Alfvén Wave Heating and Current Drive Analysis in Magnetized Plasma Structures

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In this paper a method for the analytical treatment of the dielectric permeability tensor is developed and analysis of the wave heating and current drive is carred out for magnetized plasma structures with an equilibrium field aligned current. Electron Landau and transit time magnetic pumping absorptions of waves is consiclered. Simple expressions for the current drive hy the Alfvén and fast waves are obtained. This current drive is discussed in terms of "gradient" effects, including the effect of the resonant ancl nonresonant current clrive. The infiuence of all electric fielcl on current drive is clemonstrated on the basis of numerical solutions of the Binetic equation with landau-Folilier-Planck collision operator.

I. Introduction

Nowadays, Alfvén waves (AW) are widely used for plasma production, resonant heating of magnetized plasmas and current clrive in laboratory experiments[1-3]. In the nearest future, this program will be under investigation in the TCA/Br - Brazilian tokamak with Alfvén wave heating[4].

Past years, the theory of the Alfvén waves had been intensively studied (see the initial paper[5] and reviews[2,6,7], and references therein) not only in connection with laboratory experiments, but also in space plasmas applications[8-11]. In space physics, there are some proposals about particle acceleration ancl solar chromosplieric and coronal heating by Alfvén waves[10-13].

The classical Alfvén waves[5] in homogeneous plasmas are degenerated modes. The properties of Alfvén waves depend strongly on the magnetic field structure and on the ratio of the Alfvén phase velocity to the electron thermal velocity(c_A/v_{Te})[2,6,7], where $c_A = B_0/\sqrt{4\pi N_i M_i}$, and $v_{Te} = \sqrt{T_e/m_e}$. The waves in the Alfvén waveband are divided into fast magneto-sonic waves (FMSW), global Alfvén waves (GAW), wliich depend on the magnetic field structure, kinetic (KAW) and quasielectrostatic (QEAW) (or "cold") Alfvén waves in magnetized plasmas with hot $(c_A \ll v_{Te})$ ancl cold $(c_A \gg v_{Te})$ electrons, and, finally, surface Alfvén modes (SAM), which do not have a radial wave structure. This last mode appears only in bounded or layered plasmas.

To study different applications of the Alfvén waves for plasma heating and current drive, several important questions arise and their theoretical solution is of paramount importance to understand the general AW effects in plasmas:

- What is the structure of Alfvén waves in inhomogeneous plasmas?
- How Landau damping, transit time magnetic pumping (TTMP), and collisional dissipation of AW vary with the curvature and the gradients of the ambient magnetic field, and with the changing of the plasma density and temperature profiles?
- What will be the distribution of absorbed power and current drive profile in different magnetic field configurations?

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To solve the problems mentioned above in collisional plasmas, the magneto-hydrodynamic (MHD) model of the plasma is used[7,8,14]. In general, laboratory or space plasmas are weakly collisional. It means that the electron-ion collision frequency is smaller than the wave frequency ($\nu_{ei} \ll \omega$) and the parallel wavelength along the magnetic field lines ($1/k_{\|}$) is smaller than the mean free path of electrons ($1/k_{\|} \ll v_{Te}/\nu_{ei}$). For weakly collisional plasmas most of the wave-particle interaction effects and the wave heating and current drive problems can be solved in the framework of the Vlasov-Maxwell's set of equations (see, for instance,[2,6,7])

$$\frac{\partial F_\alpha}{\partial t} + \vec{v}\frac{\partial F_\alpha}{\partial \vec{r}} + \frac{e_\alpha}{m_\alpha}\left\{\vec{E} + \frac{1}{c}\left[\vec{v}\times\vec{B}\right]\right\}\frac{\partial F_\alpha}{\partial \vec{v}} = \hat{S}t\{F_\alpha\}; \qquad (1)$$

$$curl\vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}; \qquad -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} = curl\vec{E}. \qquad (2)$$

Here, $F_\alpha, e_\alpha, m_\alpha$, are the distribution function, charge and mass of ions or electrons, $\vec{r}$, $\vec{v}$, are the configuration space vector-coordinates; $\vec{E}$, $\vec{B} = \vec{B}_0 + \tilde{\vec{B}}$, $\vec{j} = \vec{j}_0 + \tilde{\vec{j}}$, are the electric and magnetic fields, and the current density, which are divided in quasistationary and oscillating parts; $\hat{S}t\{F\}$ is the collision operator in the Landau form[14,15]. Throughout the paper the CGSM system of units is used.

In complicated magnetic field geometries (for instance, in toroidal one) the distribution function $F = F(t, \vec{r}, \vec{v})$ depends on seven variables and, moreover, it is necessary to include the specific effect of ion and electron periodic motions[16] along the magnetic field lines. The solution of this problem is a formidable task in the frame of Eqs. (1,2).

In the simplest approach, which ignores the drift motion across the magnetic surfaces and assumes that the toroidicity parameter $\epsilon = r/R \ll 1$ is very small, it is possible to use a cylindrical plasma model, which is also important for space plasma applications. If the Larmor radii of ions and electrons are small in comparison with the scale of the plasma inhomogeneity ($\rho_{i,e} = v_{Ti,e}/\omega_{ci,e} \ll l_{N,T} = N/(dN/dr), T/(dT/dr)$, where $\rho_{i,e}$ are ion and electron Larmor radii), which means that the plasma is magnetized, the problems of Alfvén wave heating and current drive may be solved in closed form. Some results were discussed earlier in References [2,6,7,17].

In the second part of this paper, an analytical method for evaluation of the dielectric permeability tensor-operator will be presented, taking into account drift corrections (related to gradients of equilibrium parameters) and the equilibrium plasma current. In the third part, the MHD approach for the dielectric permeability tensor in complicated plasma geometries will be demonstrated. In the fourth part, the Alfvén wave current drive for magnetized plasmas will be analyzed using the geometric optics approximation for the radial coordinate of the wave fields and some results of the numerical solution of the Fokker-Planck equation for electrons will be presented.

II. Dielectric permeability tensor operator of magnetized plasmas

The procedure to solve the set of the equations (1-2) can be reduced to Maxwell's equations only if we are able to find the relations between the current density oscillations $\tilde{\vec{j}}$ and the perturbed electric field $\tilde{\vec{E}}$ from Vlasov's equation. Usually, these relations ($\tilde{j}_s = (4\pi/\omega)c\epsilon_{sk}\tilde{E}_k$) are defined through the dielectric tensor components, ϵ_{sk}, which dependent on the equilibrium plasma parameters and the magnetic field configuration.

In this section, we will use the cylindrical model of the magnetized plasma with one or a few kinds of ions. The equilibrium plasma current $\vec{j}_0$ is aligned with the equilibrium magnetic field $\vec{B}_0$. Under the equilibrium conditions, we can assume that the current is formed due to the electron motion with the velocity, v_0, in a background of non-drifting ions. Therefore, the electrons are the source of the equilibrium current $\vec{j}_0$ in the plasma. Further, we will assume that v_0 is smaller or of the order of the electron thermal velocity v_{Te}. According to the electrodynamical laws, the longitudinal current produces the poloidal magnetic field $B_{0,\theta}(r)$. In this case, the magnetic surfaces may be considered as circular and concentric cylinders. For the described model, it is possible to use a cylindrical coordinate system:

$$X = r\,cos\theta, \qquad Y = r\,sin\theta, \qquad Z = z.$$

where r is the radius of a magnetic surface. The stationary magnetic field configuration $\vec{B}_0$ is chosen as in the Reference [6]:

$$B_{0,r} = 0, \qquad B_{0,\theta} = h_\theta B_0, \qquad B_{0,z} = h_z B_0.$$

where h_θ and h_z are, respectively, the poloidal and axial projections of the magnetic field unit vector: $\vec{h} = \vec{B}_0/B_0$. The helical magnetic field line is twisted over such cylindrical magnetic surfaces with radius r. This model is reasonable for tokamaks and solar loops with low pressure $\beta \ll 1$ and small toroidicity parameter $\varepsilon = r/R \ll 1$. A collisionless wave dissipation will be considered and the "plateau" conditions for tokamaks with weak collisions will be assumed

$$\sqrt{\epsilon/2}v_{Te}/R_0 q \ll \nu_{ei} \ll \omega, \qquad q = rh_z/R_0 h_\theta$$

where q is the tokamak safety parameter and R_0 is the tokamak major radius, which is equal to the length, L, of the plasma cylinder divided by 2π. Thus, the formal transformation of the wave frequency $\omega \rightarrow \Omega + i\nu_{ef}$, which means to use the Krook's approximation for the collision operator $\hat{S}t\{f\}$, can be done.

In the discussed plasma model, the radial inhomogeneity equilibrium density $N_0(r)$, temperature $T_{e,i}(r)$ of the plasma and the poloidal magnetic field $B_{0,\theta}(r)$ are taken into account. On the other hand, it is supposed that these equilibrium parameters do not depend on time t and are homogeneous in the poloidal angle, θ, and axial coordinate, z. Consequently, all dependence on time and poloidal and axial coordinates of the perturbed electric and magnetic fields ($\tilde{\vec{E}}$ and $\tilde{\vec{B}}$) in linear approximation appears through $exp[i(m\theta + kz - \Omega t)]$, due to the plasma homogeneity over these variables. Because of this homogeneity, we worked in the one-plane-wave approximation (allowed by the superposition principle valid for Maxwell equations). Here Ω is the wave frequency, m/r and k are the poloidal and the longitudinal projections of the wave vector, respectively.

