1s R a Good Cosmological Parameter?

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We argue that the cosmological density parameter RO miglit be meaningless in a spatially infinite universe. I;Qllo~vingEllis and Sclireiber's iclea of a topologically nontrivial, lumpy small universe, we construct a11 illustrative moclel that would account for discreparicies in estimates of Q0 as the result of strong inhornogeneity, in tlie same scale as the universe's size. A contribution of peculiar gravity to tlie redshift of distant images, in the line of Sachs-Wolfe effect but without the linear approximation, is derived.

I. Introduction

The Big Bang model of the universe, based on Friedmann-Lemaître-Robertson-Walker (FLRW) relativistic cosmologies, is at present wiclely accepted as to its grand design features; hut there is much uncertainty with respect to its parameters. To begin with, does one discard the cosmological constant Λ? While the correspondence principle makes h negligible[1], there are FLRW models where its contribution to Einstein equations is of the same order as the present mass density ρ_0 - see[2], for exarnple. Let us assume 11 = 0 hereafter. Hubble's parameter is $H_0 = 100h$ km s^{-1} Mpc^{-1}, with h undetermined within the range 0.3-1.0. Is ρ_0 smaller or larger than, or equal to, tlie critical density $\rho_{\rm crit}$? *Guth's inflationary scenario*[3] requires $\Omega_0 \equiv \rho_0/\rho_{\rm crit} = 1$ if A = 0. Astropl-iysical estimates seem to depend o11 the studied region: foi example, while tlie dynamics of a large gas cloud in tlie NGC 2300 group of galaxies points to $\Omega_0 = 1$[4], tlie calculations of White et al.[5], based on estimates of the Coma cluster's total mass, favor R $= 0.16h^{-1/2}/(1 + 0.19h^{3/2}) \approx 0.2$.

An operational definition of average density is no trivial matter - see[6] and references therein. Also, if the universe (or our inflationary bubble) is much bigger than its presently observable part, then the FLRW assumption of a presently uniform ρ is unverifiable. It has heen tacitly assumed that the high degree of smoothness of the cosmic background radiation (CBR) implies present homogeneity at some "reasonable" scale. But there is no proof of this, for the details of "initial" conditions at recoinbination time and of evolution since tlien are very much uncertain. For all me know, the present average density could vary so wilclly over cosmic distances that the parameter Ω_0, cleterminecl from a number of accessible patches of spacetime, would be unrepresentative of our universe.

An alternative to this undesirable scenario is provided by Ellis and Schreiber's ***lumpy small*** *universes*[7]. I-Iere we study such a model, with Einstein-de Sitter (EdŚ) metric, whose spatial section $S(t)$ has the topology of a. 3-torus T^3; $S(t)$ is constructed from a cube of side L $= 120h^{-1}$ Mpc, which is ahout Ellis & Schreiber's minimal value for a small universe; this cube, or basic cell, has opposite faces identified in orcler to represent T^3. She lumpiness refers to perturbations of tlie metric leading to strong density fluctuations on a scale of tlie same orcler as L. Since tlie basic cell is repeated in the form of multiple images of the sources in $S(t)$, up to the observations' horizon, the model predicts an apparent homogeneity on the scale of a few cells' volume while leaving room for large inhomogeneity on the scale of galaxy superclusters.

This worli does not aim at detailed realism. Its

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aim is rather to point out a possibility that has heen widely ignored, namely that the average density may vary strongly in the scale of hundreds of megaparsecs. The problem is attaclted in an simplified way, through numerical integration of a perturbed Einstein-de Sitter model.

II. A Perturhed small universe

This model is similar to a previous one[8] (henceforth referred to as FK) in many aspects. The perturbed EdS metric has the form

$$ds^2 = dt^2 - a^2(t,x)dx^2 - b^2(t,x)(dy^2 + dz^2) ,$$

with the periodicities $a(t, x+L) = a(t, x)$, $b(t+L, x) = b(t, x)$, which are interpreted in the sense of a compact universe of topology T^3. Our calculational units are $c = G = t_0 = 1$, where G = Newton's constant and $t_0 = 2/3H_0$ is the present age of the universe in EdS model; L = 0.06 in these units. There is yet no reliable evidence for such a model, but recently Biesiada[9] proposed an interpretation of the distribution of gamma-ray bursts in terms of a toroidal universe with L = 300 Mpc. On the other hand, COBE's result on the CBR seems to imply much bigger lower bounds on L - see, for example[10]. But present day cosmology is so full of tentative results, that we need not abandori tlie idea of small universes on its first confrontation with observational analyses. Besides, some properties of small universes are linked to their compact topology rather than to their smallness[7], and this paper's scheme may probably he reformulated in terms of this compactness, so as to be reconciled with those lower bounds.

