

Electroconvective Heat Transfer in a Suspension of Spherical Chromium (Hydrous) Oxide (Cr(OH)₃) Particles

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The electroconvective heat transfer coefficient has been measured in distilled water and spherical chromium (hydrous) oxide particles suspension in distilled water from a single platinum wire (diameter = 0.025 mm) mounted along the axis of a copper cylinder (diameter = 53 mm). The measurements have been carried out as a function of time, electric field, frequency, pH value, colloidal concentration and orientation. The electro-convective heat transfer coefficient exhibits a 'timing' effect as well as an 'aging' effect. An increase in heat transfer coefficient is always observed in an ac field and the increase is extremely quick for pH values largely different from the isoelectric point (i.e.p.). Similar trend is also noticed in a dc field at very weak field strength. An increase in particle concentration increases the heat transfer coefficient and similar trend is also noticed when the inclination of the cylinder is changed from horizontal to vertical position. The heat transfer data have been correlated by dimensional analysis. The data have been compared with the predicted correlations and is well correlated over the range of the Rayleigh number from 10^6 to 9×10^9 . An empirical correlation relation has also been proposed to evaluate the efficiency of convection and the results for dc and ac fields are presented.

I. Introduction

The heat transfer and dynamics of a particulate system in a liquid has been the subject of many intensive studies because of their applications in various chemical and industrial processes such as sealing, lubrication, damping, high-vacuum equipment, computer, laser systems and many others. In order to select the optimum regimes for the construction of these equipments and to determine the most effective heat transfer conditions and characteristics of fluid disperse heat carriers, it is necessary to know how electric fields influence the processes of momentum and energy transfer in such systems.

Although the electric charge on the ultrafine particles is often ignored in theoretical analysis, the electri-

fication of these particles is always possible when dispersed in an aqueous solvents. The particles used in this investigations are known to be negatively charged. As a result, interparticle repulsion prevents particle agglomeration and thus augments the heat transfer mechanism when subject to a non-uniform electric field.

A vast literature is devoted to the study of heat transfer in fluids and fluidized beds. The experiments for heat transfer into fixed beds have been carried out by Dhingra^[1]. The heat transfer in an electrostatically charged fluidized bed has also been performed by Wolney^[2]. Recently, a large amount of experimental research on radiative heat transfer in fluidized beds has been undertaken^[3], but the research of heat transfer in electrorheological (ER) fluids is insufficient. Shulman^[4]

investigated the electroconvection in ER fluids in a coaxial cylindrical channel. Other investigators theoretically analysed the heat transfer in ER fluids. Different models were made to describe the contribution due to convection. Interest has recently been stimulated to study the electro-convection in ER fluids in a cylindrical enclosure. The various characteristics of these particles such as their physicochemical, electrophysical and thermophysical properties can be determined using electric fields.

This paper investigates the convective heat transfer in ultrafine particles ($\text{Cr}(\text{OH})_3$) in the body of a carrier liquid (distilled water) where the electric fields and field gradients are relatively small. Experiments were carried out under the influence of dc and ac electric fields. The convective heat transfer coefficient has been evaluated under various experimental conditions such as time, frequency, orientation, concentration and charge of the particle. Additionally, the experimental data have been correlated by dimensional analysis. The data have been compared with the empirical correlation relation and analytical expression and is well correlated over the range of the Rayleigh number from 10^8 to 9×10^9 . An empirical correlation relation has also been proposed to evaluate the efficiency of convection and the results for dc and ac fields are presented.

Experimental Arrangement

The detailed experimental arrangement used in this investigation has been given elsewhere^[5-8]. However, a brief description of the equipment is furnished here.

The heat transfer data was evaluated from a hot wire cell made from a copper cylinder (diameter of 53 mm) with a fine platinum wire (diameter of 0.025 mm) stretched along its axis. The hot wire cell in series with a standard resistor was placed in one arm of Wheatstone bridge. This cell could be positioned at any angle between the vertical and horizontal orientations. Electric fields in the cell were created by an applied electrical potential (dc or ac) between the central wire and the surrounding cylinder. The heating current was supplied from a constant current source, while the surrounding temperature was maintained constant in a constant temperature bath. The heat transfer data was obtained by calibrating the wire as a platinum resistance thermometer and then measuring the voltage

across it and a standard resistor in series with it. If I is the current flowing through the wire in the absence of any electric field and I_{el} is the additional current necessary to keep the bridge balanced in the presence of any electric field, then the rate of heat transfer is given by $Q_{el}(2II_{el} + I_{el}^2)R$, where R is the resistance of the platinum wire. The heat transfer coefficient is then obtained by using the relation $h_{el} = Q_{el}/AT_d$, where A is the surface area of the wire and T_d is the temperature difference between the wire and the surrounding medium.

Spherical chromium (hydrated) oxide particles ($\rho = 2.42 \text{ g cm}^{-3}$) with $r_p = 0.17 \mu\text{m}$ were generated by aging chrome alum solutions^[9]. Details of the preparations and characteristics of these materials are given in the cited reference. A transmission electron micrograph of these particles used in this investigation is presented in Fig. 1. The isoelectric point (i.e.p.) of such particle was at pH 7.3. All measurements were carried out at a fixed particle concentration ($n_p \approx 2.2 \times 10^{15}$ particles/ m^3) and pH value 11, unless otherwise stated.

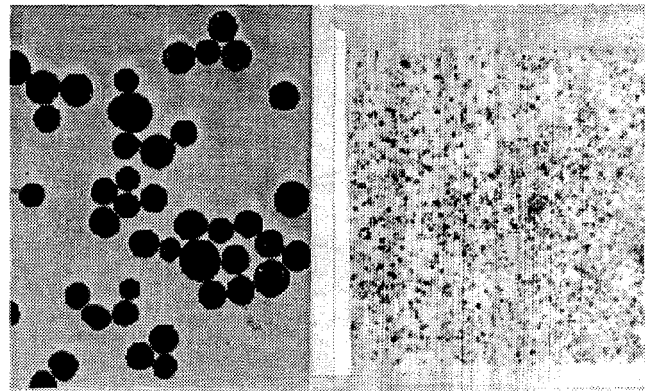


Figure 1. A transmission electron micrograph of spherical chromium (hydrated) oxide ($\text{Cr}(\text{OH})_3$) particles used in the convection experiment ($r_p = 0.17 \mu\text{m}$).

