Magnetic Fluid Free Surface Instabilities in High-Frequency Rotating Magnetic Fields

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Various complex phenomena connected with magnetic droplet behaviour under the action of the high-frequency rotating magnetic fields are considered. By virial method the stability of the oblate ellipsoid shape in high-frequency rotating field is analyzed and the possibility of the transitions "oblate - prolate - oblate" as the rotating field strength increases is shown. It is obtained that instability of the oblate shape exists at magnetic permeability of the droplet higher than a critica1 value and that it disappears at large frequency of the rotating magnetic field. On the basis of the virial method the angular velocity of the droplet rotation is calculated and it was found that to describe its power-law dependence on field frequency it is necessary to account for the distribution of the magnetization relaxation times of the concentrated phase of magnetic colloid. On the basis of a simple model - magnetic fluid cylinder in high-frequency rotating field - the instability with respect to the circunferential mode of the cylinder (leading to the "star-fish" configurations) is found and a proportionality of the number of arms to field square is shown. Peculiarities of the magnetic fluid layer undulation instability in high-frequency rotating and constant normal magnetic fields are analyzed and it is concluded that observation in experiment of the undulations of the extended prolate droplet is connected with comparable values of the rotating field period and the characteristic time of the droplet shape relaxation. Comparison with the normal field instability of the ferrosmectics is also given.

I. Introduction

Pattern formation due to the magnetic fluid free surface instabilities causes a lot of interest. Here we can point out spike instability under the action of normal field[1], labyrinthine pattern formation in the plane slots of magnetic fluid[2,3], parametric oscillations under the action of a.c. tangential magnetic field[4] and others. Behaviour of magnetic fluid drops, especially those of concentrated phase of the magnetic colloid[5], is of special interest. By studying the statics and dynamics of the elongational instability of the magnetic fluid drops under the action of the homogeneous magnetic field it is possible to determine the low values of the surface tension of concentrated phase[6,7], high values of its magnetic permeability[6,7], and also viscosity of the concentrated phase[8], which turns out to be rather high.

As it was found in Ref. [9], and will be shown also here on the basis of the detailed theoretical calculations, specific properties of the concentrated phase of the magnetic colloids allow to observe rather intricate free surface instabilities of the magnetic drops under the action of the high-frequency rotating field. Since in Ref. [9] only some remarks about the theoretical approach to the problem of the magnetic drop behavioui under the action of high-frequency rotating magnetic field were given, here we proceed with detailed calculations which confirm the possibility of "oblate - prolate - oblate" shape transitions observed in the experiment[9] and also on the basis of simple model illustrate the physical mechanisms responsible for the formation of "star-fish" configurations[9].

II. "Oblate - prolate - oblate" shape transitions

Experimentally[9] it was found that the microdrops of the concentrated phase have oblate shape at small and high rotating magnetic field strengths. At intermediate values of the field strength the microdrops have prolate wormlike configuration with rather complex dynamics. It is possible to show the existence of such sequence of shape transformation on the basis of the virial method[10]. Virial method has heen applied before for the study of the elongational instability of the microdrop in the static field[11] and its equivalence to the energetical approach[6] for static case has been illustrated[12].

Nevertheless the virial method is more general as an energetical one since besides the statics also allows to consider the dynamical phenomena. That is tlie point why virial method is applied for the study of drop behaviour under the action of high-frequency rotating field since due to the arising antisymmetrical tangential stresses the phenomena under consideration are connected with dissipation.

Virial relations are obtained on the basis of the equation of the motion for magnetic fluid, which in Stokes approximation looks like

$$\frac{\partial \vec{\sigma}_l^{(i)}}{\partial x_l} + \frac{\partial \vec{T}_l^{(i)}}{\partial x_l} = 0 , \tag{1}$$

where the stress tensor accounting for the antisymmetric tangential stresses in the droplets (η_m - viscosity of the concentrated phase) is

$$\begin{aligned} \sigma_{nm}^{(i)} &= -p\delta_{nm} + 2\eta_m 1/2 \left(\frac{\partial v_n}{\partial x_m} \right. \\ &+ \left. \frac{\partial v_m}{\partial x_n} \right) + \frac{1}{2} e_{nml} \left[\vec{M} \times \vec{H} \right]_l , \end{aligned} \tag{2}$$

and $T_{nm}^{(i)}$ - magnetic stress tensor and equation of the motion of the surrounding nonmagnetic viscous fluid

$$\frac{\partial \vec{\sigma}_l^{(e)}}{\partial x_l} = 0 \tag{3}$$

To obtain antisymmetric tangential stresses magnetization relaxation equation must be added[13], the which in the case of the weak fields when linear magnetization law in static case is valid, accounting for demagnetizing field effects) can be written as

$$\frac{d\vec{M}}{dt} - \left[\vec{\Omega}_0 \times \vec{M} \right] = -\frac{1}{\tau_B} \left(\vec{M} - \chi (\vec{H}_0 - \vec{N}\vec{M}) \right) . \tag{4}$$

Here $\vec{\Omega}_0 = 1/2 rot \vec{v}$ is the vorticity, τ_B is the Brownian relaxation time, and $\vec{N}$ is the tensor of the demagnetizing field coefficients.