We begin to find the linearized solution of the Vlasov's kinetic equation (1) for the distribution function of the electrons and ions, $F = F_0 + \tilde{f}$, as a sum of the equilibrium distribution function and a perturbed part $\tilde{f}$, which is proportional to the wave fields as $\propto \tilde{E}_{r,\theta,z} exp[i(m\theta + kz - \Omega t)]$. The distribution function depends on the radius and the velocity space variables. We use a special system of coordinates in velocity space $(v_\perp, \sigma, v_\parallel)$, where $v_\perp$ (perpendicular) and σ are local-polar velocity coordinates, and $v_\parallel$ (parallel) is the projection of $\vec{v}$ on the $\vec{B}$ lines. Unlike Ross et al.[6], we take into account, for the equilibrium distribution, the first order drift corrections to the stationary local Maxwell distribution of each plasma component α:

$$F_{0\alpha} = F_{M\alpha} + F_{b\alpha} sin\sigma, \tag{3}$$

where

$$F_{M\alpha} = \frac{N_{0\alpha}}{(2\pi v_{T\alpha}^2)^{1.5}} exp\left[-\frac{v_\perp^2 + (v_\parallel - v_{0\alpha})^2}{2v_{T\alpha}^2}\right], \qquad F_{b\alpha} = \frac{v_\perp}{\omega_{c\alpha}}\frac{dF_{M\alpha}}{dr}$$

For the set of the vectors $\vec{A} = \{\tilde{\vec{E}}, \tilde{\vec{B}}, \tilde{\vec{j}}, \vec{v}\}$, we use, conveniently, the normal A_1, binormal A_2 and parallel A_3 projections relative to the direction of the magnetic field, which are transformed to the cylindrical components A_r, A_θ, A_z through the following formulae:

$$A_1 = A_r, \quad A_2 = A_\theta h_z - A_z h_\theta, \quad A_3 = A_z h_z + A_\theta h_\theta.$$

In the velocity space the coordinates $v_{1,2,3}$ are defined through the local polar coordinates:

$$v_1 = v_\perp cos\sigma, \qquad v_2 = v_\perp sin\sigma, \qquad v_3 = v_\|,$$

wliicli are variecl in tlie region:

$$0 \le v_\perp < \infty, \qquad 0 \le \sigma < 2\pi, \qquad -\infty < v_\| < \infty.$$

Foi. future analysis we expand the perturbed distribution function in Fourier series over angle a in the velocity space,

$$\tilde{f} = f_0 + \sum_{l\neq 0}[f_{r,l}\,cos(l\sigma) + f_{b,l}\,sin(l\sigma)]\}\,exp[i(m\theta + kz - \Omega t)].$$

The first three coefficients $f_{0,r,b}$ in the above equation are most important for the dielectric tensor evaluation. f_0 is related to tlie perturbed distribution function averaged over particle gyrations in the magnetic field and f_r, f_b (where inclex "1" is omittecl) are corrections of tlie perturbed clistribution function, which are related to the particle gyration. Assuming that tlie Larmor radius is much smaller than the radial wave field inhomogeneity, we evaluate the set of the linear equations for $f_{0,r,b}$ from the Vlasov's equation (1):

$$i(v_\| k_\| - \omega)f_0 + \frac{h_\theta^2}{r}\left(v_\|^2\frac{\partial f_0}{\partial v_\perp} - v_\| v_\perp\frac{\partial f_0}{\partial v_\|}\right) + \frac{v_\perp}{2}\left(\frac{df_r}{dr} + \frac{f_r}{r} + ik_b f_b\right)$$

$$= -\frac{e}{m_\alpha}\left[E_3\frac{\partial F_M}{\partial v_\|} + \frac{E_2}{\omega_c}\frac{\partial}{\partial r}\left(F_M + \frac{v_\perp}{2}\frac{\partial F_M}{\partial v_\perp}\right) + \frac{v_\|}{cv_\perp}B_1F_b\right]; \qquad (4)$$

$$\omega_c f_b = -i(\omega - k_\| v_\|)f_r + v_\perp\frac{\partial f_0}{\partial r} - \frac{{h_\theta}^2 v_\|}{r}\left(v_\perp\frac{\partial f_0}{\partial v_\|} - v_\|\frac{\partial f_0}{\partial v_\perp}\right) +$$

$$\frac{e}{m_\alpha}\left[E_1\frac{\partial F_M}{\partial v_\perp} + \frac{B_2}{c}\left(\frac{v_\perp v_0}{v_T^2}F_M\right) - \frac{B_3}{c}F_b\right]; \qquad (5)$$

$$\omega_c f_r = i(\omega - k_\| v_\|)f_b - ik_b v_\perp f_0 - \frac{e}{m_\alpha}\left[E_2\frac{\partial F_M}{\partial v_\perp} + E_3\frac{\partial F_b}{\partial v_\perp} - \frac{B_1}{c}\left(\frac{v_\perp v_0}{v_T^2}F_M\right)\right]. \qquad (6)$$

Here, $k_\| =$ th, $+ h_\theta m/r$ and $k_b = h_z m/r - kh_\theta$ are the parallel and binormal components of the wave vector.

In the next step, combining the Equations (5,6) and (4) and taking into account only first order drift corrections $\propto \rho_\lambda/l_{N,T}$, we find expressions for f_r, f_b and f_0. These functions will be used for the calculations of the normal (j_1), binormal (j_2) and parallel (j_3) components of the oscillating current density:

$$j_1 = \sum_\alpha e_\alpha\int_0^{2\pi}d\sigma\,cos\sigma\int_{-\infty}^{+\infty}dv_\|\int_0^\infty v_\perp^2\tilde{f}_\alpha dv_\perp = \pi\sum_\alpha e_\alpha\int_{-\infty}^{+\infty}dv_\|\int_0^\infty v_\perp^2 f_r^{(\alpha)}dv_\perp; \qquad (7)$$

$$j_2 = \sum_\alpha e_\alpha\int_0^{2\pi}d\sigma\,sin\sigma\int_{-\infty}^{+\infty}dv_\|\int_0^\infty v_\perp^2\tilde{f}_\alpha dv_\perp = \pi\sum_\alpha e_\alpha\int_{-\infty}^{+\infty}dv_\|\int_0^\infty v_\perp^2 f_b^{(\alpha)}dv_\perp; \qquad (8)$$

$$j_3 = \sum_\alpha e_\alpha\int_0^{2\pi}d\sigma\int_{-\infty}^{+\infty}v_\| dv_\|\int_0^\infty v_\perp\tilde{f}_\alpha dv_\perp = 2\pi\sum_\alpha e_\alpha\int_{-\infty}^{+\infty}v_\| dv_\|\int_0^\infty v_\perp f_0^{(\alpha)}dv_\perp \qquad (9)$$

where the summation is carried out for all kinds of the plasma particles ($a =$ e, $i_1, \ldots$).

After integration of Eqs. (7-9) over the velocity space we can ohtain the dielectric tensor-operator $\hat{\epsilon}_{sk}$ using the following relations: $j_s = i(\Omega/4\pi)(\delta_{sk} - \hat{\epsilon}_{sk})E_k$, where δ_{sk} is the Kroneker symbol.

Here, we present all nine components of the dielectric permeability tensor. The most of them are differential operators on r, and they are labeled as $\hat{\epsilon}_{sk}$:

$$\hat{\epsilon}_{11} = 1 + \sum_\alpha \frac{\omega_{P\alpha}^2}{\omega_{c\alpha}^2}\left[1 + \frac{\omega^2}{\omega_{c\alpha}^2} - 2\frac{k_\| v_{0\alpha}}{\omega} + 2\frac{k_b^2 v_{T\alpha}^2}{\omega^2}(\Lambda_\alpha - 1)\right]; \tag{10}$$

$$\hat{\epsilon}_{12} = \sum_\alpha \frac{i\omega_{P\alpha}^2}{\omega\omega_{c\alpha}}\left[1 + \frac{\omega^2}{\omega_{c\alpha}^2} - \frac{k_\| v_{0\alpha}}{\omega} - \frac{(\chi_N + \chi_T)k_b v_{T\alpha}^2}{\omega\omega_{c\alpha}} + \frac{2k_b v_{T\alpha}^2}{r\omega\omega_{c\alpha}}(\Lambda_\alpha - 1)\frac{\partial}{\partial r}(r\ldots)\right]; \tag{11}$$

$$\hat{\epsilon}_{13} = -i\frac{k_b}{k_\|}\sum_\alpha \frac{\omega_{P\alpha}^2}{\omega\omega_{c\alpha}}\left\{\Lambda_\alpha - \frac{k_\| v_{0\alpha}}{\omega} - \frac{k_b v_{T\alpha}^2}{2\omega\omega_{c\alpha}}\left[\chi_T + 2\chi_N\Lambda_\alpha + \chi_T(1 + 2{Z_\alpha}^2)\Lambda_\alpha\right]\right\}; \tag{12}$$

$$\hat{\epsilon}_{21} = -\epsilon_{12} - i\sum_\alpha \frac{\omega_{P\alpha}^2}{\omega^2}\frac{k_b v_{T\alpha}^2}{\omega_{c\alpha}^2}\left\{\frac{4}{r}(\Lambda_\alpha - 1) - 2\chi_N\Lambda_\alpha - \chi_T[(1 + 2{Z_\alpha}^2)\Lambda_\alpha - 1]\right\}; \tag{13}$$