In order to test the experimental set up, firstly, the experiment was carried out in distilled water in absence of electric field. The free-convective heat transfer coefficient, h_f was determined for various heating currents. The global effect of convective heat transfer is written in terms of the Nusselt number Nu , which is the ratio of the mean heat flux to the heat flux which would exist without convection for the same temperature difference

$$Nu = \frac{h_f D}{\lambda}, \quad (1)$$

where h_f is the free-convective heat transfer coefficient, D is the diameter of the cylinder and λ is the thermal conductivity of the fluid. The Nusselt number Nu is normally plotted function of the Rayleigh number Ra (proportional to the temperature difference)

$$Ra = \frac{g\beta T_d D^3}{\nu^2} Pr, \quad (2)$$

where β is the coefficient of thermal expansion of the liquid, g is the gravitational acceleration, Pr is the Prandtl number, ν is the kinematic viscosity and T_d is the temperature difference between the wire and the surrounding medium. The value of Nu as a function of Ra (for no applied electric field) is evaluated for distilled water and they agree very well with the already known results^[10]. The good agreement between our experimental results and the classical ones constitutes a test for our experimental set-up.

III. Results and discussion

III.1 Convection in distilled water

Convection in distilled water is measured under the influence of ac and dc fields. In the case of an ac field, there exists a critical electric field ($\approx 5.0v$, where $1v = 104 \text{ V/cm}$) below which no convection is observed. When the applied electric field is greater than the critical electric field, convective heat transfer coefficient increases very slowly and then reaches a saturation value at higher electric field. Similar effect is noticed at different pH values (4 to 11), and the change in heat transfer coefficient is not all significant even at higher pH value (pH=11). In the case of a dc field, convective heat transfer coefficient decreases gradually as the field is increased. The heat transfer coefficient is negative, and the negative behaviour implies a suppression of the free-convection by electro-convection. The experimental results (ac case) are dimensionally analysed and compared with various correlation relations. The details are discussed in Section IV and the results are presented in Fig. 10.

III. Convection in spherical chromium (hydrous) oxide ($\text{Cr}(\text{OH})_3$) particle suspension in distilled water

Convection in spherical chromium (hydrous) oxide particle suspension in distilled water is measured under

various experimental parameters such as time, electric field (dc and ac), frequency, pH value, colloidal concentration and orientation. The experimental results (ac case) have also been dimensionally analysed and compared with various correlation relations, and presented in Fig. 11. (see Section 4 for details).

III.2.1. Time evolution of electroconvection

The time evolution of electroconvection for spherical chromium (hydrous) oxide particle suspension in distilled water is presented in Fig. 2. As seen in the figure, the electroconvective heat transfer coefficient exhibits a 'timing' effect. The magnitude of the convection decreases with time, the effect being significant only within the first few minutes after the field is applied, then decreases very slowly after 10 minutes and reaches a stationary value in about half an hour (curve 1 of Fig. 2). A similar effect is noticed when the measurement was done after 48 hours (curve 2), and also with different voltage (curve 4) and pH value (curve 5). The initial decrease in heat transfer coefficient may be attributed to relaxation phenomena. Before the application of the electric field, the particles are randomly distributed in the carrier liquid and experience no collisions with each other. Since the particles are charged, once the field is turned on, they undergo collisions with each other until thermal equilibrium is achieved. The presence of collisions reduce the heat transfer coefficient due to the reduction in particle momentum. The collision probability reduces gradually with time until thermodynamic equilibrium is achieved with the surroundings. Thus the heat transfer coefficient reduces gradually and remains practically constant after a while.

As time progresses, the pH value of the suspension changes gradually, which in turn causes a reduction in the surface charge of the particle. Thus the 'aging' effect is associated with particles settling, coagulation, etc., as clearly noticed in curve 2 of Fig. 2. It is expected that the surface charge of the colloidal particles plays a dominant role in the convective heat transfer mechanism.

The timing effect was also carried out under the influence of an ac field of frequency 60 Hz and with a colloidal solution of pH value 5.30. No change in heat transfer coefficient was observed during the period for which the dc fields were applied (curve 3 in Fig. 2).

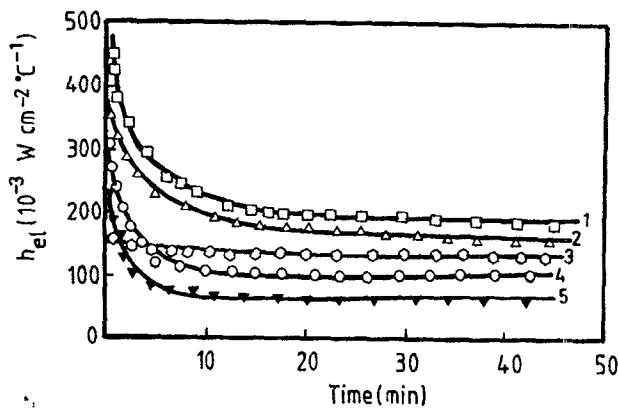


Figure 2. Time evolution of electroconvective heat transfer coefficient of chromium (hydrrous) oxide particle suspension in distilled water. Horizontal cylinder, $T_d = 8.0^\circ\text{C}$; 1. $U = 0.1$ Volt dc, pH value = 11.0; 2. $U = 0.1$ Volt d-c, pH value = 11.0, obtained after 48 hours; 3. $U = 50$ Volt a-c, 60 Hz, pH value = 5.30; 4. $U = 0.3$ Volt d-c, pH value = 11.0; 5. $U = 0.3$ Volt d-c, pH value = 10.0.

III.2.2. Effect of d-c fields on convection

The effect of d-c fields on the convective heat transfer coefficient is presented in Fig. 3. These measurements were carried out with colloidal solutions of different pH values (between 4.10 to 11.10). An enhancement in heat transfer coefficient is observed at very weak field strengths (between 0.1 to 0.2 v) and particularly significant at higher pH values away from the isoelectric point (i.e.p. of $\text{Cr}(\text{OH})_3$ is 7.3) (see curves 1, 4, 5 and 6). The heat transfer coefficient decreases rapidly to zero as the field is increased and then becomes negative, implying a suppression of the free-convective heat transfer coefficient. The heat transfer coefficient then decreases gradually further as the field strength is increased.

The electric force which acts on fluid per unit volume is expressed as [11]

$$f_e = qE_r + \text{grad} \left(\frac{1}{2} E_r^2 \rho \frac{\partial \epsilon}{\partial \rho} \right) + \frac{1}{2} E_r^2 \text{grad} \epsilon, \quad (3)$$

where the first term represents the Coulomb force (or the electrophoretic force) and the last two terms represent forces induced by the non-uniformity of the dielectric constant. The symbol q is the charge of the particles, E_r is the radial electric field, ρ is the specific mass and ϵ is the permittivity of the fluid. In the present investigation, a suspension of charge particles is considered. The contribution to convection arises mainly

due to electrophoretic force. The electrophoretic force on the colloidal fluid can be written as

$$f_{el} = qE_r n_p, \quad (4)$$

where n_p is the number of particle per unit volume. The radial electric field intensity is estimated as

$$E_r = \left(0.13 \frac{U}{r} \right) \left(\frac{\text{Volt}}{\text{rmmcm}} \right), \quad (5)$$

where r is the distance from the centre of the platinum wire and U is the voltage applied to the cylinder. The total number of colloidal particle in the cylinder is given by

$$n_T = \pi r_c^2 L n_p, \quad (6)$$

where r_c is the radius of the cylinder and L is its length.

