Plateau Regime Transport for Elongated Tokamaks with High Longitudinal Particle Flow

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The particle and heat transport in a transverse circular cross-section tokamak, with high longitudinal particle velocities, were extensively studied in many works. Here we are studying the particle and heat transverse fluxes in tokamak configurations with an elongated cross-section. This work is a generalization of the Wong and Burrel [Phys. Fluids 25, 1863 (1982)] study for this case.

I. Introduction

The neutral beam injection is one of the main methods of the auxiliary plasma heating and current drive in tokamaks[1]. Due to the beam injection, the plasma rotates in the toroidal direction and then new interesting plasma dynamics problems appear, mainly in the transport theory. Although the plasma toroidal velocity is supposed, now[2], to be smaller than the sound velocity, since it is convenient to improve the power deposition and correspondingly to increase the ion temperature, it is necessary to know the transport coefficients in a tokamak plasma with particle sound toroidal velocities. The poloidal and toroidal rotation in a tokamak have been under consideration for decades[3-7], both in collisional[3,5,6], and weakly collisional[4,7] plasmas.

The particle and heat transport in a transverse circular cross-section toroidal plasma, with particle high longitudinal velocities, were studied earlier[7-10]. Here, we are considering the particle and heat transverse fluxes in toroidal configurations with an elongated cross-section. This work is a generalization of the Wong and Burrel[7] study for this case.

II. Basic eqiiations

As in Ref. [7], we use the drift kinetic equation[6]

$$\frac{d\mathbf{r}}{dt}\cdot\nabla f+\frac{dv_\parallel}{dt}\frac{\partial f}{\partial v_\parallel}+\frac{dv_\perp^2}{dt}\frac{\partial f}{\partial v_\perp^2}=C, \tag{1}$$

where

$$\frac{d\mathbf{r}}{dt}=v_\parallel\mathbf{h}+\mathbf{V}_E+\left(\frac{v_\perp^2}{2}+v_\parallel^2\right)\frac{1}{\omega_B}[\mathbf{h}\times\nabla lnB],$$

$$\frac{dv_\parallel}{dt}=\frac{e}{M}E_\parallel+\frac{v_\perp^2}{2}div\mathbf{h}+v_\parallel\mathbf{V}_E\quad\nabla lnB,$$

$$\frac{dv_\perp^2}{dt}=-v_\perp^2(v_\parallel div\mathbf{h}+div\mathbf{V}_E+\mathbf{V}_E\cdot\nabla lnB).$$

Here $\mathbf{h}=\mathbf{B}/B$, $\mathbf{V}_E=c[\mathbf{E}\times\mathbf{h}]/B$, $\omega_B=eB/Mc$.

We wish to find a solution of Eq. (1) in the coordinate system $x^i=\{r,\theta,\zeta\}$ with the circular magnetic surfaces and with the straight magnetic field lines. In this case, the radial coordinate r is equal to $r=\sqrt{l_1l_2}$, where l_1 and l_2 are the minor and the large half-axis of the torus elliptical cross-section, respectively. The values θ and ζ are the poloidal and toroidal angles.

In this coordinate system, the magnetic field has the following form

$$B=B_s\left(1+\epsilon^*cos\theta+\frac{\epsilon^*A}{2}cos^2\theta\right), \tag{2}$$

where $A=\epsilon^*(exp(2\eta)-1)/q^2$, $\epsilon^*=\epsilon exp(-\eta/2)$, $\eta=ln(l_2/l_1)$, $\epsilon=r/R$ and R is the torus large radius. In the circular cross-section tokamak, the values of A and η are equal to zero and $\epsilon^*=\epsilon$.