$$\hat{\epsilon}_{22} = \sum_\alpha \frac{\omega_{P\alpha}^2}{{\omega_{c\alpha}}^2}\left\{1 + \frac{\omega^2}{\omega_{c\alpha}^2} - \frac{\chi_1 v_{0\alpha}\omega_{c\alpha}}{\omega^2} - 2\frac{v_{T\alpha}^2}{\omega^2}(\Lambda_\alpha - 1)\frac{\partial}{\partial r}r\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\ldots\right)\right]\right.$$

$$\left. - \frac{\chi_N v_{T\alpha}^2}{r\omega^2}\left[1 + 2(\Lambda_\alpha - 1)\frac{\partial}{\partial r}(r\ldots)\right] - \frac{\chi_T v_{T\alpha}^2}{r\omega^2}\left[1 - [1 - (1 + 2{Z_\alpha}^2)\Lambda_\alpha]\frac{\partial}{\partial r}(r\ldots)\right]\right\}; \tag{14}$$

$$\hat{\epsilon}_{23} = \sum_\alpha \frac{\omega_{P\alpha}^2}{\omega\omega_{c\alpha}}\left\{(\Lambda_\alpha - \frac{k_\| v_{0\alpha}}{\omega})\frac{1}{k_\|}\frac{\partial}{\partial r} + \frac{\chi_N}{k_\|}\Lambda_\alpha + \frac{\chi_T}{2k_\|}[1 - (1 - 2{Z_\alpha}^2)\Lambda_\alpha]\right\}; \tag{15}$$

$$\hat{\epsilon}_{31} = -\epsilon_{13} + i\frac{k_b}{k_\|}\sum_\alpha \frac{\omega_{P\alpha}^2}{\omega\omega_{c\alpha}}\left[(1 - \Lambda_\alpha)\frac{k_\| v_{0\alpha}}{\omega}\right]; \tag{16}$$

$$\hat{\epsilon}_{32} = \frac{-1}{k_\| r}\sum_\alpha \frac{\omega_{P\alpha}^2}{\omega\omega_{c\alpha}}\left\{(1 - \frac{k_\| v_{0\alpha}}{\omega})\Lambda_\alpha - \frac{k_b v_{T\alpha}^2}{\omega\omega_{c\alpha}}\{\chi_N\Lambda_\alpha + \frac{\chi_T}{2}[1 + (1 + 2Z_\alpha^2)\Lambda_\alpha]\}\right\}\frac{\partial}{\partial r}(r\ldots); \tag{17}$$

$$\epsilon_{33} = \sum_\alpha \frac{\omega_{P\alpha}^2}{k_\| v_{T\alpha}}\left\{\left(1 + \frac{k_\| voe}{\omega}\right)\Lambda_\alpha - \frac{\chi_N k_b v_{T\alpha}^2\Lambda}{\omega\omega_{c\alpha}} - \frac{\chi_T k_b v_{T\alpha}^2[1 - (i - 2{Z_\alpha}^2)\Lambda_\alpha]}{2\omega\omega_{c\alpha}}\right\}; \tag{18}$$

where $\omega_{P\alpha}^2 = 4\pi N_{0\alpha}e_\alpha^2/M_\alpha$ is the plasma frequency,

$$\Lambda_\alpha = 1 + i\sqrt{\pi}Z_\alpha W(Z_\alpha), \qquad Z_\alpha = \frac{\omega - k_\| v_{0\alpha}}{\sqrt{2}k_\| v_{T\alpha}} \tag{19}$$

and

$$W(Z_\alpha) = exp(-Z_\alpha^2)\left[1 + \frac{2i}{\sqrt{\pi}}\int_0^{Z_\alpha} exp(t^2)dt\right],$$

is the usual plasma dispersion function, and the radial inhomogeneity parameters χ_1, χ_N, χ_T are defined as

$$\chi_1 = r{h_z}^2\frac{\partial}{\partial r}\left(\frac{h_\theta}{rh_z}\right), \qquad \chi_N = \frac{\partial}{\partial r}lnN_0, \qquad \chi_T = \frac{\partial}{\partial r}lnT.$$

They are, respectively, the shear parameter of tlie helical magnetic field, χ_1, and the logarithmic derivatives of tlie equilibrium clensity N_0 and temperature T of the plasma particles. These components are necessary for the analysis of the plasma eigenmodes, wave heating and current drive prohlems.

Note that in the expressions of $\hat{\epsilon}_{ik}$ we keep only the most important terms, which are proportional to the electron current velocity ($v_0 k_{\|}/\Omega$), tlie squared gyrotropy parameter, (Ω^2/ω_{ci}^2), and tlie radial graclients of the density $(\chi_N v_T/\omega_c)$ and temperature $(v_T\chi_T/\omega_c)$. If we omit the tensor components $\hat{\epsilon}_{13}, \hat{\epsilon}_{31}, \hat{\epsilon}_{32}, \hat{\epsilon}_{23}$ and exclude the terms with χ_N, χ_T, we obtain the well-known result of References [6,17,18] for the dielectric permeahility of the cylindrical plasmas in a helical magnetic field. Moreover, we take into account Doppler shifts, $k_{\|}v_0$, in the plasma dispersion function, which can be important for the analysis of the resonance zone condition of the collisionless wave dissipation in the moving plasma with "hot" electrons. The components of the permeability tensor $\hat{\epsilon}_{ik}$ can he simplified taking into account the well known approximations of tlie function, $W(Z_\alpha)$, for $Z_\alpha \ll 1$ and $Z_\alpha \gg 1$.

III. Magneto-hydrodynamic interpretation of dielectric permeability tensor

As was shown above, tlie direct method of the evaluation of a dielectric permeability tensor was used by means of the calculation of currents in a plasma on the basis of the kinetic equation. However, it is interesting and sometimes important, to demonstrate the magneto-hydrodynamic (MHD) way of evaluating the dielectric permeability tensor and its interpretation. Such an approach helps us to gain physical understanding of the prohlem and show the macroscopic parameters of the plasma (pressure, viscosity and so on), from which the dielectric permeability tensor can be derived. With this purpose, it is convenient to proceecl from hydrodynamic equations[14,19] for the ions and electrons, which are valicl for all collisional regimes,

$$m_\alpha N_\alpha \frac{d_\alpha \vec{V}_\alpha}{dt} = -\vec{\nabla} p_\alpha - \vec{\nabla}\cdot\vec{\vec{\pi}}_\alpha + e_\alpha N_\alpha \vec{E} + \frac{e_\alpha N_\alpha}{c}[\vec{V}_\alpha \times \vec{B}] + \vec{R}_\alpha \tag{20}$$

Bere, we use the well-known designations: m_α is tlie mass, N_α is the density, $\vec{V}$ is the macroscopic velocity, p_α is an isotropic part of the pressure, $\vec{\vec{\pi}}_\alpha$ is the viscosity tensor of the α-kind particles; $\vec{R}$ is the friction force between the particles of α -kind and other particles.

In a weakly collisional plasma the friction between particles for the transverse motion is usually negligibly small and can be omitted. Definitions of p_α and $\pi_{\|\alpha}$ (see, for example,[14]), are given by expressions:

$$p_\alpha = \frac{1}{3} m_\alpha \int w^2 F_\alpha d^3v; \qquad \pi_{\|\alpha} = m_\alpha \int \left({w_{\|}}^2 - \frac{1}{3} w^2 \right) F_\alpha d^3 v. \tag{21}$$

where F_α is the distribution function of ions or electrons, which should be determined from the kinetic equation, and $w_{\|} = v_{\|} - v_0$.

Below, we will take into account only the longitudinal component of the viscosity which is important for a fully ionized and weakly collisional plasma for different problems (the transverse viscosity is proportional to a collision frequency and the oblique (magnetic) viscosity take into account tlie finite Larmor radius effects wliich are small in our case). The viscosity equation is used as in[20]:

$$\vec{\nabla}\cdot\vec{\pi}=\frac{3}{2}\left\{[\vec{h}(\vec{\nabla}\cdot\vec{h})+(\vec{h}\cdot\vec{\nabla})\vec{h}]\pi_{\|}+\vec{h}(\vec{h}\cdot\vec{\nabla})\pi_{\|}\right\}-\frac{1}{2}\vec{\nabla}\pi_{\|} \tag{22}$$

where h is tlie unit vector of the magnetic field. To simplify tlie calculation procedure of tlie dielectric permeability tensor, we need to evaluate only the scalar values p_α and $\pi_{\|\alpha}$.

Assuming that the electric field and others macroscopic values of the plasma are oscillating with the frequency ω and ions and electrons have mean velocities $v_{0\alpha}$, tlie oscillating transverse current can be found from Eq.(20), using Eq.(22):

$$\vec{j}_\perp=\sum_\alpha\frac{e_\alpha}{m_\alpha(\omega_{c\alpha}^2-\omega_\alpha^{*2})}\left(\omega_{c\alpha}[\vec{h}\times\vec{A}_\alpha]+i\omega_\alpha^*\vec{A}_\alpha\right), \tag{23}$$

where

$$\vec{A}_\alpha=-e_\alpha N_\alpha\vec{E}_\perp-\frac{e_\alpha N_\alpha}{c}[\vec{V}_{0\alpha}\times\vec{B}]+\vec{\nabla}_\perp\left(p_\alpha-\frac{1}{2}\pi_{\|\alpha}\right)+\frac{3}{2}\pi_{\|\alpha}\vec{\nabla}_\perp\ln B,$$

$$\vec{V}_{0\alpha}=v_{0\alpha}\vec{h}+(c/e_\alpha N_\alpha B_0)\nabla_r p_{0\alpha}\vec{e}_b.$$

Note that Eq.(23) is valid for an arbitrary magnetic field configuration. The term p, $-0.5\pi_{\|\alpha}$ is in fact tlie transverse pressure $p_{\perp\alpha}$ and $p_\alpha+\pi_{\|\alpha}$ is tlie longitudinal pressure $p_{\|\alpha}$, wliich is the reason the longitudinal viscosity, $\pi_{\|\alpha}$, is equal to $2(p_{\|\alpha}-p_{\perp\alpha})/3$.

