\frac{\dot{R}_3\dot{R}_6}{R_3R_6}\right] g_{mn}$$

For the total action of gravity plus matter in the background of the cosmological constant we have

$$\mathcal{L} = \frac{C^4}{16\pi G}(R+2\Lambda)\sqrt{-g} + \mathcal{L}_M + \mathcal{L}_F \ , \tag{3}$$

(G = ten dimensional gravitational constant).

Here **A** is the cosmological constant, $\mathcal{L}_M$ is the Lagrangian of matter, and $\mathcal{L}_F$ is the Lagrangian of antisymmetric tensor field generating the monopole configuration.

In 10 dimensions we have

$$\mathcal{L}_F = \frac{1}{K}F_{\mu\nu}F^{\mu\nu}\sqrt{-g} \tag{4}$$

where $F_{\mu\nu}$ is tlie antisymmetric gauge field, and $K = \frac{4}{3}\pi^4$ from normalization in 10 dimensions.

Varying Eq. (3) with respect to $g_{,,}$ gives

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \Lambda g_{\mu\nu} = -\frac{8\pi G}{C^4}T_{\mu\nu} \ , \tag{5}$$

where

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\partial\mathcal{L}_M}{\partial g^{\mu\nu}} + \frac{2}{\sqrt{-g}}\frac{\partial\mathcal{L}_F}{\partial g^{\mu\nu}} \ .$$

From Eq.4 we have for the gauge field energy momentum tensor

$$(T_{\mu\nu})_F = \frac{4}{K}F_{\mu\alpha}F^{\alpha}_{\nu} - \frac{g_{\mu\nu}}{K}F_{\alpha\beta}F^{\alpha\beta} \quad (\mu\nu = 0,...9) \ . \tag{6}$$

For tlie energy momentum tensor of matter we choose the phenomenological representation of all non-gauge matter to take on the form of the energy momentum tensor for a dissipative fluid [16]

$$T_{\mu\nu} = (\bar{P}+\epsilon)U_\mu U_\nu + g_{\mu\nu}\bar{P} - \eta K_{\mu\nu} \tag{7}$$

with

$$K_{\mu\nu} = U_{\mu;\nu} + U_{\nu;\mu} + U_\mu + U^\lambda U_{\nu;\lambda} + U_\nu U^\lambda U_{\mu;\lambda}$$

$$\bar{P} = P + \left(\frac{2}{9}\eta - \xi\right)U^a_{;a}$$

where $U^a_{;a}$ is the expansion, ξ is the coefficient of bulk viscosity and η is the coefficiente of shear viscosity.

Calculating the components of Eq. (7) we have using comoving coordinates

$$(U^0 = 1, \quad U^\alpha = 0, \alpha = 1, ...9)$$

$$T_{00} = \epsilon$$
$$T_{ij} = \bar{P} g_{ij} - 2\eta \left(\frac{\dot{R}_3}{R_3}\right) g_{if} \tag{8}$$
$$T_{mn} = \bar{P} g_{mn} - 2\eta \left(\frac{\dot{R}_6}{R_6}\right) g_{mn}$$

and

$$T = T_{\mu\nu} g^{\mu\nu} = -\epsilon + 9\bar{P} - 6\eta \frac{\dot{R}_3}{R_3} - 12\eta \frac{\dot{R}_6}{R_6} \tag{9}$$

for the trace of the energy momentum tensor of matter.