We'll find tlie solution of Eq.(1), in the r-approximation, with the equilibrium Maxwell distribution function

$$f_0 = n\left(\frac{M}{2\pi T}\right)^{\frac{3}{2}} exp\left(-\frac{Mw^2}{2T}\right), \qquad (3)$$

where $W^2 = v_\perp^2 + w_\parallel^2$, $w = v - V_\parallel$, $f = f_0 + f_1$, $V_\parallel$ is the particle mean velocity along the rnagnetic field. Using Eq.(2), we derive the solution of Eq.(1) as

$$f_1 = f_0\frac{M}{T}\left\{\frac{P}{u}\left[\frac{e}{M}\psi\left(-u+\frac{q}{\epsilon}U\right)+q\frac{\epsilon^*}{\epsilon}D\left(cos\theta+\frac{A}{4}cos2\theta\right)\right]+\right. \qquad (4)$$

$$\left.+\pi\delta(u)\frac{\partial}{\partial\theta}\left[\frac{e}{M}\left(-u+\frac{q}{\epsilon}U\right)\left(\psi_1+\frac{1}{2}\psi_2\right)+q\frac{\epsilon^*}{\epsilon}D\left(cos\theta+\frac{A}{8}cos2\theta\right)\right]\right\},$$

where

$$D = \frac{v_\perp^2}{2}U + (w_\parallel + V_\parallel)\left[w_\parallel\left(U - \frac{\epsilon}{q}V_\parallel\right) + V_\parallel\left(U - U_\theta + U_p\right)\right],$$

$$\psi = \psi_1 + \psi_2, \qquad \psi_1 = \psi_{1s}sin\theta + \psi_{2c}cos\theta, \qquad \psi_2 = \psi_{2s}sin2\theta + \psi_{2c}cos2\theta,$$

$$u = w_\parallel + \frac{q}{\epsilon}(U_\theta - U_p), \qquad U = U_\theta + U_T\left(\frac{Mw^2}{2T} - \frac{5}{2}\right),$$

$$U_p = \frac{1}{M\omega_B n}\frac{\partial p}{\partial r}, \qquad U_T = \frac{1}{M\omega_B}\frac{\partial T}{\partial r}.$$

Here tlie poloidal velocity U_θ is equal to

$$U_\theta = \frac{\epsilon}{q}\frac{V^\theta}{h^\theta},$$

V^θ and h^θ are the contravariant coinponents of tlie velocity V and tlie unit vector h, where

$$\mathbf{V} = V_\parallel\mathbf{h} + \mathbf{V}_E + \mathbf{V}_p, \qquad \mathbf{V}_p = \frac{1}{enB}[\mathbf{h}\times\boldsymbol{\nabla}p], \qquad (5)$$

The values U_p and U_T are the particle drift velocities. We suppose, also, that the electric field has the form E $= -$V\$. The expressions P/u and $\delta(u)$ in Eq.(4) denote the Cauchy principal value and the delta-function, respectively.

III. Absence of the high poloidal velocity regime

It is well-known that for a circular cross-section tokamak, the poloidal velocity U_θ is rnuch smaller than the plasma sound velocity c_s, where $c_s^2 = (T_i + T_e)/M_i$ [9], [11], for both a collisional and for a weakly-collisional regimes. We can prove that in tlie elliptical torus this value will be of the level of the drift velocities.

Suppose, that U_θ is greater than the drift velocities U_p and U_T. In this case, we can omit the drift terms in Eq.(4) and we have,

$$U = U_\theta, \qquad u = w_\parallel + \frac{q}{\epsilon}U_\theta, \qquad (6)$$

$$D = \frac{v_\perp^2}{2}U_\theta + \left(U_\theta - \frac{\epsilon}{q}V_\|\right)\left[u^2 + u\left(V_\| - \frac{2q}{\epsilon}U_\theta\right) + \frac{q}{\epsilon}U_\theta\left(\frac{q}{\epsilon}U_\theta - V_\|\right)\right].$$

Assume, also, that

$$\psi = \epsilon^*(\tilde{\psi}_{1c}cos\theta + \tilde{\psi}_{2c}cos2\theta). \tag{7}$$

We need to find the radial particle flux

$$\Gamma_{nj} = \int d\mathbf{v} < f_i \frac{dr_j}{dt} >, \tag{8}$$

where "j" denotes the kind of particle and

$$< ... >= \oint(...)\frac{dl}{B}/\oint\frac{dl}{B} \tag{9}$$

The integration here is along the magnetic field line.