For simplicity we restrict the spatial dependence of a and b to the x variable. We assume a velocity field v = (v,0,O), discarding $O(v^2)$ terms. Einstein's constraint equations (cf. [11]) are then

$$\frac{2\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2} - \frac{2b''}{a^2 b} - \frac{b'^2}{a^2 b^2} + \frac{2a'b'}{a^3 b} = 8\pi\rho ,$$

$$\frac{2\dot{b}'}{a^2 b} - \frac{2\dot{a}b'}{a^3 b} = 8\pi\rho v ,$$

where a dot means $\partial/\partial t$, a prime $\partial/\partial x$. The dynamical equations produce tlie system

$$\begin{aligned} \dot{a} &= f \\ \dot{b} &= g \\ \dot{f} &= \frac{ag^2}{2b^2} - \frac{fg}{b} + \frac{b'^2}{2ab^2} + \frac{b''}{ab} + \frac{a'b'}{a^2 b} \\ \dot{g} &= -\frac{g^2}{2b} + \frac{b'^2}{2a^2 b} , \end{aligned}$$

which was integrated numerically, as in FK, except that the evolution proceeded from $t = 0.1$ to 1.0, and two-term expansions were used, with $\delta t = 5 \times 10^{-5}$. The initial conclitions are

$$\begin{aligned} a(0.1,x) &= 0.1^{2/3}(1 + \alpha \sin qx), \\ b(0.1,x) &= 0.1^{2/3} \\ \dot{a}(0.1,x) &= (2/3) \times 0.1^{-1/3} , \\ \dot{b}(0.1,x) &= (2/3) \times 0.1^{-1/3}(1 + \beta \sin qx) , \end{aligned}$$

where $\alpha = 10^{-4}$, $\beta = 4 \times 10^{-5}$, and $q = 2\pi/L$. These conditions were chosen to serve the model's illustrative purpose; they are not strictly realistic, except for their EdS limit if α and β vanish. On the other hand, they might well be compatible with analyses of COBE's map[12] of the CBR, which would have α and $\beta \simeq 10^{-5}$ at the epoch of recombination. We did not take $t_{rec} \simeq 10^{-5}$ as initial time because it would require much refinement of the integration process, beyond the scope of this work. Note that our treatment is not perturbative: the nonlinear equations are solved directly from perturbed initial conditions.

The numerical results in the range $t = 0.5 - 1.0$ were fitted to the following approximations:

$$a_{rel}(t,x) = t^{-2/3}a(t,x) \cong 1 - 0.108t^{1.53} sin qx, \quad (1)$$

$$b_{rel}(t,x) = t^{-2/3}b(t,x) \cong 1, \quad (2)$$

$$\rho_{rel}(t,x) = 6\pi t^2 \rho(t,x) \cong A(t) + B_1 sin qx + A_2(t)\cos 2qx, \quad (3)$$

$$v(t,x) \cong 4.2 \times 10^{-4} t^{-1.34} cos qx , \quad (4)$$

where $A(t) = 1 + 0.014t^{2.31}$, $B_1(t) = 0.261t^{0.75}$, ancl $A_2(t) = -0.022t^{2.12}$. Note that $\rho)_{rel}(t, \mathrm{x})$ has a maximum of 1.297 and a minimum of 0.775 at t = 1.0, separated by x = L/2 = $60h^{-1}$ Mpc. In principle we could have started with, say, an $\Omega_0 = 0.6$ universe (but not so small; cf.[13] and disturb it to obtain a fluctuation $\delta\rho_0/\rho_{crit} = 50.4$. Qualitatively this would mimic the results of Mulchaey et al.[4] mentioned above.

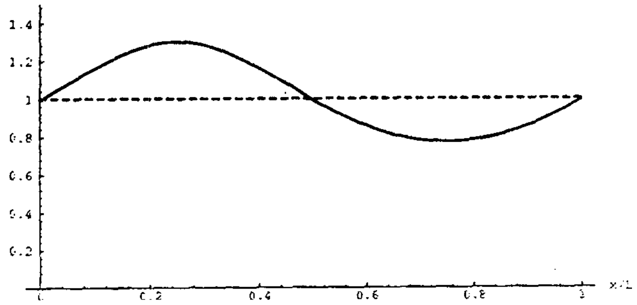

Figure 1. The relative density $\omega_0(x/L)$, L = $120h^{-1}$ Mpc.

Figure 1 is a plot of $\rho_{rel}(1, \mathrm{x})$, which me may call $\omega_0(x)$ as a variable counterpart of Ω_0. These results imply a reformulation of the relation between distance ancl Hubble's flow velocity, which will not be discussed here. (After this work was completed a preprint appeared[14], proposing that the "local universe" is a bubble of smaller density than the larger scale universe, with consequences to Hubble's law. In our terminology, they are saying that the $\omega_0(0)$ is substantially lower than the average Ω_0.)