It is important to point out some general consequences of the relaxation equation (4): well the known dependence of the magnetic susceptibility of the droplet on its shape according to the relation $\chi_e = \chi/(1+\chi N)$ and the dependence of magnetic relaxation time on the droplet shape according to the relation $\tau_e = \tau_B/(1 + \chi N)$. Both magnetic susceptibility and relaxation time are diminshing with the increase of the demagnetizing field coefficient.

Boundary conditions describing the force balance on the surface of droplet are following:

$$\begin{aligned} \sigma_{nn}^{(i)} - \sigma_{nn}^{(e)} &= -\sigma/R_c + 2\pi(\vec{M}\vec{n})^2 ; \\ \sigma_{in}^{(i)} &= \sigma_{in}^{(e)} , \end{aligned} \tag{5}$$

where $1/R_c$ is the mean curvature of the surface.

Considerable simplification of the problem occurs if one assume that the droplet shape belongs to the class of the ellipsoidal configurations which is not changing at affinity deformations. In that case the solution of the magnetostatic field problem is reduced to the accounting for shape depending demagnetizing field coefficients.

Another approximation involved in the consideration is connected with the calculation of viscous stresses of the surrounding nonmagnetic fluid. Since the viscosity of the concentrated phase of magnetic colloid η_m is much higher than the viscosity of surrounding fluid η viscous stresses arising at the droplet, rotation with respect to it are calculated as for solid ellipsoid. When considering the affinity deformations leading to the configuration change due to the same reason, viscous stresses are accounted for only inside the droplet.

At last, since the period of rotating field T in the case of our interest[9] is rnuch less than the characteristic time of the droplet shape relaxation $\eta_m R/\sigma$, the virial relation obtained is averaged with respect to the field period. To consider more general case numerical methods[14] rnust be applied. Corresponding work is under progress.

Under the assumption of the ellipsoidal shape of the droplet the magnetic field inside the droplet is homogeneous and the volume ponderomotive force in the equation (1) vanishes by multiplying the equation (1) with cartesian coordinate x_k after the integration throughout the volume of the droplet. Hence, one obtains the following virial relation

$$\begin{aligned} V_{ik} &= \int_{S_d} x_k \sigma_{il}^{(e)} n_l dS - \sigma \int_{S_d} (\delta_{ik} - n_i n_k) dS \\ &+ 2\pi \int_{S_d} x_k n_i (\vec{M}\vec{n})^2 dS \end{aligned}$$

$$+ \quad \delta_{ik} \int_{V_d} p dV - \int_{V_d} \tau_{ik} dV = 0 \ . \tag{6}$$

Here $\vec{n}$ is the normal to the interface, τ_{ik} is the viscous stresses inside the droplet, and σ is the surface tension.

Due to the approximations involved, viscous stresses from the surrounding fluid are calculated according to the following relations[15] (a_1 = a; a_2 = b; a_3 = c are the semiaxes of the general ellipsoid)

$$\sigma_{il}^{(e)} n_l = \frac{8\eta g}{abc} A_{il} \frac{x_l}{a_l^2} \ ;$$

$$g = 1/(x_i^2/a_i^4)^{1/2} \ ;$$

$$A_{il} = \frac{a_l^2 \omega_{il}}{2(a_i^2 \alpha_{i0} + a_l^2 \alpha_{l0})} \ ;$$

$$\alpha_{i0} = \int_0^\infty \frac{d\lambda}{(a_i^2 + \lambda)\sqrt{(a^2+\lambda)(b^2+\lambda)(c^2+\lambda)}} \ ;$$

$\omega_{il} = e_{ilk}\Omega_k$, and $\vec{\Omega}$ is the angular velocity of the droplet rotation.

Due to the relations

$$n_i = \frac{x_i}{a_i^2} g$$

and

$$\frac{1}{V_d} \int_{S_d} g \frac{x_k x_l}{a_l^2} dS = \delta_{kl} \ ,$$

virial relation (6) can be transformed to the following expression

$$\begin{aligned} V_{ik} &= \frac{32\pi\eta}{3} A_{ik} - \int_{V_d} \tau_{ik} dV + \delta_{ik} \int_{V_d} p dV \\ &- \sigma \int_{S_d} (\delta_{ik} - n_i n_k) dS \\ &+ 2\pi \int_{S_d} x_k n_i (\vec{M}\vec{n}) dS = 0 \ . \end{aligned} \tag{7}$$

The magnetization is calculated from the relaxation equations (4). After averaging with the respect to the field period, in the general case we have

$$\langle M_1^2 \rangle = \frac{\chi^2 H_0^2}{2((1+\chi N_1)^2 + (\omega' \tau_B)^2)} \ ; \ \langle M_2^2 \rangle = \frac{\chi^2 H_0^2}{2((1+\chi N_2)^2 + (\omega' \tau_B)^2)}$$

$$\langle M_1 M_2 \rangle = \frac{\chi^2 H_0^2 \omega' \tau_B \chi (N_2 - N_1)}{2((1+\chi N_1)^2 + (\omega' \tau_B)^2)((1+\chi N_2)^2 + (\omega' \tau_B)^2)} \ , \tag{8}$$

where N_1 and N_2 are demagnetizing field coefficients along the main axes of the ellipsoid in the plane of rotating field, and $\omega' = w - R$ is the angular velocity of the field rotation with respect to the droplet.