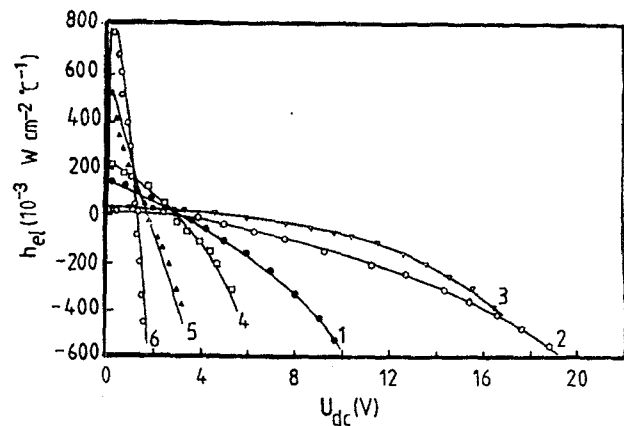


Figure 3. Dependence of electroconvective heat transfer coefficient of chromium (hydrrous) oxide particle suspension in distilled water as a function of dc voltage with different pH-values. Horizontal cylinder, $T_d = 10^\circ\text{C}$. 1. pH value = 4.10; 2. pH value = 5.30; 3. pH value = 7.20; 4. pH value = 9.0; 5. pH value = 10.10; 6. pH value = 11.10.

As the field is turned on, the convective heat transfer coefficient increases extremely quickly until the field is of the order of 0.1 to 0.2 Volt when the density inversion occurs (the number of particle per unit volume decreases). Since the cylinder is maintained at positive potential and for highly negatively charged particles (far away from isoelectric point), the electrophoretic force rises up suddenly to a very high level when the field is applied. Thus the heat transfer coefficient increases almost vertically near the vicinity of the origin. The maximum increase in heat transfer coefficient is reduced gradually as the charge of the particle is reduced. Near the isoelectric point, the increase in the

heat transfer coefficient is practically zero due to the absence of charges (curves 2 and 3). It turns out that the charge of the colloidal particles are playing a dominant role in the energy transfer mechanism.

On further increase in electric field, the heat transfer coefficient drops rather quickly and approaches a zero value. The reduction in the heat transfer coefficient observed in this case is associated with the number of particles involved in the energy transfer mechanism. As the total number of particle n_T is fixed, the number of particle present in the fluid medium will decrease gradually with increase in electric field, causing a reduction in heat transfer coefficient. As the field is further increased, the heat transfer coefficient approaches a zero value, implying a complete removal of the charged particles from the fluid medium ($n_T \rightarrow 0$). On further increase in electric field, it becomes negative and follows the trend as observed in distilled water.

III.2.3 Effect of a-c fields and frequency on convection

The effect of a-c field on the convective heat transfer coefficient for colloidal chromium (hydrous) oxide particle suspension in distilled water is presented in Fig. 4. These measurements were carried out with colloidal solutions of different pH-values, above and below the isoelectric point (i.e.p. of chromium (hydrous) oxide $\cong 7.3$). A sharp rise in the heat transfer coefficient is observed for pH values away from the isoelectric point (curves 4, 5, 6, 7 and 8), whereas the heat transfer coefficient changes very slowly and gradually for pH values close to isoelectric point (curves 2 and 3). The negligible change in heat transfer coefficient presented in curve 1 resulted from the polarity effect.

When an a-c electric field, with frequency 60 Hz is applied to a fluid, the electrophoretic (or the Coulomb) force is negligible because the electric relaxation time of most fluids is of the order of 10 - 100 sec. For a charged suspension, the electrophoretic force is found to be significant even in an a-c field, and the heat transfer coefficient rises extremely quickly for pH values away from the isoelectric point. The change in the heat transfer coefficient is negligible for pH values close to isoelectric point due to absence of charges.

The magnitude of the convection decreases gradually as the frequency of the applied field is increased (Fig. 5). It has been observed that the polarization

of the molecule in an a-c fields is less than that in d-c fields^[12]. Thus the dielectrophoretic forces should also be less in the former field and consequently, the heat transfer coefficient in the presence of a-c fields would be less than that due to d-c fields. The polarization of the molecules in the presence of an a-c field will decrease gradually as the frequency of the field is increased. As a result, dielectrophoretic forces would decrease, leading to a reduction in heat transfer coefficient.

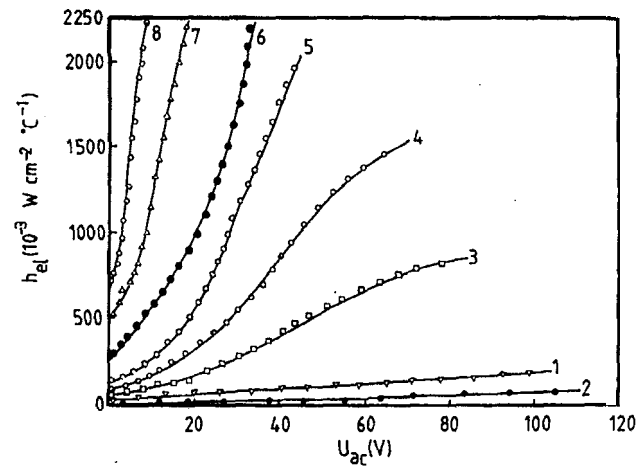


Figure 4. Dependence of electroconvective heat transfer coefficient of chromium (hydrous) oxide particle suspension in distilled water as a function of a-c voltage with different pH values. Horizontal cylinder, $T_d = 10^\circ \text{C}$. 1. pH value = 4.10; 2. pH value = 7.20; 3. pH value = 9.0; 4. pH value = 9.50; 5. pH value = 9.90; 6. pH value = 10.40; 7. pH value = 10.90; 8. pH value = 11.30.

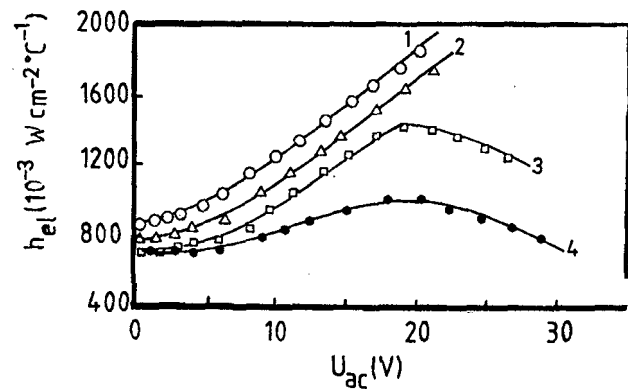


Figure 5. Dependence of electroconvective heat transfer coefficient of chromium (hydrous) oxide particle suspension in distilled water as a function of frequency (f). Horizontal cylinder, pH value = 11.0. 1. $T_d = 8.0^\circ \text{C}$, $f = 60 \text{ Hz}$; 2. $T_d = 6.0^\circ \text{C}$, $f = 60 \text{ Hz}$; 3. $T_d = 8.10^\circ \text{C}$, $f = 120 \text{ Hz}$; 4. $T_d = 8.20^\circ \text{C}$, $f = 200 \text{ Hz}$.

