# A Consistent Spectroscopical Analysis for a Generalized Gauge Model

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An Abelian gauge model based on the introduction of independent gauge connections is considered. The spectroscopy of this model shows tlie existence of two sectors, one vectorial and other scalar. It presents a massless photon accoinpanied by other gauge bosons, vector mesons and scalar particles. However the richness of this model comes from the internal consistency that it displays: the existence of different field parametrizations that allow the accommodation of different basis for calculations. This work also includes a study on the invariaiice of the quanta under field reparametrizations.

## I. Introduction

Any particle data shows different particles witli spin 1[1]. There, they appear systematized in two categories: mediators and vector mesons. However their properties such as mass, cliarge, lifetime, principal decays aiid forces, are described by qualitatively different fundamentals. While the mediator class (photon, gluons, $W^{\pm}$, $Z_0$) is associated to the gauge approach and the corresponding grandunification scheme[2], the class of the vector mesons (*p*, $K^*$, $\omega$, $\phi$, $J/\psi$, D*,Y) is associated to tlie quark content[3]. However we still consider propitious to investigate on the possibilities for including both spin-1 families together in a same model, although knowing about their quark discrepancies. Sakurai has already tried a model for different spin-1 particles through a global gauge description[4].

Consider now a set of N vector fields transforming under a commom U (1) group according to tlie following relations:

$$A_{\mu I}(x) \longrightarrow A_{\mu I}(x) + \partial_\mu \alpha(x), \qquad \text{(I.1)}$$

where $I = 1,2,...,N$ and $\alpha(x)$ is the real paraineter associated to Abelian group. The fact that tliey all transform with the real parameter $\alpha(x)$ does not prevent them from being actually N independent vector potentials that might eventually describe degrees of freedom associated to quanta with differeiit physical atrihutions, sucli as mass or some other global internal quantum numbers (flavour, eletric cliarge). Different proofs derived from Kaluza-Klein approacli and from relaxing supersymmetry constraints gives a basis to assume eq.(I.1) as involving N distinct potential fields[5]. In this regard, it is also worthy to recall that differential-geometric considerations support tlie existence of sucli N fields witli transformations as above[6]. In the fibre bundle description of gauge tlieories, as it is well-known, from tlie principal fibre bundle one derives connections. This means the possibility of adding to the connection genuine tensors over tlie principal hundle. In our case, we still maintain tlie picture of a simple connection on the U(l) - b indle, and consider the N potentials as being given by the genuine U(l) - connections to wliich one adds N independent tensors of the internal group. Historically the presence of different connections in a theory can t e reviewed through relativity. There, the Palatini tenror is added to tlie Cliristofell symbol[7]. Thus it s possible to make an analogy between a possible non-mini mal gauge model derived from eq.(I.1) and Einstein-Car ;an theory.

A model considering the presence of different connections in a single group is able to unify distinct spin families witli the saine nature (either bosons or fermions). F:om Lorentz group one gets the information that any vector potential field carries two quanta with spin-1 and spin-0. Consequently, reading off eq.(I.1) one gets the presence of two families with N-vector and N-scalar particles respectively. Then, remembering that this non-minimal gauge model contains QED as boundary condition, one expects the presence of one massless particle and one longitudinal degree of freedom to be naturally frozen. Thus the subtraction of these restri:tions coming from tlie gauge mechanism yields a spectioscopy determined by tlie following excitations: one niassless vector particle,(N-1) massive vector particles, snd (N-1) massive scalar particles. This means that eq (1.1) contains instructions to accomodate a photon, a nonet with spin 1 (vector mesons), and a nonet with zero spin (pseudo scalar mesons). Therefore one notices that this formula develops a scalar-vector model without requiring a coupling with matter fields.

String models contain many multiplets of massive vector mesons[8] as in extended supergravity models[9]. Therefore sucl: theme involving the existence of different gauge mesons is already being studied through different models. However what differentiates one model from another is its consistency in front of physical needs as renormaliza ɔility, unitarity, analiticity, and so on. In this sense, one characteristic coming from a inodel involving different fields in a same group is that it adds another test tc the field approach: the requirement of consistency in the spectroscopy analysis. This means that prior to st ıdying the renormalizability and unitarity properties cf this extended gauge model, we should first analyse whether the quantiim numbers associated to the fields are derived consistently. Therefore there are three minimal conditions to turn eq.(I.1) a candidate to build up physical models. They are the consistencies coming from the spectroscopy, the renormalizability, and the unitarity programs. The main effort of this work will be on the spectroscopy aspect.