Excluding the pressure, the virial relations (7) allow to obtain the following relations for the steady state of the axisymmetric droplet (x_1, x_2 are the main axes of the ellipsoid in the plane of the rotating field, and x_3 is the main axis along the normal to the plane of the rotating field):

$$V_{33} - 1/2(V_{11} + V_{22}) = 0 \tag{9}$$

and

$$V_{12} - V_{21} = 0 \ . \tag{10}$$

Eq. (9) gives the eccentricity of the droplet, while Eq. (10) gives the angular velocity of its rotation.

Applying the relations

$$\int_{S_d} g \left(\frac{3x_3^2}{a_3^2} - 1 \right) n_1^2 dS = \frac{3V}{4\pi} \pi \left(\frac{6(1+e^2)}{e^4} - \frac{(2e^4 + 8e^2 + 6)}{e^5} arctge \right) \ ;$$

$$\int_{S_d} (1 - 3n_1^2) dS = \left(\frac{3V}{4\pi} \right)^{2/3} \pi \frac{1}{(1+e^2)^{1/6}} \left(\frac{(2e^2+3)(1+e^2)^{1/2}}{e^2} - \frac{(3+4e^2)}{2e^3} \ln \left(\frac{e + (1+e^2)^{1/2}}{(1+e^2)^{1/2} - e} \right) \right)$$

and expressing tlie value of the magnetization averaged per period according to tlie relations (8), the following relation for the axisymmetric steady shape of the droplet is obtained

$$\frac{\chi^2 H_0^2 (3V/4\pi)^{1/3}}{\sigma} = \frac{1}{\pi(1+e^2)^{1/6}} \frac{\left(\frac{(2e^2+3)(1+e^2)^{1/2}}{e^2} - (3+4e^2)\ln\left(\frac{e+(1+e^2)^{1/2}}{(1+e^2)^{1/2}-e}\right)\right)}{\left(\frac{(2e^4+8e^2+6)}{e^5} arctge - \frac{6(1+e^2)}{e^4}\right)} \times \left((1+\chi N_1)^2 + (\omega' \tau_B)^2\right), \quad (11)$$

where $e = (a^2/c^2 - 1)^{1/2}$ is the eccentricity of the oblate ellipsoid, and

$N_1 = 4\pi\left((1+e^2)\frac{arctge}{2e^3} - \frac{1}{2e^2}\right)$ is the demagnetization coefficient of oblate shape in the plane of the rotating field.

Relation *(11)* allows the calculation of the axes ratio of the oblate shape in dependence on the rotating magnetic field strength. In the limit $\omega' \tau_B \to 0$, relation *(11)* coincides with the relation following from tlie consideration of the energy minima of the droplet in high-frequency rotating field including free surface energy and demagnetization energy, just like what happens when considering tlie elongational instability of the droplet in constant field.

After after taking average of tangential antisymmetric stresses with respect to the rotating field period in steady state, the virial relation *(10)* gives the following relation for the angular velocity of the drop rotation (torque balance equation):

$$\frac{8\pi\eta\Omega}{N_1} = \frac{\chi H_0^2 \omega' \tau_B}{(1+\chi N_1)^2 + (\omega' \tau_B)^2} \quad . \qquad (12)$$

The relations *(11)* and *(12)* allow the determination of the axes ratio of the steady oblate shape and angular velocity of the droplet rotation. As it follows from relation (12), at small $\omega \tau_B$ the angular velocity of the droplet rotation is proportional to the angular velocity of tlie rotating field. In experiment[9], a power law for the dependence of the angular velocity of the droplet rotation on field frequency is observed. As it was already remarked Ref. [9] to describe such dependence in the framework of the model proposed it is necessary to account for the distribution of the magnetic relaxation times of the real magnetic colloid, since the relaxation Eq. (4) can give quantitative results only for monodisperse colloid. To describe experimental data quantitavely it is possible to apply the following relation for the complex magnetic susceptibility of the colloid in a.c. field[16]:

$$\tilde{\chi}(\omega) = \chi/(1+(i\omega\tau_0)^{1-\alpha}) \qquad (13)$$

where a accounts for the distribution of the magnetic relaxation times, and τ_0 is the mean magnetic relaxation time. Cole - Cole plots for complex magnetic susceptibility *(13)* correspond to the arcs of circle instead of the semicircles for the monodisperse colloid and allow to determine the value of α from experimental data.

Calculation according to the relation *(13)* of the mean magnetic torque acting on the droplet in rotating field allows to obtain the torque balance condition for small angular velocities of the droplet rotation in following form

$$\frac{8\pi\eta\Omega}{N_1} = \frac{\chi(\omega\tau_0)^{1-\alpha}\cos(\pi\alpha/2)H_0^2}{1+2(\omega\tau_0)^{1-\alpha}sin(\pi\alpha/2)+(\omega\tau_0)^{2(1-\alpha)}+2\chi(1+(\omega\tau_0)^{1-\alpha}sin(\pi\alpha/2))N+\chi^2N^2} \qquad (14)$$

which generalizes in that limit the torque balance equation (12). One can see that in limit $\omega\tau_0 \to 0$ (which corresponds well to the conditions of the experiment[9]) $\Omega \sim \omega^{1-\alpha}$. Thus it is possible to describe the dependente of the angular velocity of the droplet rotation on rotating field frequency observed in experiment[9] when the distribution of the magnetic relaxation times is accounted for. The consideration which follows is based on the simple magnetic relaxation given by Eq. (4).