For further simplification, we suppose tlie tokamak to be axially-symmetric with circular magnetic surfaces (it can be also a solar plasma), and the plasma particle conclitions to be in tlie "plateau" regime (there are no trapped particles in this regime) and toroidal effects are taken into account only in the $1.5\pi_{\|\alpha}\vec{\nabla}_\perp\ln B_0$ term. It is coiivenient to proceed from tlie clrift líinetic equation (see, for example,[20]):

$$\frac{\partial F}{\partial t}+\frac{d\vec{r}}{dt}\cdot\vec{\nabla}F+\frac{dv_\perp^2}{dt}\frac{\partial F}{\partial v_\perp^2}+\frac{dv_\|}{dt}\frac{\partial F}{\partial v_\|}=\hat{S}t\{F\}; \tag{24}$$

Here

$$\frac{d\vec{r}}{dt}=\vec{u}+\frac{1}{\omega_c}\left[\vec{h}\times(\frac{\partial\vec{u}}{\partial t}+\vec{u}\cdot\vec{\nabla}\vec{u}+\frac{v_\perp^2}{2}\vec{\nabla}\ln B)\right],$$

$$\vec{u}=v_\|\vec{h}+\vec{V}_E,$$

$$\frac{dv_\perp^2}{dt}=v_\perp^2\left[v_\|\vec{h}\cdot\vec{\nabla}\ln B-\vec{\nabla}\cdot\vec{V}_E-\vec{V}_E\cdot(\vec{h}\cdot\vec{\nabla})\vec{h}\right],$$

$$\frac{dv_\|}{dt}=\frac{e}{m_\alpha}E_\|-\frac{v_\perp^2}{2}\vec{h}\cdot\vec{\nabla}\ln B+v_\|\vec{V}_E\cdot(\vec{h}\cdot\vec{\nabla})\vec{h},$$

where $\mathbf{F}=F_M+\tilde{f}_c+\tilde{f}_T$, and F_M is the Maxwell distribution function.

After some simplification, we find the expression for the oscillating part of the distribution function $\tilde{f}_{c\alpha}$ in "cylindrical" approximation and $\beta = 8\pi p_0/B_0^2 \ll 1$:

$$\tilde{f}_{c\alpha} = \frac{iF_{M\alpha}e_\alpha}{m_\alpha\omega\left(\omega_\alpha^* - k_\| w_\|\right)}\left\{\frac{ik_b\omega_\alpha^*}{\omega_{c\alpha}}\frac{m_\alpha v_\perp^2}{2T_\alpha}E_r - \frac{1}{\omega_{c\alpha}}\left[\left(\omega_\alpha^* - k_\| w_\|\right)\frac{\partial \ln F_{M\alpha}}{\partial r} + \right.\right.$$

$$\left.\left. + \omega_\alpha^*\frac{m_\alpha v_\perp^2}{2T_\alpha}\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)\right]E_b + \left(\frac{\omega m_\alpha w_\|}{T_\alpha} - \frac{k_b v_\|}{\omega_{c\alpha}}\frac{\partial \ln F_{M\alpha}}{\partial r} + \frac{k_\| v_{0\alpha}}{\omega_{c\alpha}}\frac{m_\alpha v_\perp^2}{2T_\alpha}\chi_2\right)E_\|\right\}. \tag{25}$$

where

$$\omega_\alpha^* = \omega - k_\| v_{0\alpha}, \qquad \chi_2 = -\omega_{ci}v_{0e}/c_A^2.$$

The "toroidal" oscillating part $\tilde{f}_T$ of the distribution function, which we use in the $1.5\pi_{\|\alpha}\nabla_\perp \ln B_0$ term, follows from Eq.(24):

$$\tilde{f}_{T(\pm)} = \frac{iF_M}{2R\left(\omega - k_{\|(\pm)}v_\|\right)}\frac{m_\alpha}{T}\left(v_\|^2 + \frac{v_\perp^2}{2}\right)\left(V_{Er} \pm iV_{E\theta}\right), \tag{26}$$

where

$$k_{\|(\pm)} = h_z k + h_\theta(m \pm 1)/r = \frac{1}{R}\left(n + (m \pm 1)/q\right), \tilde{f}_{T(\pm)} \sim exp\left[in\zeta + i\left(m \pm 1\right)\theta\right],$$

and ζ is a toroidal angle.

Using Eq.(22) and neglecting $v_{0\alpha}$ in comparison with the thermal velocity $v_{T\alpha}$, we find:

$$\frac{3}{2}\pi_{\|\alpha}\vec{\nabla}_\perp \ln B_0 = -\frac{icN_{0\alpha}T_{0\alpha}}{2\omega B_0 \boldsymbol{R}^2}\left\{\left(-i\vec{E}_\perp + [\hat{E}_L \times \vec{h}]\right)Q_\alpha^{(+)} + \left(i\vec{E}_\perp + [\vec{E}_\perp \times \vec{h}]\right)Q_\alpha^{(-)}\right\}, \tag{27}$$

where

$$Q_\alpha^{(\pm)} = C_\alpha^{(\pm)}\left[i\sqrt{\pi}W\left(C_\alpha^{(\pm)}\right)\left(1 + 2C_\alpha^{(\pm)4}\right) + C_\alpha^{(\pm)}(1 + C_\alpha^{(\pm)2})\right] \qquad C_\alpha^{(\pm)} = \omega/|k_{\|(\pm)}|v_{T\alpha}$$

Now, it is possible to calculate the "cylindrical" parts of p_α and $\pi_{\|\alpha}$ values. Using Eqs.(21) and (25), we find

$$p_{\perp\alpha} = \frac{ie_\alpha N_{0\alpha}v_{T\alpha}^2}{\omega\omega_{c\alpha}}\left(-i\hat{M}_{b\alpha}E_r + \hat{M}_{r\alpha}E_b + \hat{M}_{\|\alpha}E_\|\right) \tag{28}$$

Here,

$$\hat{M}_{b\alpha} = 2k_b\left(\Lambda_\alpha - 1\right),$$

$$\hat{M}_{r\alpha} = 2\left(\Lambda_\alpha - 1\right)\left(\frac{\partial}{\partial r} + \frac{1}{r}\right) - \chi_N - \chi_T,$$

$$\hat{M}_{\|\alpha} = -\frac{\omega_{c\alpha}\omega\Lambda_\alpha}{k_\| v_{T\alpha}^2} + \frac{k_b\omega\chi_n}{k_\|\omega_\alpha^*}\left(\Lambda_\alpha - \frac{k_\| v_{0\alpha}}{\omega}\right) +$$

$$+ \frac{k_b\chi_T}{2k_\|}\left[1 - \frac{k_\| v_{0\alpha}}{\omega_\alpha^*} + \frac{\omega\Lambda_\alpha}{\omega_\alpha^*}\left(1 + 2Z_\alpha^2\right)\right] - 2\chi_2\frac{k_\| v_{0\alpha}}{\omega_\alpha^*}\left(\Lambda_\alpha - 1\right).$$

Using Eqs.(23) and (28), we find as a result the expression for the transversal current $\vec{j}_\perp$,

$$\vec{j}_\perp = \frac{\omega}{4\pi i}\sum_\alpha \frac{\omega_{0\alpha}^2}{\omega^2(\omega_{c\alpha}^2 - \omega_\alpha^{*2})}\{(\omega_\alpha^*\vec{e}_r - i\omega_{c\alpha}\vec{e}_b)[(1 + D_\alpha^{(-)})\omega_\alpha^* E_r - \tag{29}$$

$$-\frac{v_{T\alpha}^2(\chi_N + \chi_T)}{\omega_{c\alpha}}\left(k_b E_r + i\frac{1}{r}\frac{\partial(rE_r)}{\partial r} + i\chi_2 E_\|\right) + i\left(E_b(\chi_1 v_{0\alpha} + D_\alpha^{(+)}\omega_\alpha^*) - v_{0\alpha}\frac{\partial E_\|}{\partial r}\right)\Bigg] +$$

$$+ (i\omega_{c\alpha}\vec{e_r} + \omega_\alpha^*\vec{e_b})\left[\left((1 + D_\alpha^{(-)}\right)E_b - iE_r D_\alpha^{(+)})\omega_\alpha^* + k_b v_{0\alpha}E_\|\right] + \frac{\omega}{e_\alpha N_{0\alpha}}\left(\vec{e_r}\hat{L}_{1\alpha} + i\vec{e_b}\hat{L}_{2\alpha}\right)p_{\perp\alpha}\},$$

where

$$D_\alpha^{(\pm)} = \frac{v_{T\alpha}^2}{2R^2\omega_{c\alpha}\omega_\alpha^*}(Q_\alpha^{(+)} \pm Q_\alpha^{(-)}), \qquad \hat{L}_{1\alpha} = \omega_{c\alpha}k_b - \omega_\alpha^*\frac{\partial}{\partial r}, \qquad \hat{L}_{2\alpha} = \omega_{c\alpha}\frac{\partial}{\partial r} - \omega_\alpha^* k_b.$$

Note also that in the case $k_{\|(\pm)}v_{Ti} \ll \omega \ll k_{\|(\pm)}v_{Te}$ we have $\tilde{\pi}_{\|e} \simeq -\tilde{p}_e$, i.e. the longitudinal pressure $\tilde{p}_{\|}$ is approximately equal to zero if we take into account only parts of $\tilde{p}_e$ and $\tilde{\pi}_{\|e}$, depending on Landau damping. The corresponding parts of $\tilde{p}_i$ and $\tilde{\pi}_{\|i}$ are small as compared with $\tilde{p}_{\|e}$ and $\tilde{\pi}_{\|e}$ and are omitted. We can see that the Landau damping in the transverse current is derived from the pressure $\tilde{p}_e$ and the viscosity $\tilde{\pi}_{\|e}$.