As it can be seen, from Eq.(1), the radial velocity is

$$\frac{dr_j}{dt} = \frac{\epsilon^* c}{\epsilon RB}\left[\tilde{\psi}_{1c}sin\theta + 2\tilde{\psi}_{2c}sin2\theta + \frac{M_j}{e_j}\left(\frac{v_\perp^2}{2} + v_\|^2\right)\left(sin\theta + \frac{A}{2}sin2\theta\right)\right]. \tag{10}$$

Using now Eqs.(4), (8)-(10), we ohtain

$$\Gamma_{nj} = -\frac{\epsilon^{*2}}{\epsilon^2}\frac{qn\sqrt{\pi}}{R\omega_{Bj}v_{tj}^3}U_\theta exp\left(-\frac{q^2U_\theta^2}{\epsilon^2 v_{T_j}^2}\right)\left\{\frac{T_j^2}{M_j^2}\left(1 + \frac{A^2}{4}\right) + \left[\frac{e_j}{M_j}\tilde{\psi}_{1c} + \frac{T_j}{M_j} + \right.\right. \tag{11}$$

$$\left.\left. + \left(V_\| - \frac{q}{\epsilon}U_\theta\right)^2\right]^2 + 2\left[\frac{e_j}{M_j}\tilde{\psi}_{2c} + \frac{AT_j}{4M_j} + \frac{A}{4}\left(V_\| - \frac{q}{\epsilon}U_\theta\right)^2\right]^2\right\},$$

where $v_{T_j}^2 = 2T_j/M_j$, q is the safety factor, $q = \phi'/\chi'$, ϕ and χ are the toroidal and poloidal inagnetic fluxes. From Eq.(11), we see, that the ambipolarity condition

$$\sum_j e_j\Gamma_{nj} = 0 \tag{12}$$

can not be fulfilled, as we have in Eqs.(11),(12) the sum of the squared values. Consequently, the stationary poloidal velocities around the sound velocities in the elliptic toltamalt, as in a circular cross-section tokamalt, do not exist.

IV. Electric field potential

We use now that $U_{\theta j} \ll$ c,. In this case, we can derive the perturbed particle densities

$$\tilde{n}_j = \int d\vec{v}f_{1j} = -\frac{e_j n}{T_j}\tilde{\psi} - \frac{\epsilon^* n M_j}{T_j}V_\|^2\left(cos\theta + \frac{A}{4}cos2\theta\right). \tag{13}$$

I-Iere, the small terms, that are of the same order of magnitude as the drift terms, are omitted.

From the quasineutrality condition, we have

$$\sum_j e_j n_j = 0\ . \tag{14}$$

After some algebra, the potential ψ can be calculatecl

$$\tilde{\psi} = -\epsilon^*\frac{\sum_j e_j n_j \frac{M_j V_{j\|}^2}{T_j}}{\sum_j \frac{e_j^2 n_j}{T_j}}\left(cos\theta + \frac{A}{4}cos2\theta\right). \tag{15}$$

Now, we can write the values f_{1j} and dr_j/dt in the form

$$f_{1j} = -\frac{\epsilon^*}{\epsilon}\frac{q\sqrt{\pi}}{v_{Tj}}\delta(u)\left[U_{\theta j} + U_{Tj}\left(x - \frac{5}{2}\right)\right](x + G_j)\left(sin\theta + \frac{A}{4}sin2\theta\right), \quad (16)$$

$$\frac{dr_j}{dt} = \frac{\epsilon^*}{\epsilon}\frac{cT_j}{e_j RB}(x + G_j)\left(sin\theta + \frac{A}{2}sin2\theta\right). \quad (17)$$