Experimental results on the oblate shape axis ratio correspond well to relation (11) in the limiting case $\omega'\tau_B \to 0$, which corresponds to the quasistatic conditions.

The next problem which should be adressed in the framework of the model considered concerns the stability of the oblate shape since the experiment shows that axisymmetric oblate shape is unstable at intermediate values of field strength. For that we will consider the stability of the oblate shape with the respect to the deformation to general ellipsoid. Our approach will be based on the consideration of affinity deformations when Lagrange displacements of the material points are linear functions of the cartesian coordinates. In particular case the Lagrange displacements can be chosen in following way:

$$\xi_1 = L_1(t)x_1; \xi_2 = -1/2L_1(t)x_2; \xi_3 = -1/2L_1(t)x_3$$

leading to a spontaneous symmetry breaking in the plane of the rotating field. An equation for the $L_1(t)$ can be obtained looking for the variation of virial relation (7). For that, the following relations for Lagrange variation of the surface elements are applied[11]:

$$\delta_L n_i = -\frac{\partial\xi_j}{\partial x_i}n_j + n_i n_j n_l \frac{\partial\xi_j}{\partial x_l} \ ;$$

$$\delta_L(n_i dS) = -\frac{\partial\xi_l}{\partial x_i}n_l dS \ ;$$

$$\delta_L(dS) = - - n_i n_k \frac{\partial\xi_k}{\partial x_i}dS \ .$$

Excluding pressure for the variation of the virial relations,

$$\delta_L(V_{11} - 1/2(V_{22} + V_{33})) = 0$$

and

$$\sigma L_1 \int_{S_d} \left(\frac{9}{4}n_1^4 - 3n_1^2 - \frac{1}{4}\right) dS - 4\pi L_1 \frac{3}{4} \int_{S_d} g\left(\frac{3x_1^2}{a_1^2} - 1\right) n_1^2 n_3^2 dS < M_1^2 > +$$
$$+2\pi \int_{S_d} \frac{g}{2}\left(\frac{3x_1^2}{a_1^2} - 1\right) 2(\vec{M}\vec{n})(\Delta_L \vec{M}\vec{n})dS - 3\eta_m V \cdot L_1 = 0 \ ,$$

where $\Delta_L \vec{M}$ is the magnetization variation at affinity deformations calculated according to the relations (8).

The variation of the angular velocity of the droplet rotation is calculated according to the relation (10). As a result, one obtains

$$\delta(\omega'\tau_B) = -\frac{\chi H_0^2 \tau_B \omega'\tau_B(\delta N_1 + \delta N_2)(1 - \chi^2 N_1^2 + (\omega'\tau_B)^2)}{4\eta N_1 F((1+\chi N_1)^2 + (\omega'\tau_b)^2))} \ ,$$

$$F = \frac{4\pi}{N_1} + 2\frac{\chi H_0^2 \tau_B}{4\eta} \frac{((1+\chi N_1)^2 - (\omega'\tau_B)^2)}{((1+\chi N_1)^2 + (\omega'\tau_B)^2)^2} \ ;$$

Variations of the demagnetization field coefficients are expressed in the following way:

$$\delta N_1 = 4\pi L_1 \left(\frac{33e^2 + 27}{16e^4} - \frac{(15e^2 + 27)(1 + e^2)}{16e^5} arctge\right) = \pi q_1 L_1 \ ;$$

$$\delta N_2 = 4\pi L_1 \left(\frac{(9 + 3e^2)}{16e^4} + \frac{(3e^2 - 9)(1 + e^2)}{16e^5} arctge\right) = \pi q_2 L_1$$

Accounting for the following expressions of the moments

$$\int_{S_d}\left(\frac{9}{4}n_1^4-3n_1^2-\frac{1}{4}\right)dS=\left(\frac{3V}{4\pi}\right)^{2/3}\frac{\pi}{(1+e^2)^{1/6}}\left(\frac{(-8e^6+94e^4+183e^2+81)}{16e^4(1+e^2)^{1/2}}-\right.$$
$$\left.-\frac{(104e^4+156e^2+81)}{32e^5}\right)\ln\left(\frac{e+(1+e^2)^{1/2}}{(1+e^2)^{1/2}-e}\right)=\pi\left(\frac{3V}{4\pi}\right)^{2/3}p_1\ ;$$

$$\int_{S_d}g\left(\frac{3x_1^2}{a_1^2}-1\right)n_1^2n_3^2dS=$$
$$\frac{3V}{4\pi}\pi\frac{(1+e^2)}{4e^4}\left(\frac{(5e^4+42e^2+45)}{e^3}\mathrm{arctg}e-\frac{(27e^2+45)}{e^2}\right)=\frac{3V}{4\pi}\pi p_2\ ;$$

$$\int_{S_d}g\left(\frac{3x_1^2}{a_1^2}-1\right)n_2^2dS=-\frac{3V}{4\pi}\pi\frac{1}{2e^2}\left(\frac{(e^4-2e^2-3)}{e^3}arctge+\frac{(3+e^2)}{e^2}\right)=-\frac{3V}{4\pi}\pi p_3\ ;$$

$$\int_{S_d}g\left(\frac{3x_1^2}{a_1^2}-1\right)n_1^2dS=\frac{3V}{4\pi}\frac{\pi}{2e^2}\left(\frac{(5e^4+14e^2+9)}{e^3}\mathrm{arctg}e-\frac{(11e^2+9)}{e^2}\right)=\frac{3V}{4\pi}\pi p_4\ .$$