III. The perturbed redshifts

Now we calculate the influence of the above perturbations on tlie redshifts. For simplicity both the observer and the source are placed on the x axis, with the former at x = 0; the lattei is at x = x,, and (potentially) has repeated images at x $= x_e = x_s + nL$, where n is an integer ancl $|x_e| < 0.62 \cong 10L$ (to make lookback time ≤ 0.5). Let the source emit a pulse of wavelengtli λ_e, during the time interval t_e to $t_e +$ A,, througl-i such a path as to make an image at x,. This image may be treated as if it weie a source at x_e, with peculiar velocity v, $= v(t_e,$ le,), for the apparent light path results from "unwinding" the real path; so it moves from x_e to $x_e + \lambda_e v_e$ in the period A,. This pulse is observed by us in tlie interval 1 to $1 + \lambda_0$, while Earth moves from 0 to $\lambda_0 v_0$, $v_0 = v(1, 0) = 0.00042$. If $t(x)$ is the light path from $(t_e,$ x,) to (1,0), we get

$$\int_{t_e+\lambda_e}^{1+\lambda_0} t^{-2/3}dt - \int_{t_e}^{1} t^{-2/3}dt = \int_{\lambda_0 v_0}^{x_e+\lambda_e v_e} a_{rel}[t(x) + \lambda(x), x]dx - \int_0^{x_e} a_{rel}[t(x), x]dx \ ,$$

$$\lambda_0 - t_e^{-2/3}\lambda_e = \int_0^{x_e} (\partial a_{rel}/\partial t)[t(x), x]\lambda(x)dx + a_{rel}(t_e, x_e)\lambda_e v_e - \lambda_0 v_0 \ .$$

Approximating the path by its EdS equation, $t(x) = (1 - x/3)^3$ (thus possibly neglecting a significant correction to the redshift and Hubble's law; it will be the subject of further study), and substituting $\lambda_0/\lambda_e = 1 + Z$, $t_e^{-2/3} = 1 + Z_{EdS}$, $\lambda(x) = \lambda_e t_e^{-2/3} t(x)^{2/3}$, and Eqs. (1) and (4) in the above expression, we get

$$\delta Z \equiv Z - Z_{EdS} = (1 + Z_{EdS})(P + \mathrm{V} + Z_0), \qquad (5)$$

where

$$\mathrm{P} = -0.165 \int_0^{x_e} (1 - x/3)^{3\ 59} sin\ qx \ \mathrm{dx} \ ,$$

$$V = (av)(t_e, x_e) = 0.00042 t_e^{-0.67} \cos qx_e (1 - 0.108 t_e^{1.53} \sin qx_e) ,$$

ancl $Z_0 = -v_0$ is a Doppler shift due to tlie observer's motion. One of Harrison's[19] formulas for composite redshifts would give $1+z = (1+z_R)(1+z_0+z_C+z_{grav})$. Since $z_R \cong Z_{EdS}$, this expression agrees with Eq. (5) if we identify z_C with our "streaming velocity" V, and the peculiar gravity shift z_{grav} with P, as already done in FK. **P** can be said to generalize the Sachs-Wolfe effect[15] in our limited context, since tlie present treatment is not a linear approximation.

Fig. 2 is a plot of V, P, and δZ as functions of x_e/L.

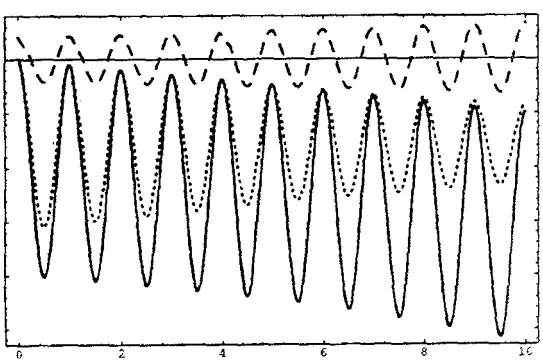

Figure 2. Plots of V (dashed), P (dotted), and δZ (full), for an image at position X in nnits of L = 120 h^{-1} Mpc.

IV. Final remarks

In our small universe the scale of density inhomogeneities is about as large as it can be, that is, of the same order as the universe's size. This cell repeats itself in the form of multiple images, thus partly mimicking the *infinity* of EdS model. But the pattern of matter distribution in observable space is determined by the evolution of the basic cell's pattern. Ω_0 is now a. density average over the real world of sources (i.e., over the basic cell). It remains useful in tlie present context, as we proceeded by perturbation of an FLRW universe, with one of its allowed topologies; cf.[16]. Ellis & Schreiber[7] pose the question of whether this is usually the case for lumpy small universes. A conjecture hy Thurston[17] points to an answer by reversing the question: given a closed space with an inhomogeneous metric, thc latter can be smoothed out into a piecewise homogeneous metric. (See also the one-but-last paragraph in[18].) The strong density contrast gives rise to a measurable contribution of peculiar gravity to the redsliift. This shift is ignored by Harrison[19] as negligible, and apparently so also in studies of large scale structures - see the last paragraph in[8] - where it may have been misinterpreted as part of the velocity field. So it seems desirable to investigate it in more realistic simulations.

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