Here, $x = M_j v_\perp^2/2T_j$ and

$$G_j = \frac{M_j V_{\parallel j}^2}{T_j}\left(1 - \frac{e_j}{M_j}\frac{\sum_k \frac{e_k n_k M_k}{T_k}}{\sum_k \frac{e_k^2 n_k}{T_k}}\right). \quad (18)$$

V. **Particle and heat** fluxes.

Now, we can get the radial particle and heat fluxes. The last one is defined below

$$\Gamma_{Tj} = \int d\mathbf{v} < f_j \frac{M_j w^2}{2}\frac{dr_j}{dt} > \quad (19)$$

Using Eqs.(16), (17) and the definitions Eqs.(8) and (19), we find

$$\Gamma_{nj} = -\frac{n_j v_{Tj} q\sqrt{\pi}}{2R\omega_{Bj}}\frac{\epsilon^{*2}}{\epsilon^2}\left(1 + \frac{A^2}{16}\right)\left[U_{\theta j}\left(1 + G_j + \frac{1}{2}G_j^2\right) + \frac{U_{Tj}}{2}\left(1 - G_j - \frac{3}{2}G_j^2\right)\right], \quad (20)$$

$$\Gamma_{Tj} = -\frac{n_j v_{Tj} q T_j\sqrt{\pi}}{2R\omega_{Bj}}\frac{\epsilon^{*2}}{\epsilon^2}\left(1 + \frac{A^2}{8}\right)\left[U_{\theta j}\left(3 + 2G_j + \frac{1}{2}G_j^2\right) + \frac{U_{Tj}}{2}\left(9 + 2G_j - \frac{1}{2}G_j^2\right)\right]. \quad (21)$$

If we make the ellipticity equal to zero, i.e. $\epsilon^* = \epsilon$ and $A = 0$, we obtain the Wong and Burrel [7] expressions for the fluxes.

VI. **Conclusion.**

As it follows from tlie ambipolarity condition, Eq.(12), and from tlie particle fluxes, Eq.(20), this condition is the same, as in tlie circular cross-section tokamak, and hence the ambipolar poloidal velocity value is the same. The ellipticity influences only the values of the fluxes. For tlie usual value $A \approx 1$ (as in JET - Joint European Torus), this influence is given by the ratio ϵ^{*2}/ϵ^2, which equals the ratio l_1/l_2. This is the decreasing factor of the neoclassical fluxes in the elliptical tokamak. We see, also, that the fluxes are proportional to the temperature gradients T_j'. This can be shown by using tlie ambipolarity condition, Eq.(12), and the substitution of the obtained expressions for $U_{\theta j}$ into Eqs.(20), (21).

References

1. S. D. Scott, M. Bitter, R. J. Fonk, R. J. Goldston et al., Plasma Phys. and Contr. Nucl. Fusion Res., 1988, Proc. 12th Int. Conf. Nice, 1, 655, IAEA, Vienna (1989).
2. W. B. F. Core, P. Van belle, G. Sadler, Proc. 14th Int. Conf. Madrid,.**II.D.**, part I, 49, IAEA, Vienna (1987).
3. T. Stringer, Phys. Rev. Lett. 22, 770, (1969).
4. R. D. Hazeltine, F. L. Hinton, Phys. Fluids 16, 1838, (1973).
5. R. D. Hazeltine, Phys. Fluids 17, 961, (1974).

6. A. B. Mikhailovskii, V. S. Tsypin, Sov. Phys. JETP. 56, 75, (1982).
7. S. K. Wong, K. H. Burrel, Phys. Fluids 25, 1863, (1982).
8. S. K. Wong, F. L. Hinton, Phys. Rev. Lett. 52, 827, (1984).
9. V. S. Tsypin, Sov. J. of Plasma Phys. 11, 201, (1985).
10. V. S. Tsypin, Sov. J. of Plasma Phys. 14, 466, (1987).
11. A. B. Mikhailovskii, Sov. J. of Plasma Phys. 9, 346, (1982).