As a result, the following relaxation equation for the deformation rate $L_1(t)$ of the oblate shape is obtained:

$$\dot{L}_1=\frac{\alpha}{\tau_d}L_1\ ,$$

where the dimensionless growth rate of the oblate shape is expressed as

$$\alpha=p_1-\frac{\pi Bm}{((1+\chi N_1)^2+(\omega'\tau_B)^2)^2}\left(\frac{3p_2}{2}+\pi p_4(\chi(1+\chi N_1)q_1-\right.$$
$$\left.-Bm\frac{\tau_B}{\chi\tau_f}\frac{(\omega'\tau_B)^2(q_1+q_2)(1-\chi^2N_1^2+(\omega'\tau_B)^2)}{N_1F((1+\chi N_1)^2+(\omega'\tau_B)^2)^2}\right)-\pi p_3\left(\chi(1+\chi N_1)q_2-\right.$$
$$\left.\left.-Bm\frac{\tau_B}{\chi\tau_f}\frac{(\omega'\tau_B)^2(q_1+q_2)(1-\chi^2N_1^2+(\omega'\tau_B)^2)}{N_1F((1+\chi N)^2+(\omega'\tau_B)^2)^2}\right)\right)\ . \tag{15}$$

Here, the following characteristic times are introduced - the deformation time of the droplet: $\tau_d=4\eta_m(3V/4\pi)^{1/3}/\sigma$ and the characteristic time $\tau_f=\eta\tau_d/\eta_m$ determined by the viscosity of the surrounding fluid.

Relation (15) allows to determine the growth increment of the nonaxisymmetric deformations of the oblate shape as a function of its axis ratio, which according to the relation (11) increases with the increase of magnetic Bond numher (magnetic field strength). For that dependence, the magnetic Bond number and the droplet angular velocity are determined according to the relations (11) and (12). Results for $\omega\tau_B=1$ and $\tau_B/\chi\tau_f=10^{-2}$ are shown on Fig. 1 for several values of the magnetic permeability. One can see that if the magnetic permeability of the droplet is rather high, there exists a range of axis ratio of oblate shape at which the axisymmetric oblate shape is unstable to symmetry destroying perturbations (Fig. la). That result is in accordance with experimental observations[9] which show that at intermediate values of the magnetic field strength, the oblate shape is unstable and wormlike extended droplet configurations are developing. As one can see from Fig. 1b the critica1 value of magnetic permeability at which such phenomenon can be observed is about 11. Thus high magnetic susceptibility of the concentrated phase of magnetic colloid is essential to observe such instability.

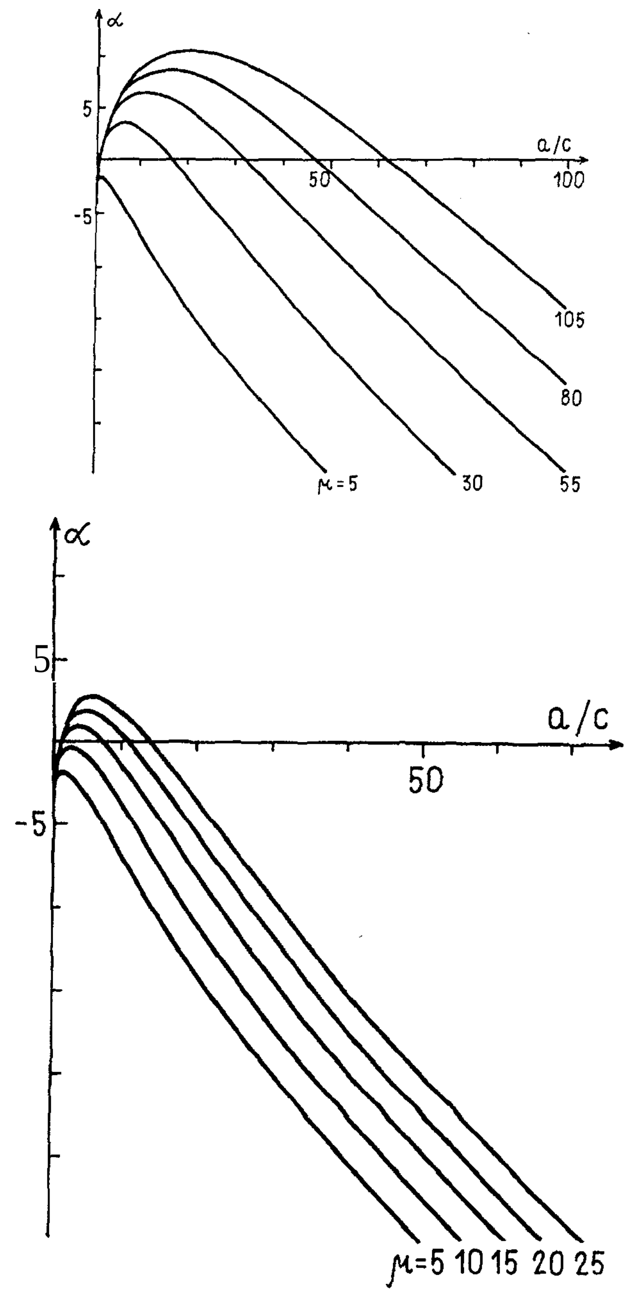

Figure 1: "Oblate - prolate" shape transition growth increment α versus axes ratio a/c of the oblate ellipsoid at different values of the magnetic permeability. $\tau_B\chi^{-1}\tau_f^{-1} = 10^{-2}$.