Substitution of Eq.(28) into Eq.(29) results in the components of the dielectric permeability tensor:

$$\hat{\epsilon}_{11} = 1 + \sum_{\alpha} \frac{\omega_{0\alpha}^2}{\omega^2 (\omega_{c\alpha}^2 - \omega_\alpha^{*2})} \{\omega_\alpha^{*2} \left(1 + D_\alpha^{(-)}\right) - \frac{k_b \omega_\alpha^*}{\omega_{c\alpha}} v_{T\alpha}^2 (\chi_N + \chi_T) + $$

$$+ \omega_{c\alpha}\omega_\alpha^* D_\alpha^{(+)} + \frac{1}{N_{0\alpha}} \hat{L}_{1\alpha} \left(\frac{N_{0\alpha} v_{T\alpha}^2}{\omega_{c\alpha}} \hat{M}_{b\alpha}\right)\}, \tag{30}$$

$$\hat{\epsilon}_{22} = 1 + \sum_{\alpha} \frac{\omega_{0\alpha}^2}{\omega^2 (\omega_{c\alpha}^2 - \omega_\alpha^{*2})} \{\omega_\alpha^{*2} \left(1 + D_\alpha^{(-)}\right) - v_{T\alpha}^2 (\chi_N + \chi_T)(\frac{\partial}{\partial r} + \frac{1}{r}) - $$

$$- \frac{1}{N_{0\alpha}} \hat{L}_{2\alpha} \left(\frac{N_{0\alpha} v_{T\alpha}^2}{\omega_{c\alpha}} \hat{M}_{r\alpha}\right) + + \omega_{c\alpha}(\chi_1 v_{0\alpha} + \omega_\alpha^* D_\alpha^{(+)})\}, \tag{31}$$

$$\hat{\epsilon}_{12} = i \sum_{\alpha} \frac{\omega_{0\alpha}^2}{\omega^2 (\omega_{c\alpha}^2 - \omega_\alpha^{*2})} \{\omega_\alpha^* \omega_{c\alpha} \left(1 + D_\alpha^{(-)}\right) - \frac{\omega_\alpha^*}{\omega_{c\alpha}} v_{T\alpha}^2 (\chi_N + \chi_T)(\frac{\partial}{\partial r} + \frac{1}{r}) + $$

$$+ \frac{1}{N_{0\alpha}} \hat{L}_{1\alpha} \left(\frac{N_{0\alpha} v_{T\alpha}^2}{\omega_{c\alpha}} \hat{M}_{r\alpha}\right) + + \omega_\alpha^* (\chi_1 v_{0\alpha} + \omega_\alpha^* D_\alpha^{(+)})\}, \tag{32}$$

$$\hat{\epsilon}_{21} = i \sum_{\alpha} \frac{\omega_{0\alpha}^2}{\omega^2 (\omega_{c\alpha}^2 - \omega_\alpha^{*2}))} \left\{-\omega_\alpha^* \omega_{c\alpha} \left(1 + D_\alpha^{(-)}\right) + k_b v_{T\alpha}^2 (\chi_N + \chi_T) - \right.$$

$$\left. - \omega_\alpha^{*2} D_\alpha^{(+)} + \frac{1}{N_{0\alpha}} \hat{L}_{2\alpha} \left(\frac{N_{0\alpha} v_{T\alpha}^2}{\omega_{c\alpha}} \hat{M}_{b\alpha}\right)\right\}, \tag{33}$$

$$\hat{\epsilon}_{13} = i \sum_{\alpha} \frac{\omega_{0\alpha}^2}{\omega^2 (\omega_{c\alpha}^2 - \omega_\alpha^{*2})} \left\{v_{0\alpha} \hat{L}_{1\alpha} + \frac{1}{N_{0\alpha}} \hat{L}_{1\alpha} \left(\frac{N_{0\alpha} v_{T\alpha}^2}{\omega_{c\alpha}} \hat{M}_{\|\alpha}\right)\right\}, \tag{34}$$

$$\hat{\epsilon}_{23} = - \sum_{\alpha} \frac{\omega_{0\alpha}^2}{\omega^2 (\omega_{c\alpha}^2 - \omega_\alpha^{*2})} \left\{v_{0\alpha} \hat{L}_{2\alpha} + \frac{1}{N_{0\alpha}} \hat{L}_{2\alpha} \left(\frac{N_{0\alpha} v_{T\alpha}^2}{\omega_{c\alpha}} \hat{M}_{\|\alpha}\right)\right\} \tag{35}$$

The operators $\hat{L}_{1\alpha}$, $\hat{L}_{2\alpha}$, $\hat{M}_{r\alpha}$, $\hat{M}_{b\alpha}$, $\hat{M}_{\|\alpha}$ are applied to all quantities, which are in the right hand side of them, including the electric field components.

To derive the remaining cornponents of the dielectric permeability tensor, $\hat{\epsilon}_{3\alpha}$, it is necessary to take into account drift terms in the drift kinetic equation, i.e., F_α has to be replaced by $F_\alpha \rightarrow F_\alpha + \Delta F_\alpha$, where

$$\Delta F_\alpha = \frac{ick_b v_\perp^2}{2B_0 \omega \omega_{c\alpha} (\omega_\alpha^* - k_\| w_\|)} \frac{\partial F_{M\alpha}}{\partial r} \{ - ik_b E_r + \left(\frac{\partial}{\partial r} + \frac{1}{r}\right) E_b\}. \tag{36}$$

We proceed from expressions Eqs.(9) and (24). As a result, we find

$$\hat{\epsilon}_{31} = \frac{ik_b}{k_\|} \sum_{\alpha} \frac{\omega_{0\alpha}^2}{\omega \omega_{c\alpha}} \left\{\Lambda_\alpha - \frac{k_\| v_{0\alpha}}{\omega} - \frac{k_b v_{T\alpha}^2}{\omega \omega_{c\alpha}} [\chi_N \frac{\omega}{\omega_\alpha^*} \left(\Lambda_\alpha - \frac{k_\| v_{0\alpha}}{\omega}\right) + \right.$$

$$+\frac{\chi_T}{2}\left[\frac{\omega}{\omega_\alpha^*}\Lambda_\alpha\left(1+2Z_\alpha^2\right)+1-\frac{k_\parallel v_{0\alpha}}{\omega_\alpha^*}\right]]\Big\}, \tag{37}$$

$$\hat\epsilon_{32}=\sum_\alpha\frac{\omega_{0\alpha}^2}{\omega\omega_{c\alpha}}\left\{\frac{\chi_N v_{0\alpha}}{\omega}-\frac{\omega}{k_\parallel\omega_\alpha^*}\left(\Lambda_\alpha-\frac{k_\parallel v_{0\alpha}}{\omega}\right)\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)+\right.$$

$$\left.+\frac{v_{T\alpha}^2k_b}{\omega_\alpha^*\omega_{c\alpha}k_\parallel}[\chi_N\left(\Lambda_\alpha-\frac{k_\parallel v_{0\alpha}}{\omega}\right)+\frac{\chi_T}{2}[\frac{\omega_\alpha^*-k_\parallel v_{0\alpha}}{\omega}+\Lambda_\alpha\left(1+2Z_\alpha^2\right)]]\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)\right\}, \tag{38}$$

$$\hat\epsilon_{33}=1+\sum_\alpha\frac{\omega_{0\alpha}^2}{k_\parallel^2v_{T\alpha}^2}\left\{\Lambda_\alpha+\frac{k_bv_{T\alpha}^2}{\omega_\alpha^*\omega_{c\alpha}}[\chi_N\left(\frac{k_\parallel^2v_{0\alpha}^2}{\omega^2}-\Lambda_\alpha\right)-\frac{\chi_T}{2}[1+\right.$$

$$\left.+\Lambda_\alpha\left(2Z_\alpha^2-1\right)]]+\frac{v_{0\alpha}\chi_2k_\parallel^2v_{T\alpha}^2}{\omega\omega_\alpha^*\omega_{c\alpha}}\left(\Lambda_\alpha-\frac{k_\parallel v_{0\alpha}}{\omega}\right)\right\}. \tag{39}$$

In conclusion to this section, we can say that the MND method of the dielectric permeability tensor evaluation may be used as an alternative to the method of the direct solution of the kinetic equation, which is developed in Section 2. This method can help us to clarify the underlying physics in an arbitrary geometry of the magnetic field. The tensor components, which are obtained by means of both methods, coincides. The MHD method can simplify the calculations of the dielectric permeability tensor in complicated magnetic fields because we need to find only three scalar perturbed values in MHD equations: $p_{\perp\alpha}, p_{\parallel\alpha}$ and $j_{\parallel\alpha}$. The toroidal terms in Eqs. (30) - (35) (connected with the $D_\alpha^{(\pm)}$ terms) can be used in the investigation of the global Alfvén waves in a complicated geometry (for example, in tokamaks).