For the monopole configuration we choose [17]

$$F_{i\mu} F^\mu_j = 0, \quad F_{0\mu} F^\mu_i = F_{0\mu} F^\mu_0 = 0, F_{i\mu} F^\mu_m = 0,$$
$$F_{m\mu} F^\mu_0 = 0, \quad F_{m\mu} F^\mu_n = f_6^2 g_{mn} \quad (f_6 = const) \tag{10}$$

Using Eq. (6) we have

$$T_{00} = \frac{6}{K} f_0^2$$
$$T_{ij} = \frac{6}{K} f_0^2 g_{if} \tag{11}$$
$$T_{mn} = \frac{2}{K} f_6^2 g_{mn}$$

with trace

$$T = T_{\mu\nu} g^{\mu\nu} = -\frac{36}{K} f_6^2 \tag{12}$$

where we use the summation $F_{\mu\nu} F^{\mu\nu} = 6 f_6^2$. Eq. (5) can be written as

$$R_{\mu\nu} = -k \left[T_{\mu\nu} - \frac{1}{8} T g_{\mu\nu}\right] - \frac{1}{4} \Lambda g_{\mu\nu} \tag{13}$$

where T is the sum of Eq. (9) and Eq. (12) and $k = 8\pi G_{10}/C^4$.

When the sum of Eq. (8) and Eq. **(11)** is substituted in Eq. (13) we obtain with the use of Eq. **(2)**

$$\frac{3\ddot{R}_3}{R_3} + \frac{6\ddot{R}_6}{R_6} = -k \begin{bmatrix} \frac{6}{K} f_6^2 + \epsilon \\ -\frac{1}{9}\left(-\frac{36}{K} f_6^2 - \epsilon + 9\bar{P} - 6\eta\frac{\dot{R}_3}{R_3} - 12\eta\frac{\dot{R}_6}{R_6}\right)(-1) \end{bmatrix} + \frac{\Lambda}{4} \tag{14}$$

$$-\left[\frac{2K_3}{R_3^2} + \frac{\ddot{R}_3}{R_3} + 2\left(\frac{\dot{R}_3}{R_3}\right)^2 + \frac{6\dot{R}_6\dot{R}_3}{R_6 R_3}\right] = -k \begin{bmatrix} \frac{6}{K} f_6^2 + \bar{P} - 2\eta\frac{\dot{R}_3}{R_3} \\ -\frac{1}{9}\left[-\frac{36}{K} f_6^2 - \epsilon + 9\bar{P} - 6\eta\frac{\dot{R}_3}{R_3} - 12\eta\frac{\dot{R}_6}{R_6}\right] \end{bmatrix} - \frac{\Lambda}{4} \tag{15}$$

$$-\left[\frac{5K_6}{R_6^2} + \frac{\ddot{R}_6}{R_6} + 5\left(\frac{\dot{R}_6}{R_6}\right)^2 + \frac{3\dot{R}_3\dot{R}_6}{R_3 R_6}\right] = -k \begin{bmatrix} \bar{P} - 2\eta\frac{\dot{R}_6}{R_6} \\ -\frac{2}{K} f_6^2 \\ -\frac{1}{9}\left(-\frac{36}{K} f_6^2 - \epsilon + 9\bar{P} - 6\eta\frac{\dot{R}_3}{R_3} - 12\eta\frac{\dot{R}_6}{R_6}\right) \end{bmatrix} - \frac{\Lambda}{4} \tag{16}$$

using

$$\bar{P} = P + \left(\frac{2}{9}\eta - \xi\right)\left(\frac{3\dot{R}_3}{R_3} + \frac{6\dot{R}_6}{R_6}\right) \tag{17}$$

and the equation of state $P = \epsilon/9$ for the massless string modes. We find after eliminating P and ϵ from Eq. (14), Eq. (15) and Eq. (16),

$$\frac{\ddot{R}_6}{R_6} = \frac{1}{54} \begin{bmatrix} -\frac{180K_6}{R_6^2} + \frac{18K_3}{R_3^2} - 180\left(\frac{\dot{R}_6}{R_6}\right)^2 + 18\left(\frac{\dot{R}_3}{R_3}\right)^2 - 54\frac{\dot{R}_3\dot{R}_6}{R_3 R_6} \\ +k\left(\frac{84 f_6^2}{K} - 36\eta\frac{\dot{R}_6}{R_6} + 36\eta\frac{\dot{R}_3}{R_3} + 9\xi\frac{\dot{R}_3}{R_3} + 18\xi\frac{\dot{R}_6}{R_6}\right) \end{bmatrix} + \frac{5}{36}\Lambda\,, \tag{18}$$