III.2.4. Effect of pH-values on convection

Fig. 6a shows the plot of the heat transfer coefficient as a function of pH values and at a fixed dc field ($U = 0.1$ V), while the corresponding results in an ac field ($U = 8.0$ V) is shown in Fig. 6b. As observed in both cases, the change in heat transfer coefficient is negligible for pH values close to the isoelectric point due to absence of charges. The change in heat transfer coefficient is also negligible for pH values below the isoelectric point due to polarity effect. For pH values above the isoelectric point, the particles are highly negatively charged, and as a result, the heat transfer coefficient increases extremely quickly. The increase is almost vertical for pH values close to 11. The phenomenon is attributed to a 'charge effect'.

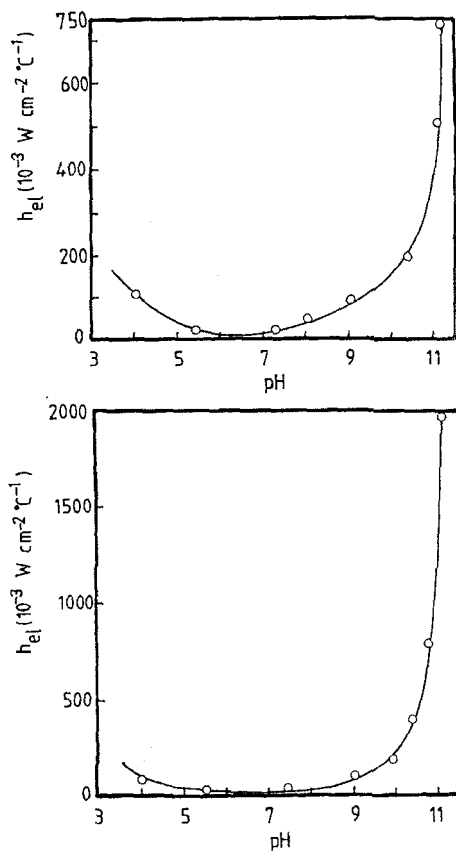


Figure 6. Dependence of electroconvective heat transfer coefficient of chromium (hydrous) oxide particle suspension in distilled water as a function of pH values. Horizontal cylinder. a. $U = 0.10$ V dc, $T_a = 10^\circ\text{C}$; b. $U = 8.0$ V ac, $T_d = 10^\circ\text{C}$.

III.2.5. Effect of suspension concentration on convection

An increase in heat transfer coefficient is also noticed when concentration of the suspension is increased (Fig. 7). An increase in concentration increases the particle number in the fluid medium. As a result, the electrophoretic force acting on the particle increases, and this in turn enhances the convection.

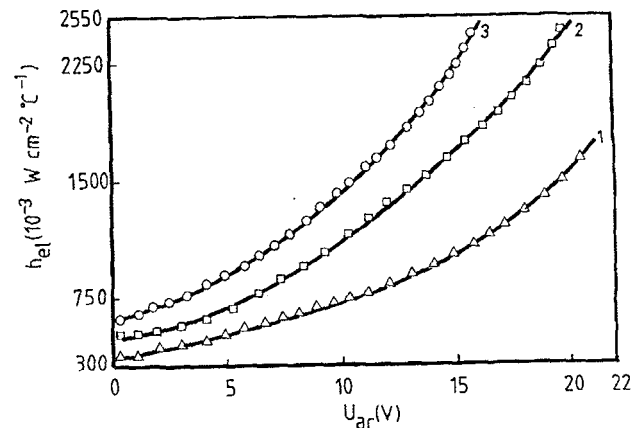


Figure 7. Dependence of electroconvective heat transfer coefficient of chromium (hydrous) oxide particle suspension in distilled water as a function of ac voltage with different colloidal concentration. Horizontal cylinder, $T_d = 8.5^\circ\text{C}$. 1. $n_p = 2.2 \times 10^{15}$ particles/ m^3 ; 2. $n_p = 4.4 \times 10^{15}$ particles/ m^3 ; 3. $n_p = 6.6 \times 10^{15}$ particles/ m^3 .

III.2.6. Effect of cylinder orientation on convection

The effect of cylinder orientation on convection for chromium (hydrous) oxide particle suspension in distilled water is presented in Fig. 8. In absence of any electric field, the free-convective heat transfer coefficient always decreases as the cylinder is moved from horizontal to vertical position^[13]. Similar effect is also noticed in chromium (hydrous) oxide particle suspension in distilled water (curve 1). An increase in concentration of the suspension increases the magnitude of the convection, but the trend for increased inclination remains the same (curve 2). However, in presence of electric field, an increase in heat transfer coefficient is noticed as the inclination of the cylinder is increased. The increase is significant for inclination greater than about 50° and becomes maximum when the cylinder is kept in a vertical position (curves 3, 4, 5, 6 and 7).

Curves 3 and 4 resulted from the measurements at a d-c field of 0.3 and 0.1 Volt respectively, while curves 5 and 6 represent the measurements in an a-c field of 7 and 10 Volt respectively. In both cases, the trend for increased inclination remains the same. Similarly, curve 7 is obtained by increasing the pH value of the suspension and again the general trend remains the same.