IV. Current drive analysis

In this section, using the cylindrical plasma model (see Section 11), we analyze the current drive produced by Alfvén waves. Density, temperature, and current profile are assumed to be diffusive along the radial variable and homogeneous along the magnetic field lines; and the plasma is magnetized, so that the Larmor radii of the electrons and ions are smaller than the radial inhomogeneous plasma parameters ($\chi_{N,T}$).

The modeling of kinetic effects produced by Coulomb collisions have wide applications in plasma physics problems connected with the current drive, RF plasma heating in closed and open magnetic devices. For the mathematical description of those problems, nonlinear integro-differential spatially uniform kinetic equation of Landau-Fokker-Planck (LFP) type is usually used (see References [14,15]). To take into account weak collision effects, $\nu_{ei} \ll$ R, the Landau form of the collision operator in the kinetic Eq. (1) is considered for electrons. We suppose that the collision frequency is sufficient ($\nu_{ei} \gg \tilde\omega_b$, where $\tilde\omega_b$ is the electron bounce frequency in the wave field) to neglect the nonlinear effects of electrons captured by the wave field[22].

IV.1 Analytical treatment

To describe the plasma interaction of a wave packet absorbed by electrons due to Landau dumping, we proceed from Eqs.(1,2). We represent the distribution function as a sum $(F_0 + \tilde f)$ of the quasistationary and perturbed parts as discussed in Section 2. The perturbed part is proportional to the wave field amplitude. Averaging Eq. (1)for electrons and ions over time or a spatial period of Alfvén waves along the magnetic field lines, multiplying these equations by the momentum $m_{e,i}v_\parallel$, integrating in velocity space, and combining the equations for electrons and ions, we obtain the average current density equation:

$$< j_z >= \frac{e^2}{m_e \nu_{ei}} \langle \tilde{E}_z (\tilde{N}_e - \tilde{N}_i \frac{m_e}{m_i}) - \frac{1}{c \mid e \mid} \left[(\tilde{\vec{j}}_e - \tilde{\vec{j}}_i \frac{m_e}{mi}) \times \tilde{\vec{B}} \right]_z \rangle. \tag{40}$$

In the above equation the brackets < ... > denote averaging over a period of the wave oscillations. Note that above equation was obtained ill the two fluid magneto-hydrodynamic model by Klima[23] and that equation follows from the generalizecl Ohm's law[14] for weakly collisional plasmas. Further, the corrections associated with oscillations of the ions in the wave fielcls will be neglected because of the small electron-ion mass ratio m_e/m_i. The oscillations of plasma density and current are proportional to the oscillations of tlie RF fields, which is chosen in one mode approximation:

$$\tilde{E}_{r,\theta,z} = E_{r,\theta,z}(\mathrm{r}) \exp i(m\theta + kz - \mathrm{R}t). \tag{41}$$

where (m, k) are poloidal and axial wave numbers, respectively.

As the next step, we find the relation between the density and current oscillations from the equation of continuity, and the magnetic and electric field oscillations from the induction equation:

$$\tilde{N}^{(e)} = -\frac{i}{e\Omega} div\, \tilde{\vec{j}}^{(e)}, \qquad \tilde{\vec{B}} = -\frac{ic}{\mathrm{R}} rot \tilde{\vec{E}}; \tag{42}$$

where $\tilde{j}_\alpha^{(e)} = -(i\Omega/4\pi)\varepsilon_{\alpha q}^{(e)} \tilde{E}_q$ and $\varepsilon_{\alpha a}^{(e)}$ is the electrori part of the dielectric permeability tensor for low frequencies $\mathrm{R} \ll \omega_{ci}$, see Eqs. (10-18).

Substituting Eqs. (41) and (42) into Eq. (40), we obtain the value of the parallel current:

$$< j_\| > \approx -\frac{|e|k_\|}{\mathrm{me\,veia}} \left\{ P_e + \frac{1}{2k_\| r} \frac{\partial}{\partial r} \left[Im(r \tilde{j}_r^{(e)} \tilde{E}_\|^*) \right] \right\}. \tag{43}$$

where $P_e = Re(j_q^{(e)} \tilde{E}_q^*)/2$ is the density of the dissipated power as defined by analogy to the homogeneous plasma case $(\bar{P}_e)$. For homogeneous plasmas, the first term of Equation (43) is the driven current, calculated in[24,25], which is proportional to the dissipated power $\bar{P}_e$ N $(\Omega/8\pi) Im(\epsilon_{33}) |E_\||^2$. The second term is the gradient current[23,24 connected to the gradient of the density over the radius and clecreasing the wave amplitude due to the wave dissipation. This current contains the helicity injection current[24,25,26].

$$j_h = -\frac{|e|}{8\pi\nu_{ei} m_e} Im \left[\frac{i\epsilon_{12}^{(e)}}{r} \frac{\partial}{\partial r} (r E_b E_\|^*) \right]. \tag{44}$$

Furthermore, we assume the geometric optic approximation over the radial coordinate to study the wave polarization:

$$E_{r,b,\|} = E_{r,b,\|}^{(0)}\, exp(i \int_0^r k_r dr) \tag{45}$$

Substituting Eq.(45) into the Maxwell's equations (2), we obtain the ratio of the parallel and binormal electric field components for low frequency waves $(\mathrm{R} \ll \omega_{ci})$:

$$\frac{E_\|}{E_b} \approx \frac{(\epsilon_{11} - N_\|^2)(\epsilon_{22} - N_r^2 - N_\|^2) - N_b^2(\epsilon_{22} - N_\|^2)}{N_r^2 N_b N_\| + \epsilon_{23}(N_h^2 + N_\|^2 - \epsilon_{11}) + \epsilon_{13} N_r N_\|} \tag{46}$$

where $\vec{N} = \vec{k}c/\Omega$ is refractive index, which will be used in the next equation only. This formula will be useful for current drive analysis.

For kinetic Alfvén wave (KAW) we find the dispersion relation (see, for example[2]):

$$N_r^2 \approx (\epsilon_{11} - N_\|^2)\epsilon_{33}/\epsilon_{11}; \qquad E_\| \approx -E_b \left[(\epsilon_{11} - N_\|^2) N^2 + i N_b^2 Im(\epsilon_{22}) \right] / N_r^2 N_b N_\| \tag{47}$$

Using the ahove equatioii we estimate the KAW current drive (relatecl to the wave dissipation hy electron) as

$$< j_{\|} >\approx j_{cd}\left[1+\frac{2\omega_{ci}Re(k_r)k_b^3v_{Te}^2}{\Omega Re(\epsilon_{33})\Omega^2}Im(\Lambda_e)\right], \quad (48)$$

where simple current drive is presented by the expression:

$$j_{cd}=-\frac{|e|k_{\|}\bar{P}_e}{m_e\nu_{ei}\Omega}$$

The first term in the Equation (48) is tlie simple current drive (j_{cd}) and the second one is the helicity injection current. If the liinetic Alfvén wave is a standing wave along toroidal and poloidal directions, we should consider the current drive from waves with $k_{\|}=\pm|k_{\|}|$ and $k_b=\pm|k_b|$ and finally we obtain only the helicity injection (or gradient) current. The local driving efficiency (the ratio of current to dissipated power) of KAW current drive is higher hy one order of magnitude than the simple current drive efficiency of travelling KAW ($j_h/\bar{P}_e\approx 10j_{cd}/\bar{P}_e$).

For global Alfvén wave (GAW) and fast magnetosonic wave (FMSW)(see, for example[2], the value of the parallel oscillatory current will be approximately equal zero, due to a high parallel conductivity. Based on this condition, we evaluate:

$$\epsilon_{33}E_{\|}\approx-\hat{\epsilon}_{32}E_b \quad (49)$$

After suhstituting this equation into Eq. (43), we obtain:

$$< j_{\|} >\approx j_{cd}\left[1-\frac{Re(1-\text{A},)}{Im(\Lambda_e)}\frac{\chi_N-2Im(k_r)}{Re(k_r)}\right] \quad (50)$$

Where χ_N is the radial inhomogeneous density parameter, A, $=1-\eta$. The first term in Eq. (50) is the simple current drive and the second one is the gradient (and helicity) current drive. If the GAW are standing waves ($k_{\|}=\pm|k_{\|}|$ and $k_r=\pm|k_r|$) the first term of Eq.(50) will disappear and only tlie gradient current drive will exist.

The value of the GAW gradient (nonresonant) current drive will be

$$< j_{\|} >\approx\frac{|e|k_{\|}\bar{P}_e}{m_e\nu_{ei}\Omega}\frac{(1-Re\Lambda_e)}{Im(\Lambda_e)}\frac{\chi_N}{k_r} \quad (51)$$

So, the local efficiency of Alfvén wave current drive can be increased by one order of magnitude due to nonresonant gradient forces when ICAW phase velocity is small, $c_A\ll v_{Te}$, and GAW phase velocity is large, $c_A>v_{Te}$. This additional nonresonant current drive will not depend on trapped particle effects, which is supposed to reduce strongly the Alfvén current drive in tokamaks[2].