It should be mentioned that the value of magnetic permeability chosen for the plot on Fig. 1b corresponds to the value of real experimental sample[9]. The value $\tau_B/\chi\tau_f = 10^{-2}$ for $\eta = 10^{-2}$ P, R $= 15\mu$m, $\mu = 25$ corresponds to the Brownian relaxation time 4 10^{-2} sec - a quite reasonable value for the concentrated phase of the magnetic colloid.

Values of the critical axes ratio at which according to the data on *Fig.* 1 shape transitions "oblate - prolate" and "prolate - oblate" occur are correspondingly near 1 and of the order of magnitude of 20, what corresponds quite well to experimental observations.

It is of the interest to rernark that the range of oblate shape axis ratio unstable with respect to symmetry *destroying* perturbations *shrinks* with the increase of the frequency. That is illustrated on Fig. 2, where growth increment of nonsymmetric perturbations is plotted at the experimental situation value of magnetic permeability $\mu = 25$ for $\omega\tau_B = 10^{-2}$ and $\omega\tau_B = 3$ $(\tau_B/\chi\tau_f = 10^{-2})$. At high-frequencies of the rotating field the instability disappears at all. That is connected with the decrease of magnetization value of the droplet which follows the increase of rotating field frequency.

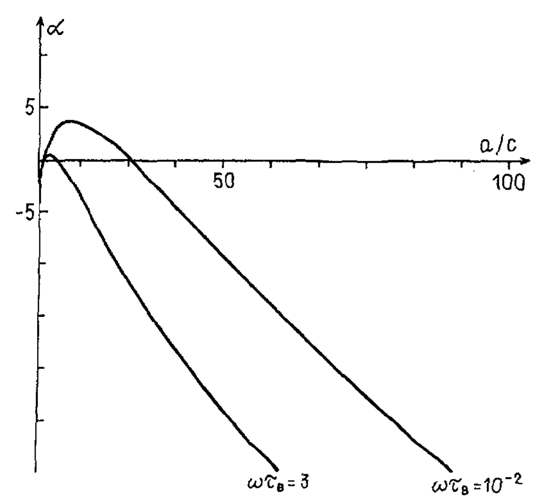

Figure 2: "Oblate - prolate" shape transition growth increment versus axes ratio a/c of the oblate ellipsoid at magnetic permeability $\mu = 25$ for two values of $\omega\tau_B$. $\tau_B\chi^{-1}\tau_f^{-1} = 10^{-2}$.

III. Formation of the "star-fish" configuration

The next point to be described concerning intricate behaviour of the droplets in high- frequency rotating fields is connected with the formation of the "star-fish" *configurations*[9]. As it follows from experimental observations the number of the arms of the "star-fishes" is proportional to the square of the field strength. The mechanism of their development as well as the main peculiarities of their behaviour can be understood on the basis of a simple model, in which the infinite cylindrical volume of magnetic fluid is considered under the action of the rotating in a plane normal to its axis magnetic field. As above, we assume that the characteristic time of the droplet shape relaxation is much larger than the period of the rotating field and time averaging with respect to rotating field period is possible. That means that symmetric configuration of cylinder corresponds to the possible figure of equilibrium.

Let us consider its shape stability with respect to perturbations in polar coordinates described as $r = \zeta(\phi) = R + \delta\zeta(\phi) = R + a_n \cos(n\phi)$, where R is the radius of unperturbed cylinder. The instantaneous value of magnetostatic potential $(\mathbf{H} = \nabla\psi)$ in the case of unperturbed shape in rotating field with cartesian components $(H_0\cos(\omega t),\ H_0\sin(\omega t))$ is given as (I - region occupied by droplet, II - outside it).

$$\psi_I^0 = \frac{2H_0}{\mu+1}\cos(\omega t) r\cos\phi + \frac{2H_0}{\mu+1}\sin(\omega t) r\sin\phi\ ; \tag{16}$$

$$\psi_{II}^0 = H_0\cos(\omega t) r\cos\phi + H_0\sin(\omega t) r\sin\phi - \frac{H_0\cos(\omega t)(\mu-1)R^2\cos\phi}{(\mu+1)r} - \frac{H_0\sin(\omega t)(\mu-1)R^2\sin\phi}{(\mu+1)r}$$

The perturbation of the magnetostatic potential arising at shape perturbations is found as a solution of the Laplace equations inside and outside the droplet obeying the following boundary conditions at unperturhed surface of the cylinder

$$\psi_I' - \psi_{II}' = \delta\zeta\left(\frac{\partial\psi_{II}^0}{\partial r} - \frac{\partial\psi_I^0}{\partial r}\right)\ ;$$

$$\mu\frac{\partial\psi_I'}{\partial r} - \frac{\partial\psi_{II}'}{\partial r} = \frac{\delta\zeta'}{R^2}\left(\mu\frac{\partial\psi_I^0}{\partial\phi} - \frac{\partial\psi_{II}^0}{\partial\phi}\right) + \delta\zeta\frac{\partial^2\psi_{II}^0}{\partial r^2}\ .$$