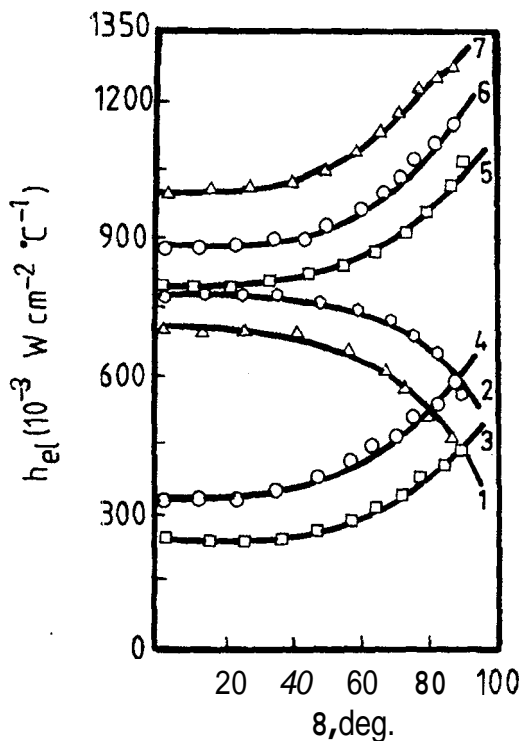


Figure 8. Effect of inclination to the horizontal plane on the electroconvective heat transfer from a cylinder in chromium (hydrous) oxide particle suspension in distilled water. pH value = 10.70 (1-6); 1. h_f , $T_d = 10^\circ\text{C}$, $n_p = 1.1 \times 10^{15}$ particles/ m^3 ; 2. h_f , $T_d = 10.10^\circ\text{C}$, $n_p = 2.2 \times 10^{15}$ particles/ m^3 ; 3. h_{el} , $T_d = 9.1^\circ\text{C}$, $U = 0.3$ V d-c, $n_p = 2.2 \times 10^{15}$ particles/ m^3 ; 4. h_{el} , $T_d = 9.1^\circ\text{C}$, $U = 0.1$ V d-c, $n_p = 2.2 \times 10^{15}$ particles/ m^3 ; 5. h_{el} , $T_d = 10^\circ\text{C}$, $U = 7$ V a-c, $n_p = 2.2 \times 10^{15}$ particles/ m^3 ; 6. h_{el} , $T_d = 10.1^\circ\text{C}$, $U = 10$ V a-c, $n_p = 2.2 \times 10^{15}$ particles/ m^3 ; 7. h_{el} , $T_d = 10^\circ\text{C}$, $U = 7$ V a-c, $n_p = 2.2 \times 10^{15}$ particles/ m^3 , pH value = 11.20.

The difference in heat transfer coefficient between horizontal and the vertical cylinders is presented in Fig. 9, where curve 1 is for a horizontal cylinder while curve 2 is for vertical cylinder. As seen in the figure, the heat transfer coefficient for the vertical cylinder is found to be higher than the horizontal cylinder. The difference in the two cases may be due to gravitational effect. In the horizontal position, the gravitational effect is non-symmetrical with respect to each half of the fluid separated by the platinum wire. In the up-

per half of the fluid, the hot volume of the liquid is moving in the upward direction, while the gravity acts down wards. Consequently, while the hot volume of the liquid is moving in the downward direction, gravity enhances its flow. The net effect is that the convective heat transfer coefficient in a horizontal cylinder is reduced as compared to a vertical cylinder where the gravity acts symmetrically with respect to the platinum wire which is mounted along the cylinder axis. Thus the heat transfer coefficient is reduced when the cylinder is kept in a horizontal position.

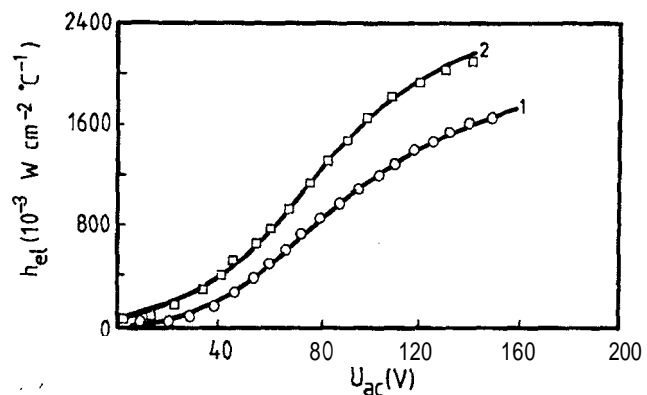


Figure 9. Dependence of electroconvective heat transfer coefficient of chromium (hydrous) oxide particle suspension in distilled water as a function of ac voltage in horizontal and vertical cylinders. pH value = 9.0. 1. Horizontal cylinder, $T_d = 9.10^\circ\text{C}$; 2. Vertical cylinder, $T_d = 9.0^\circ\text{C}$.

III.2.7. Efficiency of convection in presence of electric field

The efficiency of free-convection has been defined as the rate of transformation electrical power supplied to the system kinetic energy of convective motion^[14]. In the present study the view was taken so that the measured heat transfer coefficient in presence of electric field represented a useful conversion of additional electrical energy into the rate of generation of the kinetic energy of convective motion. In presence of electric field, the efficiency of convection can be calculated using

$$\gamma = \frac{h_{el}}{h_f + h_{el}}, \quad (7)$$

where h_f is the free-convective heat transfer coefficient and h_{el} is the electroconvective heat transfer coefficient.

The efficiency of convection for spherical chromium (hydrous) oxide particle suspension in distilled water is presented in Table I (dc field). The results for distilled water and chromium (hydrous) oxide particle suspension in distilled water is presented in Table II (ac field). In each case, the efficiency of convection has been calculated for various pH values away from the isoelectric point (i.e.p. of $\text{Cr}(\text{OH})_3 \cong 7.3$). In the case of dc field, a sharp rise in efficiency is observed at very weak field strengths and at higher pH values away from the isoelectric point. As the field is further increased, the efficiency approaches a zero value and then becomes negative. A zero value of efficiency implies a complete removal of the charged particles from the carrier liquid, while a negative value implies a suppression of the free-convection by electroconvection. The negative behaviour of γ is similar to that observed in the carrier liquid only. The maximum efficiency obtained in a dc field is 68%, corresponding to a pH value of 11.10 and an electric field gradient of 10.40 (V/cm) ($\cong 0.1\text{V}$), while an efficiency of 55% corresponds to an electric field gradient of 8.32 (V/cm) ($\cong 0.08\text{V}$).

A sharp rise in efficiency is also observed in ac fields, but in a slightly larger field and field gradients. More significantly, the efficiency increases extremely quickly when the surface charge of the colloidal particle is increased. The sharp rise in efficiency at higher pH values is thus attributed to 'charge-effect'. For higher surface charged particle (pH = 11.20), an efficiency of 54% is obtained at an electric field gradient of 5.2×10^2 (V/cm) ($\cong 5.0\text{V}$), while the maximum efficiency (73%) corresponds to an electric field gradient of 12.48×10^2 (V/cm) ($\cong 12\text{V}$). Thus the electric field gradient required for an ac convection is roughly 63 times larger than the dc convection, and this is due to relaxation phenomena associated with the a-c field.

Working correlations for electroconvection

One of our primary objectives is also to find a relationship between the dimensionless quantities of Nusselt number, Rayleigh number and an appropriate geometric parameter. The mean Nusselt number and Rayleigh number are defined by

$$(Nu_{el})_D = \frac{h_{el}D}{\lambda}, \quad (8)$$

and

$$Ra_D = \frac{g\beta T_d D^3}{\nu^2} Pr, \quad (9)$$

where D is the diameter of the cylinder and λ is the thermal conductivity of the fluid. The fluid properties were evaluated at the film temperature given by

$$T_f = \frac{T_w + T_B}{2}, \quad (10)$$

where T_w is the temperature of the wire and T_B is the temperature of the surrounding medium.