IV.2 Numerical treatment

In this subsection, for modeling of the current drive and RF plasma heating in tokamaks and space plasma configurations, we shall consider the two dimensional in tlie velocity space and spatially uniform LFP kinetic operator in tlie approximation of the isotropic Rosenblutli potentials (see Ref. [15]). We assume that the magnetic surfaces in a tokamak are circular and coaxial. The equation under consideration has been obtained from the drift ltinetic equation with the collision integral and under the quasi-linear assumption for narrow $\Delta c_{ph}\ll v_{ph}$ Alfvén wave packet[27–34], where Δc_{ph} is the width of the packet, c_{ph} is the phase velocity, and v_{T_e} is the tlieimal electron speed.

It is assumecl that the action of the waves, with the phase velocity considerably exceeding the thermal ion speed ($c_{ph}\gg v_{T_i}$), leads to a slight and unimportant distortion of the ion distribution function. Therefore, the ion distribution is not changed very much and can be chosen as Maxwellian with a fixed initial temperature $T=T_i(0)=T_e(0)$. The RF field action on the quasistationary distribution function can be described in the quasilinear approximation. The wave absorption due to Landau damping (for example, kinetic Alfvén waves) is talten into account hy tlie quasi-linear operator, $\hat{D}_zf$, in tlie form,

$$\hat{D}_zf=\partial/\partial v_z(D_0\partial f/\partial v_z),$$

Further, this operator will be written down in spherical coordinates. We confine ourselves to tlie approximation (see[27,31]) where the quasi-linear diffusion coefficient is constant within the phase resonance region and equals to zero in the other part of the velocity space:

$$D_0 = \begin{cases} \text{const.}, & \text{if } |v_z - c_{ph}| \leq \Delta c_{ph} \\ 0, & \text{otherwise.} \end{cases}$$

To present the form of the lrinetic equation we shall malie the usual normalization procedure for the velocity, the time and distribution function:

$$\tilde{v} = v/v_{Te(t=0)}; \qquad \tilde{t} = t/t_N, \quad t_N = \frac{v_{Te}^3 m_e^2}{4\pi e^4 N_e ln(\Lambda_{Coul})}; \qquad f = \frac{2\pi v_{T_e}^3 F}{N_e}.$$

As usual, the notations $N_{e,i}, e, m_e$ denote electron and ion densities, charge and mass of electrons, respectively; $ln\Lambda_{Coul}$ is the Coulomb logarithm.

Now, omitting the sign 'tilde', tlie kinetic equation for the new variables is

$$\frac{\partial f}{\partial t} = \hat{S}t\{f\} + \hat{D}f - \hat{E}f; \quad t > 0, \tag{52}$$

where $f(v,\mu,t)$ is the electron distribution function, and the specific type of the collision operator in the right part is presented in the form[32]:

$$\hat{S}t\{f\} = \frac{1}{v^2}\frac{\partial}{\partial v}\left[\frac{1}{v}\frac{\partial W(f;v)}{\partial v}\right] + \frac{1}{v^2}C(f;v)\frac{\partial}{\partial \mu}\left[(1-\mu^2)\frac{\partial f}{\partial \mu}\right]. \tag{53}$$

Here,

$$W(f;v) = \sum_{\alpha=e,i}\left\{\int_0^v (\frac{(}{m_e})(m_\alpha)F_\alpha(x,t)\left[p(x,\mu) - p(v,\mu)\right]x^2dx \right.$$

$$\left. - \int_0^v P_\alpha(x)\left[f(x,\mu) - f(v,\mu)\right]x^2dx\right\};$$

$$C(v) = \frac{1}{2v}\sum_{\alpha=e,i}\left(N_\alpha(v) - \frac{R_\alpha(v)}{v^2} + vP_\alpha\right);$$

$$p_\alpha(v,\mu) = \int_v^\infty F_\alpha(x,\mu)xdx, \quad F_\alpha(v) = \int_{-1}^1 F_\alpha(v,\mu)d\mu, \quad N_\alpha(v) = \int_0^v F_\alpha(x)x^2dx,$$

$$P_\alpha(v) = \int_{-1}^1 p_\alpha(v,\mu)d\mu, \quad R_\alpha(v) = \int_0^v P_\alpha(x)x^2dx$$

where v = $|\vec{v}|$ - modulus of the velocity; $\mu = \vec{v}\cdot\vec{B}/|\vec{v}||\vec{B}|$ and $-1 \leq \mu = v_{\|}/v \leq 1$.

The initial electron distribution has the Maxwellian form. The distribution function moment $N_\alpha(v=\infty)$ corresponds to tlie density of particles, the energy of particles and the parallel current of tlie system are defined in spherical coordinates as follows:

$$\mathcal{E}_\alpha = \int_0^\infty dvv^4\int_{-1}^1 d\mu F_\alpha(v,\mu,t), \quad j = \int_0^\infty dvv^3\int_{-1}^1 d\mu\mu f(v,\mu,t).$$

To investigate the influence of an external electrical field we include in Equation (52) the operator $\hat{E}f$[31], which has the following form:

$$\hat{E}f = \gamma\frac{\partial f}{\partial v_z} = \gamma\frac{1}{v^2}\left\{\frac{\partial}{\partial v}\left[fv^2\mu\right] + \frac{\partial}{\partial \mu}\left[(1-\mu^2)vf\right]\right\},$$

where γ is the ratio between the electrical field E and the so called Dreicer field, $E_{Dr} = 4\pi e^3 nln(\Lambda_{Coul})/T_e$; for tokamak plasmas y < 0.01 as a rule.

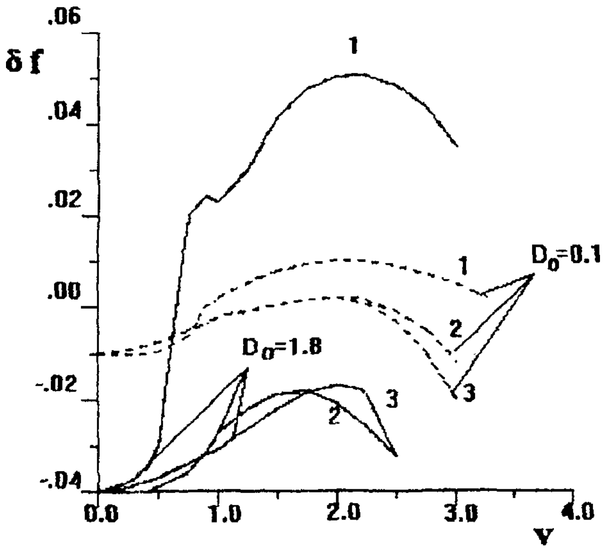

Figure 1. Plot of the deviation of the distribution function from Maxwell distribution δf over module of the normalized velocity v, for the parallel $\mu = 1(1)$, antiparallel $\mu = -1$ (3) and perpendicular $\mu = 0$ (2) directions, relatively to the magnetic field in the velocity space, wlien tlie normalized quasi-linear coefficient D_0 is equal to 1.8 (solid line) and 0.1 (dashed line) when the normalized phase velocity is $c_A/v_{Te} = 0.6$.

In the computer solution of the problem, the numerical algorithms, which are based upon the completely conservative finite difference scheme [32,33], are used. If the discrete model (difference scheme) possesses only approximate analogs of the conservation laws, then this can lead to the accumulation of errors in the analysis of non-stationary and nonlinear problems. The completely conservative difference scheme reflects some symmetry properties of the nonlinear kinetic equation in a discrete case. This scheme maintains two distribution function moments (integrals), which are correspond to the density (N_e) and energy (I)n the plasma system.

Under the influence of the RF diffusion operator, the initial distribution changes its form. The distribution function takes an anisotropic shape and the current hegins to he driven. The increasing current saturates in time. Up to this moment of time, N $10 t_N$, the current reaches the most high value j_D while the quasilinear operator forms "plateau" in the distribution function in the velocity region v, $\sim c_{ph}$. The most intensively Coulomb diffusion affects in the thermal velocity region, $c_{ph} \approx v_{Te}$, see Fig.1, that has result in increasing of the maximum of the dissipated power and the possibly achievable current.

Certainly, the value of the current depends on the phase velocity, c_{ph}, the magnitude of the diffusion coefficient D_0 and the width of wave packet Δc_{ph}, and rises with their increasing. The current dependence on the coefficient D_0 and width Δc_{ph} is essentially nonlinear as shown in Fig.2.

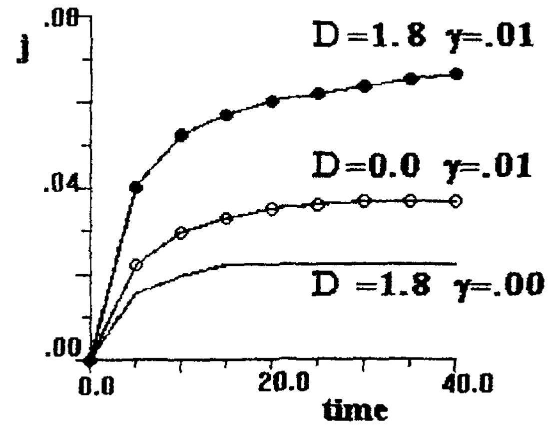

Figure 2. The evolution of the normalized current density in normalized time for the following parameters: phase velocity $c_{ph} = 0.6$, electrical field $\gamma = E/E_{Dr} = .01$ (dotted lines) and 0.0 (solid Iine), quasilinear diffusion coefficient $D_0 = 1.8$ (solid circle) and 0.0 (empty circle).

In the super thermal velocity region v $> v_{th}$, the electrical field influence on the electron distribution function, leads to the creation of distribution tails, which have substantial nonmaxwellian character. The value of the Ohmic current is $j_{||} = \sigma_{OH} E_{||}$, where σ_{OH} and E are the plasma conductivity and electrical field, respectively.