A better understanding for eq.(I.1) instructions can be obtained through the following field reparametrizations:

$$\begin{aligned} D_\mu(x) &\longrightarrow D_\mu(x)+\partial_\mu\alpha(x) \\ X_{\mu i}(x) &\longrightarrow X_{\mu i}(x)\ , \end{aligned}$$

where

$$\begin{aligned} D_\mu(x) &= A_{\mu 1}(x)+A_{\mu 2}+\ \dots\ +A_{\mu N}(x) \\ X_{\mu 1}(x) &= A_{\mu 1}(x)-A_{\mu 2}(x) \\ \\ X_{\mu(N-1)} &= A_{\mu 1}(x)-A_{\mu N}(x)\ . \end{aligned} \qquad (I.2)$$

Tlius eq.(I.2) shows that there is only one genuine gauge field, $D_\mu(x)$, wliile the fields $X_{\mu i}(x)$ are gauge-singlets (for the Abelian case). Geometrically the potential fields $X_{\mu i}(x)$ arise from tlie torsion tensor of tlie higher - dimensional manifold that spontaneously coinpactify to $M_4$ x$B_k$, where $M_4$ is tlie Minkowski space-time and $B_k$ some K-dimensional internal space. Neverthless although tliere is a geometric origin for $X^i_\mu$ fields we still slioiild argue about their distinction from tlie Proca case. We should discuss that such proposed non-minimal gauge model does not represent a combination between the usual gauge theory written for a $D_\mu(x)$ field with a Proca model containing (N-1) massive potential fields. Eq.(I.2) offers gauge instructions as gauge fixing term and Ward identity, which iiiclude $X_{\mu i}(x)$ fields on their respective mechanism; another difference comes from the fact that the $X_{\mu i}(x)$ fields longitudinal sector propagates.

This work is organized as follows. In Section II the structure of this generalized gauge model is presented. Then it is observed the existence of different field parametrizations to be analysed. This fact motivates Section III, which studies a $\Omega$ matrix that regulates such field hasis transformations. Then it is left for Section IV the model spectroscopical analysis. In order to explore a little more about this possibility of having different field frameworks wliich preserve quanta invariance, we work out at Section V a longitudinal diagonalized basis. An Appendix follows for showing explicit calculations involving two potential fields in a same group.

## II. Field parametrizations

Symmetry can be dressed through different basis, the field parametrizations. Tlie simplest case is when one derives the symmetry messages through the $\{D, X_i\}$ - basis which is defined in Eq.(II.2). It is called the constructor set. It yields the general Lagrangian

$$\mathcal{L} = Z_{\mu\nu}Z^{\mu\nu} + Z_{\mu\nu}\widetilde{Z}^{\mu\nu} + m^2_{ij}X^i_\mu X^{\mu j} + \mathcal{L}_{G.F.} \quad \text{(II.1)}$$

Decomposing tlie generalized field strenght, $Z_{\mu\nu}$, in antisymmetric and symmetric pieces, one gets

$$Z_{\mu\nu} = Z_{[\mu\nu]} + Z_{(\mu\nu)} \; , \quad \text{(11.2)}$$

where

$$Z_{[\mu\nu]} = dD_{\mu\nu} + \alpha_i X^j_{\mu\nu} + \gamma_{[ij]}X^i_\mu X^j_\nu \; , \quad \text{(II.3)}$$

and

$$\begin{aligned} Z_{(\mu\nu)} &= \beta_i \Sigma^i_{\mu\nu} + \rho_i g_{\mu\nu}\Sigma^{i\alpha}_\alpha + \gamma_{(ij)}X^i_\mu X^j_\nu + \\ &+ \tau_{ij} g_{\mu\nu} X^i_\alpha X^{\alpha j} \; , \end{aligned} \quad \text{(II.4)}$$

with

$$\begin{aligned} D_{\mu\nu} &= \partial_\mu D_\nu - \partial_\mu D_\nu \\ X^i_{\mu\nu} &= \partial_\mu X^i_\nu - \partial_\nu X^i_\mu \\ \Sigma^i_{\mu\nu} &= \partial_\mu X^i_\nu + \partial_\mu X^i_\nu \; , \end{aligned} \quad \text{(II.5)}$$

and

$$\widetilde{Z}_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma}Z^{\rho\sigma} \; . \quad \text{(II.6)}$$

As a first product derived from this symmetry extension, there appears the so-called free coefficients. They are coefficients associated to every Lorentz and gauge scalar developed by this extended model. As an exainple take $d^2$, $d\alpha_i$, $m^2_{ij}$, and so on. Thus there is a total of $\frac{N(N-1)}{2}$ free coefficients present in (11.1). They are numbers which can take any real value. Their main conseqiieiice is on the pliysics dependence on their values. This means that such Lorentz and garige scalars contain possibilities of parametrizing the physical entities that symmetry organizes. For instance, a quantum number such as the physical mass will be determined through these free coefficients, and so can take different values without breaking gauge syminetry as explicit calculations iii Appendix A are shows.