Many previous analysis^[15-17] of the enclosure free-convective heat transfer data have shown that correlations of the form

$$Nu_L = C_1 Ra^{c_2}, \quad (11)$$

could correlate extremely well vertical cylindrical data and plates. Experimental correlations of heat transfer data for air as the test fluid reported by Warrington and Powe^[18] are given by

$$Nu_L = 0.479 Ra_L^{0.171}. \quad (12)$$

In the present study, the heat transfer is described in terms of the average Nusselt number for some typical Rayleigh number which corresponds to the linear part of the experimental data. The experimental data have been correlated by dimensional analysis. The data have been compared with the empirical correlations and analytical expressions, and the correlation used for electroconvection is the same used for free-convection.

In Figs. 10 and 11, the heat transfer correlation of $(Nu_{el})_D$ and Ra_D is plotted for both the experimental results and the predicted values of various correlations for distilled water and chromium (hydrous) oxide particle suspension in distilled water. The analytical results are also shown on the same plot. The experimental values in this case predicted slightly higher Nu_{el} number than the predicted correlations for $Ra > 3.11 \times 10^8$ for distilled water. The corresponding value for chromium (hydrous) oxide particle suspension in distilled water is about 2.5×10^8 . Curve 1 in Figs. 10 and 11 are the predicted correlation obtained by using the equation

$$(Nu_{el})_D = m(Ra_D)^n, \quad (13)$$

where the value of m for distilled water and chromium (hydrous) oxide particle suspension in distilled water is

Table I. The efficiency of convection in spherical chromium (hydrous) oxide (Cr(OH)₃ particle suspension in distilled water under the influence of dc field.

$E_s (dc) \left(\frac{V}{cm}\right)$	Cr(OH) ₃ in Distilled Water			
	pH=4.10 γ (%)	pH=9.0 γ (%)	pH=10.10 γ (%)	pH=11.10 γ (%)
0	0	0	0	0
1.04	0.5	1.0	4.50	10.00
2.08	1.00	2.0	8.00	16.00
3.12	2.00	4.00	12.00	20.00
4.16	4.50	6.00	16.00	25.00
5.20	7.00	11.00	19.00	33.00
6.24	11.00	14.00	24.00	42.00
7.28	13.00	19.00	30.00	47.00
8.32	17.00	24.00	37.00	55.00
9.36	19.00	26.00	42.00	63.00
10.40	24.00	30.00	50.00	68.00
20.80	23.00	27.00	45.00	60.00
41.60	20.00	25.00	43.00	53.00
62.40	19.00	22.00	39.00	49.00
83.20	17.00	20.00	35.00	31.00
104.0	16.00	19.00	30.00	25.00
124.80	14.00	17.00	26.00	12.00
145.60	13.00	13.00	22.00	0
166.40	11.00	12.00	16.00	-13.0
187.20	10.00	11.00	10.00	-30.00
208.00	4.00	8.00	0	-
260.00	3.00	5.00	-8.00	-
312.00	2.00	0	-36.00	-
364.00	0	-14.00	-	-
416.00	-9.00	-25.00	-	-
468.00	15.00	-50.00	-	-
520.00	18.00	-	-	-

0.32, and the corresponding exponent takes the values 0.362 and 0.405 respectively.

Al-Arabi and Khamis^[19] have correlated heat transfer data for cylinders of various lengths, diameters and angles of inclination from the vertical. Their results are of the form

$$N\bar{u}_L = m(Gr_L Pr)^n,$$

where m and n are functions of the cylinder diameter and angle of inclination from the vertical, θ . In the laminar regime, they found

$$N\bar{u}_T = [2.9 - 2.32(\sin\theta)^{0.8}](Gr_D)^{-1/12} (Gr_L Pr)^{[\frac{1}{4} + (\frac{1}{12})(\sin\theta)^{1.2}]}, \quad (14)$$

where the Grashof number based in cylinder diameter is restricted to the range $1.08 \geq Gr_D \geq 6.9 \times 10^5$. For horizontal cylinder, $\theta = 90^\circ$ and equation (14) becomes

$$(N\bar{u})_D = (0.58) \left\{ (Gr_D)^{-1/12} (Ra_D)^{1/4} \right\}. \quad (15)$$

The empirical correlation for free-convection can be used to fit the experimental data for electroconvection if the $\frac{1}{4}$ th power law in Rayleigh is changed. For distilled water, the exponent of the Rayleigh has been adjusted to 0.375, the corresponding value for chromium (hydrous) oxide particle suspension in distilled water is 0.42. This result is represented by curves 2 in Figs. 10 and 11. Curves 3 in Figs. 10 and 11 resulted from the predicted correlation proposed by Churchill and Chu^[20]. For horizontal cylinder we have

Table II - The efficiency of convection in distilled water and spherical chromium (hydrous) oxide particle suspension in distilled water under influence of ac field.

$E_s(ac) \left(\frac{V}{cm}\right) 10^2$	Dis- tilled Water pH=5.6 $\gamma(\%)$	Cr(OH) ₃ in Distilled Water					
		pH=5.6 $\gamma(\%)$	pH=7.2 $\gamma(\%)$	pH=9.0 $\gamma(\%)$	pH=10.40 $\gamma(\%)$	pH=10.90 $\gamma(\%)$	pH=11.2 $\gamma(\%)$
0	-	-	-	0	0	0	0
0.10	-	-	-	1.0	1.00	7.00	8.0
0.21	-	-	-	1.0	2.5	13.00	15.0
0.31	-	-	-	2.0	5.0	18.00	25.00
0.42	-	-	-	2.5	7.0	27.00	31.00
0.52	-	-	-	3.5	9.0	32.00	36.00
1.04	-	-	-	4.0	10.5	34.00	39.00
2.08	-	-	-	5.0	11.0	36.00	41.00
3.12	-	-	-	5.5	13.0	38.00	44.00
4.16	0	2.0	-	6.0	14.0	40.00	51.00
5.20	1.0	3.0	-	6.0	15.50	42.00	54.00
6.24	2.0	4.0	-	7.5	16.00	44.50	56.00
7.28	4.5	5.0	-	7.5	17.00	46.00	60.00
8.32	5.5	6.0	-	8.0	18.00	49.00	64.00
9.36	6.0	7.0	-	8.5	19.00	53.00	66.00
10.40	7.0	7.5	-	9.0	21.00	56.00	69.00
12.48	7.5	8.0	-	10.0	22.50	60.00	73.00
15.60	8.0	9.0	-	11.5	24.00	64.00	-
18.72	9.5	10.0	-	12.5	26.00	68.00	-
20.80	10.0	11.5	0	15.0	32.00	-	-
26.00	11.0	12.5	0.50	18.0	40.00	-	-
31.20	12.0	14.0	0.60	20.0	50.00	-	-
41.60	13.5	17.0	0.80	22.0	-	-	-
52.00	17.0	22.0	0.90	25.0	-	-	-
72.80	20.0	26.0	1.00	32.0	-	-	-
83.20	22.0	30.0	2.00	-	-	-	-
93.60	26.0	34.0	4.00	-	-	-	-
104.00	31.0	37.0	7.00	-	-	-	-
110.25	36.0	41.0	9.0	-	-	-	-
114.40	39.0	47.0	10.0	-	-	-	-

$$(Nu)_D = \left\{ 0.60 + 0.378(Ra_D)^{1/6} \left[1 + \left(\frac{0.559}{Pr} \right) 9/16 \right]^{-8/27} \right\}^2 \quad (16)$$