Note that the total current density in plasma system, presented in Fig.2, does not equal the sum of currents, which are induced separately by different mechanisms. The additional significant current appeared in plasma due to plasma conductivity stimulation by RF field under its combined action with the stationary electric field. That result is not evident for Alfvén waves heating (in comparison with the lower hybrid current drive, see[29]) because of large distance between the active regions of influence of the electric field operator and the quasilinear operator. The total current can be represented as a sum of different currents:

$$j_t = j_D + (\sigma_{OH} + \sigma_{QL})E + j_N,$$

Here, j_D is the driven current, which is equal to ηP, where η is current drive efficiency, that is defined as ratio of the current drive to the dissipated power, P; $\sigma_{QL}E$ is the current stimulated by Alfvén waves in the quasi-linear approximation and j_N is the nonlinear current[34].

To take into account tlie influence of trapped particles on the Alfvén current drive, we acld to the right hand part of Eq.(53) the angle diffusion operator as in[29,34], which represents a simplified form of the trapped particle collision opcrator, see Ref.[35]:

$$\hat{I}_{tr} f \sim \nu_b \frac{\partial}{\partial \mu}\left[(1-\mu^2)\frac{\partial f}{\partial \mu}\right] \qquad (54)$$

This operator acts within the region $v_\perp > v\cos\mu \approx v\sqrt{r/R}$ occupied by the trapped particles, where tlie bounce frequency $\nu_b \gg$ v,,. The toroidicity parameter E $= \sqrt{r/R}$ corresponds to the trapped angle of electrons; for example, if $\epsilon = 0.04$ then $\Delta\mu$ N 0.2.

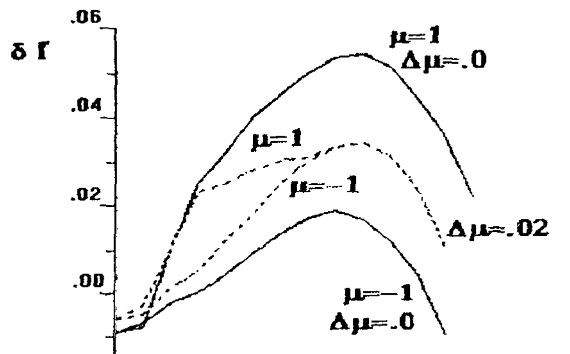

Figure 3. Plot of the deviation of the distribiition function from Maxwell distribution δf over module of the normalized velocitg v, for tlie parallel ($\mu = 1$) and antiparallel ($\mu = -1$) directions, relatively to magnetic field in tlie velocity space, when tlie normalized quasilinear coefficient D_0 is equal 1.8 (dashed line is for the trapped electron case) and tlie phase velocity $c_{ph} = 0.6$.

The current is basically carried by superthermal electrons in the absence of trappecl particles. The result of the action of trappecl particles is the isotropization of the distribution function for velocities $v > c_{ph}\sqrt{r/R}$.

Thanlis to that fact, thc value of the current decreases, which is directly related to tlie anisotropy of tlie distribution function. In Fig.3, tlie electron distribution splittirig over "directions" μ is shown in the case of the trapped particle to be ahsent and present.

Note that in Fig.2 and Fig.3 the deviations, $\delta f =$ (f $- f_m)/f_m$, of the electron clistrihution, $f(v,\mu,t)$, from Maxwellian form are shown. In spite of the strong decreasing of the current magnitude in the case of trapped particles, the stimulated plasma conductivity effect under combined action of electrical and RF fields is kept in this case too[34]. The presence of the trapped particles and the electric field modify tlie scaling of tlie current drive efficiency.

5. Conclusions

In this paper a method of the evaluation of the dielectric perineability tensor in a magnetized plasma is demonstrated in the cases of the large aspect ratio tokamak (the "plateau" regime) and cylindrical magnetic field configurations with inhomogeneous plasmas. The magneto-hydrodynamic and direct kinetic approaches are used, and their equivalence are shown. The toroidal corrections of tlie permeability tensor are fonnd using tlie MHD approach.

A general proceclure of thc current clrive calculation is developed for the cylindrical plasma moclel. The analytical expression for the longitudinal current drive by time - averaged electromagnetic forces is obtainecl and the influence of the plasma inhomogeneity (the "gradient" effect) on the current drive is discussed. It is shown that the efficiency of KAW and GAW current drive due to helicity injection and "gradient" effects are higher by one order of the magnitude in comparison with the current driven by travelling KAW and GAW.

The numerical investigation of the Alfvén current drive is carried out on the base of the drift ltinetic equation with the Landau - Fokker - Planck collision integral. It is shown that the additional increasing of the current (synergistic effect) in the plasma appears due to the plasma conductivity induced by the RF field, which action is combined with that of the electrical field.

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References

1. G. A. Collins, et. al.,: Phys. Fluids, 29, 2260 (1986).
2. A. G. Elfimov, A. G. Kirov and V. P. Sidorov, in *High-Frequency* Plasma Heating, edited by A. G. Litvak (American Institute of Physics, Translation Series, N.Y. 1992), 239.
3. A. I. Lisojvan, V. E. Moiseenko, O. M. Shvets and K. N. Stepanov: Nuclear Fusion, **32,** 1361 (1992).
4. L. Ruchlto, M.C.R. Andrade, R. M. O. Galvão and I.C. Nascimento, in: Proceedings of Intern. Conf. on Plasma Physics, Foz do Iguaçu (Brazil) (1994), Vol. **1**, p. 365.
5. H. Alfvén: Nature, **160**, 405 (1942).
6. D. W. Ross, G. L. Chen and S. M. Mahajan, Phys. Fluids, ***25,*** 652(1982).
7. J. Vaclavic and K. Appert, Nuclear Fusion, **31**, 1945 (1991).
8. E. Priest, Solar Magnetohydrodynamics (D. Reidel Publ. Co., Dordrecht, Holland 1982).
9. J. V. Hollweg, in Advances in Space Plasma Physics, edited by B. Buti (Word Scientific, Singapore, 1985) p. 77.
10. C. A. de Azevedo, A. S. de Assis, H. Shigueolta and P. H. Saltanaka,: Solar Phys., **136**, 295 (1991).
11. C. A. de Azevedo, A. G. Elfimov and A. S.de Assis, Solar Physics, **153**, 205 (1994).
12. A. S. de Assis and K. H. Tsui, Astrophys. J., 366, 1, 324 (1991).
13. L. C. Jafelice, R. Opher, A. S. de Assis and J. Busnardo-Neto, Astrophys. J., **61**, 384 (1990).
14. S. I. Braginskii, in.: Rev. Plasma Phys. (Consultants Bureau, New York, 1965) Vol. **1**, p. 205.
15. B. A. Trubnikov, in. Rev. Plasma Phys. (Consultants Bureau, New York, 1965) Vol. **1**, p. 120.
16. F. M. Nekrasov, Sov. Journ.Plasma Phys., **18**, 520 (1992).
17. V. P. Sidorov et al., Nuclear Fusion, 27, 1411 (1989).
18. M. V. Dmitrieva, A. G. Elfimov, K. G. Komoslivili, F.M. Nekrasov, V. P. Sidorov, V. F. Tishliim and A. P. Favorskii, *Preprint N4 of Keldysh* Institute *of Applied* Mathematic, USSR Academy *of Science*, Moscow (1984).
19. S. P. Hirshman and D. J. Sigmar, Nuclear Fusion. 21, 1079, (1981).
20. G. L Cliew, M. L. Goldberger and F. E. Lom, Proc. Royal Soc. A 236, 112 (1956).
21. A. B. Milthailovskii and V. S. Tsypin, Sov. Phys. JETP, 83, 75, (1982).
22. A. G. Elfimov, Sov. J. Plasma Phys., 9 , 403, (1983).
23. R. Klima, Czech. Jorn. Phys., **B30**, 874, (1980).
24. A. G. Elfimov, Sov. Journ. Plasma Phys., 17, 223, (1991).
25. T. Ohkawa, Comments Plasma Phys. Controlled Fusion, 12, 165 (1989).
26. J. B. Taylor, Phys. Rev. Lett., 63, 1384 (1989).
27. N. J. Fisch and Ch. F. F. Karney, Phys. Fluids, **24,** 27 (1981).
28. J. A. Cordey, J. Eldington and D.F.H. Start, Plasma. Phys., 24, 73 (1982).
29. Yu. N. Dnestrovskij et al. Nuclear Fusion, 28, 267 (1988).
30. L. I. Rudakov and R. Z. Sagdeev, *Plasma Phys.* and *Problem of Thermonuclear* Reactions. AS USSR, 3, 268 (1958).
31. A. G. Elfimov, I. F. Potapenko, V. P. Sidorov:

Soviet Journal Plasma Physics, 15, 684 (1989).

32. I. F. Potapenko, V. A. Chuyanov U.S.S.R. Comput. Maths. and Math. Phys., 20, 249, (1910).

33. I. F. Potapenko, In.: Proceedings of *17th International Symposion on Rarefied Gas* Dynamics, Aachen, Germany, **2,** 431 (1990).

34. I. F. Potapenko, A. G. Elfimov, In.: *Proceed. of* 16 *European Conference* CONTROLLED *FUSION AND* PLASMA PHYSICS, Venice, March 1989, **1,** 18 (1989).

35. M. N. Rosenbluth, D. W. Ross, D. P. Kostomarov, Nuclear Fusion, 12, 3 (1972).