The contribution coming from $\widetilde{Z}_{\mu\nu}$ is called a semi-topological Lagrangian[10]. It is a particularity from this generalized gauge model. This is due to the fact that even in four dimensions tlie $Z_{\mu\nu}$ tensor appears contributing to the interaction sector, altliough it does not to the kinetic sector.

From Lorentz group representations, one gets instructions where spin-1 sector is localizecl in **(11.3)** and (11.4) while spin-0 part will be only in (11.4): $Z_{[\mu\nu]}$ and $\widetilde{Z}_{\mu\nu}$ belongs to (0,1) e (1,0), while $Z_{(\mu\nu)}$ belongs (0,0) e (1,1). Tliis prediction can be directly tested by analysing that the covariant field strengtli $\Sigma^i_{\mu\nu}$ contributes to botli spin sectors when eq.(II.1) is organized in terms of transverse and longitudinal operators. Froni Poincaré group, one expects that representations with different spins will present different masses for the transverse and longitudinal sectors.

Being a gauge theory, this generalized gauge model requires a gauge fixing terin. Giveii the presence of only one gauge group we have just one gauge fixing term to fix the potential field orbits. From[11], the most general gauge fixing term involving such N-potential fields for the covariant case is

$$\mathcal{L}_{G.F.} = \frac{1}{2\alpha}\left[\partial.(D + \sigma_i X^i)\right]^2 \quad \text{(II.7)}$$

Observe the inclusion of $\sigma_i$ parameters. They are not necessary to fix a garige. However these parameters are allowed by the gauge mechanism, and so we have to include them in the most general gauge fixing form which theory provides. Indeed there is nothing new in the $\sigma_i$ parameters inclusion, and they can be compared with the $\beta$ - parameter which writes a QED with the following gauge-breaking term, $\frac{1}{2\alpha}\left[\partial_\mu A^\mu + \beta A_\mu A^\mu\right]^2$.

In order to derive the Lagrangian spectrum more explicitly, eq.(II.3) should be rewritten in terms of transverse and lcngitudinal propagators. Defining

$$V_\mu^t \equiv (D_\mu, X_\mu^t) \ , \tag{11.8}$$

it yields,

$$\mathcal{L} = \mathcal{L}_k + \mathcal{L}_{\text{int}} \ , \tag{II.9}$$

where

$$\mathcal{L}_K = \frac{1}{2} V_\mu^t \left[(\Box K_T + M^2)P_T^{\mu\nu} + (B\Box + M^2)P_L^{\mu\nu}\right] V_\nu \ , \tag{II.10}$$

with

$$K_T = 4\begin{pmatrix} d^2 & d\alpha_1 & \dots & d\alpha_j & \dots \\ d\alpha_1 & \alpha_1^2+\beta_1^2 & \dots & & \\ \vdots & & & & \\ d\alpha_i & \alpha_i\alpha_j + \beta_i\beta_j & \dots & & \\ \vdots & \vdots & \vdots, & & \end{pmatrix} \tag{II.11}$$

and

$$B = K_L + G_F \ , \tag{II.12}$$

with

$$K_L = 8\begin{pmatrix} 0 & 0 & \dots & 0 \\ 0 & (\beta_1+\rho_1)^2 + 3\rho_1^2 & \dots & \beta_1\beta_{N-1} + \rho_1\rho_{N-1} + 2\beta_1\rho_{N-1} \\ \vdots & \vdots & & \vdots \\ 0 & \begin{matrix}\beta_{N-1}\beta_1 + 4\rho_{N-1}\rho_1 + 2\beta_{N-1}\rho_1 \\ (\beta_{N-1}+\rho_{N-1})^2 + 3\rho_{N-1}^2\end{matrix} & \dots & \end{pmatrix} , \tag{II.13}$$

$$G_F = \frac{1}{\alpha}\begin{pmatrix} 1 & 2\sigma \\ 2\sigma^t & \sigma^t\sigma \ , \end{pmatrix} \tag{II.14}$$

and

$$M^2 = \begin{pmatrix} 0 & 0 & \dots & 0 & \dots & 0 \\ 0 & m_{11}^2 & \dots & m_{ij}^2 & \dots & m_{1(N-1)}^2 \\ \vdots & \vdots & & \vdots & \vdots & \\ 0 & m_{(N-1)1} & & \dots & & m_{(N-1)(N-1)}^2 \end{pmatrix} \tag{II.15}$$

Two basi: quantum numbers emerge from the kinematics of such generalized gauge model. They are the spin and the mass. For the vector family, the corresponding physical masses are eigenvalues of the matrix $(K_T^{-1}M^2)$, and, for the scalar family one gets that physical masses will be the $(B^{-1}M^2)$ eigenvalues.