Equation (16) can be used to fit the experimental data if the $\frac{1}{6}$ th power in Rayleigh is changed to 0.20 and 0.228 for distilled water and chromium (hydrous) oxide particle suspension in distilled water, respectively. The experimental data have also been compared with the analytical expression available for free-convection^[21]. For horizontal cylinder, we obtain

$$(Nu)_D = 0.637(Ra_D)^{1/4} \left(1 + \frac{0.861}{Pr} \right)^{1/4} \quad (17)$$

$$(Nu)_D = 0.678(Ra_D)^{1/4} \left(1 + \frac{0.952}{Pr} \right)^{1/4} \quad (18)$$

The experimental data for electroconvection can be fitted with the analytical expression if the $\frac{1}{4}$ th power in Rayleigh is changed to 0.33 and 0.37 for distilled water and chromium (hydrous) oxide particle suspension in distilled water, respectively. This result is shown in curves 4 and 5 in Figs. 10 and 11 respectively. Fig. 12 represents the predicted correlation for experimental results presented in Fig. 8. The experimental result has been analysed using relation proposed by Fujii and Imura^[22]. They found that the equation

$$(N\bar{u})_L = 0.56(Gr_L Pr \cos \theta)^{1/4} \quad (19)$$

applies in the range

$$10^5 < Gr_L Pr \cos \theta < 10^{11} \quad \text{and} \quad 0 \geq \theta \geq 89^\circ,$$

where θ is measured from the vertical. The experimental data for electroconvection can be fitted with the predicted relation if the $\frac{1}{4}$ th power in Rayleigh is changed to 0.38 and Raleigh coefficient to 0.32 respectively. This gives a predicted correlation of

$$Nu_{el} = 0.32(Ra \cos \theta)^{0.38} \quad (20)$$

It is seen that the predicted results fall below the measurements for Rayleigh number greater than 8.70×10^9 , but there is nevertheless a good agreement to about 80%.

Table III - A comparison of the exponent of the Rayleigh number for various predicted relations used for distilled water and spherical chromium (hydrous) oxide particle suspension in distilled water.

Curves in Figs. 10 and 11	Exponent of Ra_D in distilled water	Exponent of Ra_D in $Cr(OH)_3$ in distilled water
1	0.362	0.405
2	0.375	0.420
3	0.20	0.228
4	0.33	0.370
5	0.325	0.368

Turning now to the overall heat transfer from the heated platinum wire as shown above, the predicted values of the average Nusselt number for Rayleigh numbers from 10^8 to 9×10^9 agrees very well with the experimental value ($\cong 80\%$). The experimental result predicts a steeper rise in Nusselt number with increasing Rayleigh number, than the predicted ones. For distilled water, the exponent of Ra_D lies between 0.20 and 0.375. However, the exponent of Rayleigh number is increased when the chromium (hydrous) oxide particle is added to distilled water. The result is summarized in Table III.

The increase in exponent of the Rayleigh with respect to free-convection indicates that the efficiency of convection increases dramatically when an electric field is applied to free-convection motion. The efficiency of convection is further increased when chromium (hydrous) oxide particle is added to distilled water and

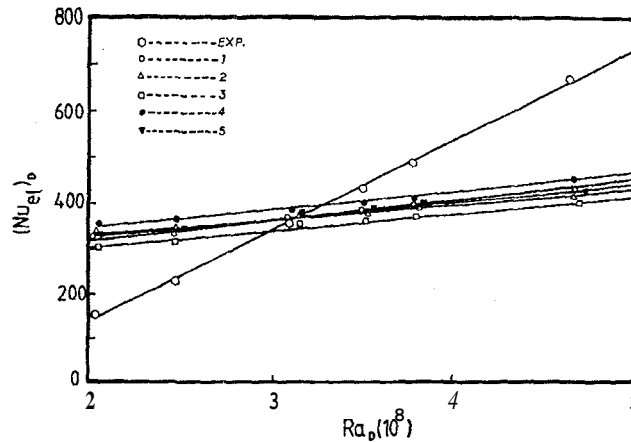


Figure 10. Correlation of data for electroconvective heat transfer from horizontal cylinder in distilled water. pH value = 5.60.

1. $(Nu_{el})_D = 0.32(Ra)^{0.362}$
2. $(Nu_{el})_D = (0.58) \left\{ \frac{1}{(Gr_D)^{0.05}} (Ra_D)^{0.375} \right\}$
3. $(Nu_{el})_D = \left\{ 0.60 + 0.387(Ra_D)^{0.20} \left[1 + \left(\frac{0.559}{Pr} \right)^{0.56} \right]^{-0.30} \right\}$
4. $(Nu_{el})_D = 0.637(Ra_D)^{0.33} (1 +$
5. $(Nu_{el})_D = 0.678(Ra_D)^{0.325} (1 +$

also, when the surface charge of the particle is increased.

V. Conclusion

Electroconvective heat transfer coefficient has been measured in spherical chromium (hydrous) oxide particle suspension in distilled-water under the influence of ac and dc fields. An increase in heat transfer coefficient is observed in an a-c field and the increase is extremely quick for pH values away from the isoelectric point (i.e.p. of $Cr(OH)_3 \cong 7.3$). Similar trend is also noticed in a dc field at very weak field strength. An increase in concentration increases the heat transfer coefficient and a similar effect is also noticed when the inclination of the cylinder is changed from horizontal to vertical position. The experimental data for horizontal cylinder have been correlated by dimensional analysis. The data have been compared with the empirical correlations and analytical expressions and a good agreement ($\cong 80\%$) is obtained for Rayleigh number up to 9×10^9 . It is observed that the empirical correlation and analytical expression for electroconvection follow laws analogous to free convection with the exception that the $\frac{1}{4}$ th power in Ra_D has been changed and adjusted between 0.20 and 0.42. The higher power

law in Rayleigh suggest that the efficiency of convection increases dramatically when an electric field is applied to free-convection. The efficiency of convection is further increased when the surface charge of the particle is increased. An efficiency of 55% is observed in a dc field corresponding to pH value of 11.10 and an electric field gradient of 8.32 (V/cm)(= 0.08 V). In the case of an ac field, an efficiency of 54% is observed corresponding to an electric field gradient of 5.20×10^2 (V/cm)(= 5V) and for a pH value of 11.20. Thus the electric field gradient required for an ac convection is roughly 63 times larger than the dc convection, and this is due to the relaxation phenomena associated with the ac field. Nevertheless, convection in charged suspension can be induced by an extremely low fields and field gradients and this low value eliminates any 'stirring effect' which may result due to electrolysis.