$$\frac{\ddot{R}_3}{R_3} = \frac{1}{54}\left[\begin{array}{c} -\frac{90K_3}{R_3^2} + \frac{90K_6}{R_6^2} - 90\left(\frac{\dot{R}_3}{R_3}\right)^2 + 90\left(\frac{\dot{R}_6}{R_6}\right)^2 - 216\frac{\dot{R}_3\dot{R}_6}{R_3R_6} \\ -k\left(\frac{132f_6^2}{K} + 72\eta\frac{\dot{R}_3}{R_3} - 72\eta\frac{\dot{R}_6}{R_6} - 9\xi\frac{\dot{R}_3}{R_3} - 18\xi\frac{\dot{R}_6}{R_6}\right) \end{array}\right] + \frac{5}{36}\Lambda \tag{19}$$

We now consider the special cases of Eq. (18) and Eq. (19)

Case I

$K_3 = K_6 = 0,\ R_3 = R_{30}e^{\alpha t},\ R_6 = R_{60}\ e^{\beta t}(\beta =$ 0, a > O), (expanding three space and static six spaces)

Eq. (2.18) and Eq. (2.19) give

$$O = \frac{1}{54}\left[18\alpha^2 + k\left(\frac{84}{K}f_6^2 + 36\eta\alpha + 9\xi\alpha\right)\right] + \frac{5}{36}\Lambda$$

$$\alpha^2 = \frac{1}{54}\left[-90\alpha^2 - k\left(\frac{132}{K}f_6^2 + 72\eta\alpha + 9\xi\alpha\right)\right] + \frac{5}{36}\Lambda$$

and equating the values of a^2 in both equations we find

$$\alpha = \frac{\frac{135}{16}\Lambda + \frac{135}{2}\frac{f_6^2}{K}}{-27\eta k - \frac{63}{8}\xi k}\ . \tag{20}$$

Thus if

$$\frac{135}{16}\Lambda + \frac{135f_6^2}{2K} < 0$$

(negative cosmological constant) we wili get inflation in the three space for a > 0.

Case II

$K_3 = K_6 = 0,\ R_3 = R_{30}e^{\alpha t}(\alpha =$ O)$, R_6 = R_{60}e^{\beta t}(\beta <$ O), (compactification of six space).

Eq. (18) and Eq. (19) give

$$\beta^2 = \frac{1}{54}\left(-180\beta^2 + k\left(\frac{84f_6^2}{K} - 36\eta\beta + 18\xi\beta\right)\right] + \frac{5\Lambda}{36}$$

$$0 = \frac{1}{54}\left(-90\beta^2 - k\left(\frac{132f_6^2}{K} - 72\eta\beta + 18\xi\beta\right)\right] + \frac{5\Lambda}{36}$$

and equating the two values of β^2 we obtain

$$\beta = \frac{\frac{15}{2}\left(\frac{1}{90} - \frac{1}{234}\right)\Lambda - \frac{kf_6^2}{K}\left(\frac{132}{70} + \frac{84}{234}\right)}{-\cancel{234} - \frac{72}{90}\eta k + \frac{18}{234}(k\xi) - \frac{18}{90}k\xi} \tag{21}$$

For $\beta < 0$, the cosmological constant is

$$\Lambda > \frac{\frac{kf_6^2}{K}\left(\frac{132}{90} + \frac{84}{234}\right)}{\frac{15}{2}\left(\frac{1}{90} - \frac{1}{234}\right)}$$

If However

$$\Lambda < \frac{\frac{kf_6^2}{K}\left(\frac{132}{90} + \frac{84}{234}\right)}{\frac{15}{2}\left(\frac{1}{90} - \frac{1}{234}\right)}\ ,$$

$\beta > 0$ and we get expansion of the six space.