As a result the following expression for the perturbed potential inside the droplet is obtained:

$$\psi_I' = \frac{2H_0\cos(\omega t)(\mu-1)r^{n-1}\cos(n-1)\phi)a_n}{(\mu+1)^2R^{n-1}} - \frac{2H_0\sin(\omega t)(\mu-1)r^{n-1}\sin(n-1)\phi)a_n}{(\mu+1)^2R^{n-1}} \tag{17}$$

Let us find an growth increment of the surface perturbations for ideal magnetic fluid. In that case the Cauchy - Lagrange integral for the velocity potential on unperturbed surface of the cylinder accounting for capillary forces and surface magnetic forces allows to obtain

$$\rho\frac{\partial\phi}{\partial t} + \frac{\sigma}{R_c} - \frac{(\mu-1)}{8\pi}(\mu H_n^2 + H_t^2) = const\ . \tag{18}$$

Relation (18) taking into the account kinematic condition $\partial\delta\zeta/\partial t = \partial\phi/\partial r$ and calculating the magnetostatic field strength according to (16) and (17) after time averaging when following relations are obtained:

$$\langle H_n^0 H_n'\rangle = \frac{2H_0^2(\mu-1)}{(\mu+1)^3R}a_n(n-1)\cos(n\phi)\ ;$$

$$\langle H_t^0 H_t'\rangle = -\frac{2H_0^2(\mu-1)}{(\mu+1)^3R}a_n(n-1)\cos(n\phi)\ ;$$

allows to obtain the following equation for the amplitude of the circunferential mode:

$$\frac{d^2a}{dt^2} + n\left(\frac{\sigma}{\rho R^3}(n^2-1) - \frac{(\mu-1)^3H_0^2(n-1)}{2\pi(\mu+1)^3\rho R^2}\right)a_n = 0\ . \tag{19}$$

As follows from (19), symmetrical configuration is unstable with respect to the circumferential mode at $H_0^2 > N^2(n)$:

$$H^2(n) = \frac{2\pi\sigma(n+1)(\mu+1)^3}{R(\mu-1)^3} . \qquad (20)$$

We can remark that according to the analysis in the first part also in the simple model considered there exists instability with respect to the transition to nonsymmetric shape with elliptic crossection whicli takes place at critical field strength $H^2(2) = \sigma\pi\sigma(\mu+1)^3/R(\mu-1)^3$, which by the order of magnitude quite well corresponds to the experimental data.

From equation (21) it is possible to obtain the number n_* of the dynamically most unstable mode, which have the greatest growth rate. For large n_* it is obtained as

$$n_* = \frac{1}{3\pi}\frac{(\mu-1)^3}{(\mu+1)^3}\frac{H_0^2 R}{\sigma} \qquad (21)$$

Linear dependence of the number of arms of the rotating "star- fish" on parameter $H_0^2 R/\sigma$ is observed in experiment[9]. Thus the simple model considered allows to explain the "star-fish" configurations observed as due to the development of specific magnetic instability in high-frequency rotating field.

More realistic model of the dynamics of the "star-fish" formation must include viscosity effects. If the viscosity of the surrounding fluid is neglected, the boundary conditions

$$-p + 2\eta_m\frac{\partial v_r}{6r} - 2\pi(\vec{M}\vec{n})^2 + \frac{\sigma}{R_c} = \text{const} \qquad (22)$$

$$v_r = \frac{\partial\zeta}{\partial t} \qquad (23)$$

and the equation of the viscous magnetic fluid motion

$$\rho\frac{\partial\vec{v}}{\partial t} = -\nabla p + \eta_m\Delta\vec{v} + \frac{(\mu-1)}{8\pi}\nabla\vec{H}^2 \ ; \ div\ \vec{v} = 0$$

allow to obtain the following equation for the growth increment of the circunferential mode of cylinder ($z = (\rho\lambda R^2/\eta_m)^{1/2}$) :

$$\lambda^2 + \frac{2\eta_m}{\rho R^2}n(n-1)\lambda\left(1 + \frac{I_n(z) - 2nI_{n+1}(z)/z}{I_n(z) - 2I_{n+1}(z)/z}\right) + n\left(\frac{\sigma}{\sigma R^3}(n^2-1) - \frac{(\mu-1)^3 H_0^2(n-1)}{2\pi(\mu+1)^3\rho R^2}\right) = 0$$

where I_n is the modified Bessel function of the first kind.

IV. Stability of the magnetic fluid layers

Magnetic free surface instabilities are observed also in rather specific lamellar systerns - ferrosmectics[17]. In Ref. [18] such instability under the action of the normal to ferosmectic layers magnetic field was described as an undulation instability of the multilayer magnetic media arising due to the tendency to decrease the demagnetizing field energy. From the point of view of the complex magnetic fluid dynamics due to specific magnetic instabilities, it is of interest to consider the stability of the single magnetic fluid layer under the action of the high-frequency rotating magnetic field.