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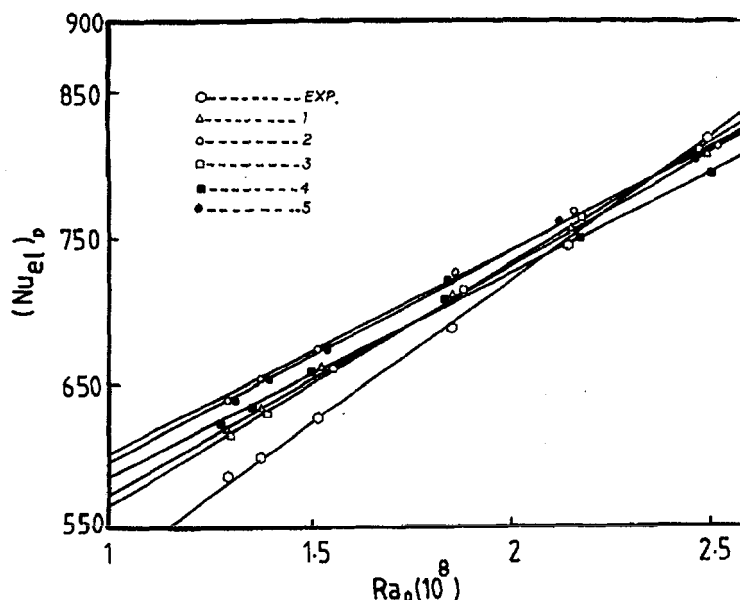


Figure 11. Correlation of data for electroconvective heat transfer from horizontal cylinder in chromium (hydrated) oxide particle suspension in distilled water. pH value = 11.20.

$$1. (Nu_{el})_D = 0.32(Ra_D)^{0.405}$$

$$2. (Nu_{el})_D = \left\{ (0.58) \frac{1}{(Gr_D)^{0.08}} (Ra_D)^{0.42} \right\}$$

$$3. (Nu_{el})_D = \left\{ 0.60 + 0.387(Ra_D)^{0.228} \left[1 + \left(\frac{0.559}{Pr} \right)^{0.56} \right]^{-0.30} \right\}^2$$

$$4. (Nu_{el})_D = 0.637(Ra_D)^{0.37} \left(1 + \frac{0.861}{Pr} \right)^{-0.25}$$

$$5. (Nu_{el})_D = 0.678(Ra_D)^{0.368} \left(1 + \frac{0.952}{Pr} \right)^{-0.25}$$

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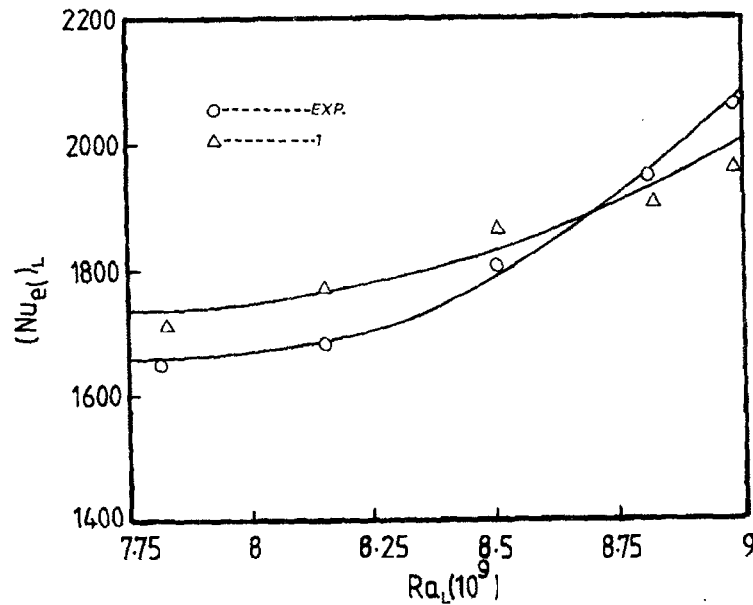


Figure 12. Correlation of experimental data presented in Fig. 8. The data refer to $U = 0.1$ v dc, $\text{pH} = 10.70$; 1. $(Nu_{el})_L = 0.32(Ra_L \cos \theta)^{0.38}$, $10 \leq \theta \leq 90$.

Nomenclature

A	=	surface area of the platinum wire
$\text{Cr}(\text{OH})_3$	=	chromium (hydrous) oxide particle
Cu	=	copper
D	=	diameter of the cylinder
U	=	voltage applied to the cylinder
E_r	=	radial electric field
E_s	=	electric field on the surface of the pt-wire
f_{el}	=	electrophoretic force
g	=	gravitational acceleration vector
Gr	=	Grashof number, $g\beta T_d D^3 / \nu^2$
h_f	=	free-convective heat transfer coefficient
h_{el}	=	electroconvective heat transfer coefficient
I	=	current through the pt-wire in absence of E
I_{el}	=	current through the pt-wire in presence of E
i.e.p.	=	isoelectric point
L	=	length of the cylinder
Nu	=	Nusselt number, $h_f D / \lambda$
Nu_{el}	=	electric Nusselt number, $h_{el} D / \lambda$
n_p	=	number of $\text{Cr}(\text{OH})_3$ particles/ m^3
Pr	=	Prandtl number, ν / α
Pt	=	platinum wire
q	=	charge of the $\text{Cr}(\text{OH})_3$ particle
Q_{el}	=	rate of heat transfer in presence of E
r_c	=	radius of the cylinder
r_p	=	radius of the $\text{Cr}(\text{OH})_3$ particle
R	=	resistance of the pt-wire
Ra	=	Rayleigh number, $g\beta T_d D^3 / \nu^2 Pr$
T_d	=	temperature difference between the pt-wire and the surrounding medium
T_b	=	temperature of the surrounding medium
T_w	=	temperature of the pt-wire

Greek Symbols

α	=	thermal diffusivity
β	=	thermal expansion coefficient
θ	=	angle of inclination from the vertical
ν	=	kinematic viscosity
μ	=	absolute viscosity
ρ	=	mass density
ρ_p	=	radius of the $\text{Cr}(\text{OH})_3$ particle
ϵ	=	permittivity of the fluid
λ	=	thermal conductivity
γ	=	efficiency of electroconvection