Case III

$K_3 = K_6 = 0,\ R_6 = R_{60}e^{\beta t}, R_3 = R_{30}e^{\alpha t}, (\eta = \xi =$ 0, no viscous effects) we have from Eq. (18) and Eq. (19)

$$\beta^2 = -\frac{180}{54}\beta^2 + \frac{18}{54}\alpha^2 - \alpha\beta + \frac{84kf_6^2}{54K} + \frac{5}{36}\Lambda\ , \tag{22}$$

and

$$\alpha^2 = -\frac{90}{54}\alpha^2 + \frac{90}{54}\beta^2 - \frac{216\alpha\beta}{54} - \frac{132kf_6^2}{54K} + \frac{5}{36}\eta\ . \tag{23}$$

From Eq. (22) and Eq. (23) we have

$$\beta^2\left(\frac{1026}{54}\right) - \alpha^2(4) = \frac{468f_6^2k}{54K} + \frac{15}{36}\Lambda$$

$$\beta^2 = \alpha^2\left(\frac{4\times 54}{1026}\right) + \frac{54}{1026}\left(\frac{468f_6^2k}{54K} + \frac{15}{36}\Lambda\right) \tag{24}$$

$$\beta = \pm\sqrt{\alpha^2\left(\frac{4\times 54}{1026}\right) + \frac{54}{1026}\left(\frac{468f_6^2k}{54K} + \frac{15}{36}\Lambda\right)}$$

Substituting Eq. (24) back into Eq. (23) we may solve for a which will generate positive a for a range of the parameters f_6 and **A**.

We note that positive a with negative β (taking - sign in Eq. (24) will generate an expanding three space and contracting six space. We also have positive a with positive β which will generate expanding three space and six space.

Case IV

For the most general situation K_3, $K_6 \neq 0$ we may solve Eq. (18) and Eq. (19) by a power series about t_0.

Given $R_3(t_0), \dot{R}_3(t_0), R_6(t_0), \dot{R}_6(t_0)$, we may find $\ddot{R}_3(t_0)$, $\ddot{R}_6(t_0)$ from Eq. (18) and (19).

We may also find the higher derivatives of R_3 and R_6 at t_0 by differentiation of Eq. (18) and Eq. (19) and write

$$\begin{aligned} R_3(t) &= R_3(t_0) + \dot{R}_3(t_0)(t-t_0) + \ddot{R}_3(t_0)\frac{(t-t_0)^2}{2!} \\ R_6(t) &= R_6(t_0) + \dot{R}_6(t_0)(t-t_0) + \ddot{R}_6(t_0)\frac{(t-t_0)^2}{2!} \end{aligned} \tag{25}$$

We note that a positive A will have a tendency to expand R_6 and R_3 from Eq. (18) and Eq. (19). Also from Eq. (18) and Eq. (19) the magnitude of ξ and η will determine the evolution of R_6 and R_3. If ξ and η depend on the curvature squared[12,13], then the evolution of R_6 and R_3 will be modified so as to be dominated by tlie curvature dependent viscosity term.

III. Conclusion

The combined system of viscous fluid, monopole configuration and the cosmological constant represents a moclel that in a phenomenological manner can model various features of the early universe. We note also that by adding a second monopole configuration of a second gauge field we have more arbitrariness in tlie parameters f_6, h, ξ and η tliat would lead to a compactification of the six space. From Eq. (18) and Eq. (19) we note that the monopole configuration impeded the contraction of the six space and also impedes the expansion of the three space in the second derivatives of the scale factors. The only way to follow the evolution of R_3 and R_6 in the general case is through a numerical analysis given the initial conditions. If this is done the large time evolution would be found from Eq. (25) with the liigher derivatives calculated from Eq. (18) and Eq. (19) at $t = t_0$.

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