Three facts are showing that theory does not depend on $\sigma_i$ parameters. First, it is because the physical entities should not depend on it. Then, one can show that it does not suffer any renormalization process and that the theory stability does not suffer its influence[12]. Tliird, the gauge fixing term does not require it in order to the propagator to have an inverse. Frorn (II.10), the existence condition for the longitudinal propagator, $< V_\mu V_\nu >_L$, is to have an invertible rnatrix $Q = \left[(K_L + \sigma^t\sigma)\Box + M^2\right]$. For this, the condition of liaving a matrix $\sigma^t$ a invertible will be not essential. Consequently, one can conclude from these three aspects that the paranieters $\sigma_i$ should be only interpreted as anotlier family of free coefficients.

Although it is an Abelian case, sucli extended model has a gauge invariant interaction part. It is written as

$$\mathcal{L}_{int} = \mathcal{L}_{int}^{(3)} + \mathcal{L}_{int}^{(4)} , \tag{11.16}$$

for

$$\begin{aligned} \mathcal{L}_{int}^{(3)} &= z^t\partial^\mu V^\nu [V_\mu^t \lambda V_\nu + \epsilon_{\mu\nu\rho\sigma} V^{\rho t}\lambda V^\sigma] + \\ &+ t^t\partial^\mu V^\nu [V_\mu^t \Lambda V_\nu] + \\ &+ \omega^t\partial_\mu V^\mu [V_\nu^t \Lambda V^\nu] + v^t\partial_\mu V^\mu [V_\nu^t \Theta V^\nu] , \end{aligned}$$

where

$$z = \begin{pmatrix} d \\ \alpha_i \end{pmatrix} , \quad t = \begin{pmatrix} 0 \\ 0\beta_i \end{pmatrix}$$
$$( \quad i \quad ) \qquad v = \begin{pmatrix} 0 \\ \beta_i + 4\rho_i \end{pmatrix} , \tag{II.17}$$

and

$$\begin{aligned} \mathcal{L}_{int}^{(4)} &= [V^{\mu t}\Theta V_\mu][V^{\nu t}\Gamma v_\nu] + [V^{\nu t}\Sigma V_\mu]^2 + \\ &+ \varepsilon^{\mu\nu\rho\sigma}[V_\mu^t \lambda V_\nu][V_\rho^t \lambda V_\sigma] . \end{aligned} \tag{II.18}$$

Matrices A, A, Θ, Γ, C are given by

$$\lambda = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & \gamma_{[12]} & \cdots & \gamma_{[1,N-1]} \\ 0 & \gamma_{[21]} & 0 & \cdots & \gamma_{[2,N-1]} \\ \vdots & \vdots & \vdots & & \\ 0 & \gamma_{[N-1,1]} & \gamma_{[N-1,2]} & \cdots & 0 \end{bmatrix}$$

$$\Lambda = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & \gamma_{(11)} & \gamma_{(12)} & \cdots & \gamma_{(1,N-1)} \\ 0 & \gamma_{(21)} & \gamma_{(22)} & \cdots & \gamma_{(2,N-1)} \\ \vdots & \vdots & \vdots & & \\ 0 & \gamma_{(N-1,1)} & \gamma_{(N-1,2)} & \cdots & \gamma_{(N-1,N-1)} \end{pmatrix}$$

$$\Theta = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & \tau_{11} & \tau_{12} & \dots & \tau_{1,N-1} \\ 0 & \tau_{21} & \tau_{22} & \dots & \tau_{2,N-1} \\ \vdots & \vdots & \vdots & & \\ 0 & \tau_{N-1,1} & \tau_{N-1,2} & \dots & \tau_{N-1,N-1} \end{pmatrix}$$

$$\Gamma = \begin{pmatrix} 0 & 0 & \dots & 0 \\ 0 & \gamma_{(11)} + 2\tau_{12} & \dots & \gamma_{(1,N-1)} + 2\tau_{1,N-1} \\ 0 & \gamma_{(21)} + \tau_{21} & \dots & \gamma_{(2,N-1)} + \tau_{2,N-1} \\ \vdots & \vdots & & \vdots \\ 0 & \gamma_{(N-1,1)} + 2\tau_{N-1,1} & \dots & \gamma_{(N-1,N-1)} + 2\tau_{N-1,N-1} \end{pmatrix}$$

$$\Sigma = 2 \begin{pmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & \gamma_{11} & \gamma_{12} & \dots & \gamma_{1,N-1} \\ 0 & \gamma_{21} & \gamma_{22} & \dots & \gamma_{2,N-1} \\ \vdots & \vdots & \vdots & & \\ 0 & \gamma_{N-1,1} & \gamma_{N-1,2} & \dots & \gamma_{N-1,N-1} \end{pmatrix} . \tag{II.19}$$