Let us take an infinite magnetic layer with thickness 2h and with boundaries in unperturbed state paralell to xy plane. Let us consider a perturbation of layer periodical in the x direction when the equations of boundaries can be written in the following way $z = \pm h + \zeta(x)(\zeta(x) = \zeta\cos(kx))$, which corresponds to the undulation mode of the layer. Perturbation of the potential of the magnetostatic field in the general case of an homogeneous field oblique to the layer $(H_{0x}, 0, H_{0z})$ is found as solution of the Laplace equation at following boundary conditions at layer interfaces (1 - magnetic fluid region, 2 - region $z > h$, 3 - region $z < -h$)

$$\psi_1' - \psi_2' = \zeta(\mu-1)H_{0z}/\mu$$

$$\mu\frac{\partial\psi_1'}{\partial z} - \frac{\partial\psi_2'}{\partial z} = (\mu-1)\zeta' H_{0x}$$

$$\psi_1' - \psi_3' = \zeta(\mu-1)H_{0z}/\mu$$

$$\mu\frac{\partial\psi_1'}{\partial z} - \frac{\partial\psi_3'}{\partial z} = (\mu-1)\zeta' H_{0x} \ .$$

As a result for tlie magnetostatic potential perturbation in the magnetic fluid we have

$$\psi_1' = \frac{(\mu-1)\zeta H_{0z} ch(kz)\cos(kx)}{\mu(ch(kh)+\mu sh(kh))} - \frac{(\mu-1)\zeta H_{0x} sh(kz)\sin(kx)}{(sh(kh)+\mu ch(kh))}$$

Thus the dynamical boundary condition p $= \sigma/R_c - 2\pi(\vec{M}\vec{n})^2$ and the Cauchy-Lagrange integral for the ideal magnetic fluid motion ($\vec{v} = \nabla\phi$)

$$\rho\frac{\partial\phi}{\partial t} + p - \frac{(\mu-1)\vec{H}^2}{8\pi} = const \ ,$$

accounting for kinematic condition on the layer interfaces

$$\frac{\partial\zeta}{\partial t} = \frac{\partial\phi}{\partial z} \ ,$$

after time averaging with respect to high-frequency rotating field ($H_{0x} = H_0\cos(\omega t)$, $H_{0z} = H_0\sin(\omega t)$) allow the following equation for growtli increment of the undulation ($H_{0e} = H_0/\sqrt{2}$)

$$\lambda^2 = -cth(kh)\left(\frac{\sigma}{\rho}k^3 + \frac{(\mu-1)^3(\mu+1)H_{0e}^2k^2}{4\pi\rho\mu(\mu+cth(kh))(1+\mu cth(kh))}\right) \ . \qquad (24)$$

From relation (23) it follows that the layer is stable under the action of the high-frequency rotating field. This is quite natural since opposite to the case of a constant magnetic field normal to the layer, when for the growth increment we have

$$\lambda^2 = cth(kh)\left(\frac{(\mu-1)^2H_0^2k^2}{4\pi\rho\mu(\mu+cth(kh))} - \frac{\sigma}{\rho}k^3\right) \ , \qquad (25)$$

due to the symmetry in high-frequency field there does not exist a preferred layer orientation. In the case of the constant normal field the situation is different and a preferred from the point of view of the demagnetization field energy orientation of the layer along the field direction can be achieved due to the undulation instability of the layer. A critical value for that instability of tlie magnetic field strength H_{0*} in the limit $kh \to 0$ which corresponds to the rotation of the layer as a whole can be found from the relation (28) and is determined by

$$H_{0*}^2 = \frac{4\pi\mu\sigma}{h(p-1)^2} \ .$$

There we can mention the result of Ref. [18] and discuss the peculiarities for normal field undulation instability of ferrosmectics where due to the vanishing of the surface tension restoring force arising at deformation of layer boundaries is caused by bencling elasticity and long-range magnetic interaction between layers is important. In that case the finite thickcness D of the sample leads to a finite value of the critical wavelength which turns out to be[18]

$$\lambda = 2\pi\left(\frac{2K_b}{B2d}\right)^{1/4} D^{1/2}$$

K_b is the bending elasticity constant, B is the compression modulus of smectic layers, and $2d$ is the thickness of the magnetic layers in ferrosmectic.

The critical value of the magnetic field strength for the development of the undulation instability in that case is expressed in following way[18]:

$$H_{0c} = \mu\left(\frac{8\pi K_1}{D\lambda}\right)^{1/2}\frac{(1+l/d/\mu)^{1/2}}{(l/d)^{1/2}(\mu-1)} \ , \qquad (26)$$

where $K_1 = 2K_b/2d$ in the orientation elasticity constant, $\lambda = (K_1/B)^{1/2}$ a thickness of 21 [19] is the penetration length of smectic liquid crystal, water layers in ferrosmectic.

The relation (25) accounting for $l/d \sim \phi^{-1/2}$ [17] allows to conclude that the critical magnetic field strength is proportional to $\phi^{-3/4}$ (ϕ - volume fraction of the magnetic particles in ferrosmectic layers. Evidently to achieve a correspondence with experimental result[17] $H_{0c} \sim \phi^{-1/2}$ it is necessary to know the variation of the material parameters of the ferrosmectic (K_1 is the orientation elasticity constant and B is the compression modulus) with volume fraction of magnetic particles.

Concerning single magnetic layer undulation instability in high-frequency rotating and normal magnetic fields described by relations (23) and (24), respectively one can conclude that the observations of small undulations of extended prolate shapes in experiment[9] are connected with comparable values of the field period and characteristic time of the prolate droplet shape deformations. Work is under progress to simulate those instabilities numerically[14] as well as to account for viscosity and droplets finite dimensions effects on undulation instability.

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