The corresponding N-equations of motion derived from (11.9) are

$$K_{\mathrm{T}}\partial_\mu V^{\mu\nu} + \frac{1}{4}M^2 V^\nu = J^\nu_{\mathrm{v}} , \tag{II.20}$$

where $V_{\mu\nu}$ is a (N x N) matrix given by

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu , \tag{11.21}$$

and $J^\nu$ is a (N $\otimes$ 1) column vector originated from tlie kinetic scalar part and from tlie iiiteraction part. It is given by

$$\begin{aligned} J^\nu_{\mathrm{v}} &= B\partial^\nu(8.V) - \mathrm{wd}''[V^t(\Theta+\Sigma)V] + \\ &+ [z^t V^{\Theta\nu} + V^{t\Theta}\lambda V^\nu + 4\varepsilon^{\mu\eta\nu\Theta} V^t_\mu \lambda V_\eta]\lambda V_\Theta + \\ &+ \partial_\mu[z V^{\mu t}\lambda V^\nu + t[g^{\mu\nu} V^t \Theta V + V^{\mu t}\lambda V^\nu] + 4\varepsilon^{\mu\nu\eta\sigma} z V^t_\eta \lambda V_\sigma] \\ &- [t^t(\partial_\mu V^\nu + \partial^\nu V_\mu) + 2g^\nu_\mu \omega^t \partial_\alpha V^\alpha + g^\nu_\mu V^t_\alpha \Theta V^\alpha + V^t_\mu \lambda V^\nu]\lambda V^\mu \\ &- [V^t_\mu(\Theta+\Sigma)V^\mu + 2(\omega+t)^t \partial_\mu V^\mu]\Theta V^\nu . \end{aligned} \tag{11.22}$$

Eq. (11.20) works as anotlier proof of tlie presence of N independent fields in a same gauge group. This is so because it shows the existence of N independent equations derived from Eq. (1.1). Observe that the equations of motion for fields $V^\mu_o \equiv D_\mu$ and $V^\mu_i \equiv X^\mu_i$ differ basically on tlie gauge fixing terin and on presence of tlie current $J^\mu_i$.

Deriving Eq. (11.20) we obtain the followiig set of (N-1) equations

$$\partial_\mu J^\mu_i = \frac{1}{4}M^2_{iI}\partial.V^I . \tag{II.23}$$

Eq. (11.23) shows that (N-1) scalars are iiot decoupled unless one is able to prove tlieir curreiit conservation.

Another natural invariance from a gauge tlieory is tlie Noether theorem. For tliis extended case the difference is that it will relate all fields transforming under a same gauge parameter, $\alpha(x)$. It yields

$$\partial_\nu N^\nu = 0 ,$$

with

$$N^\nu = \frac{\delta\mathcal{L}}{\delta\partial_\nu V^\mu_I}\delta V^\mu_I . \tag{11.24}$$

Substituing (II.24) in (11.23) oiie gets

$$B_{oI}\partial.V^I = 0 . \tag{II.25}$$

Tlierefore $J^\mu_i$ currents will suffer contrihutions only from elements $B_{iI}$ in (11.12).

After an initial study tlirougli tlie construtor set, {V}, let us diagonalize the transversal sector. Then one gets the {G} set, called as pliysical field parametrization. This terminology is due to the fact that the transverse physical masses will correspond to tlie poles of their-two point Green's functions. This set is defined through the following transformation:

$$V \equiv \Omega_T G \;, \qquad (11.26)$$

wliere

$$\Omega^t_T K_T \Omega_T = 1$$
$$\Omega^t_T M^2 \Omega_T = m_t^2 \qquad \text{(diagonal)}$$

$$\Omega^t_T B \Omega_T = \widetilde{B} \;. \qquad (II.27)$$

Substituing Eqs. (II.26), (11.27) in Eqs. (11.10) and (II.16), we have

$$\mathcal{L}(G) = \mathcal{L}_K(G) + \mathcal{L}_{int}(G) \;,$$

with

$$\mathcal{L}_K(g) = \frac{1}{2} G^t_\mu \left[ (\Box + m_T^2) P_T^{\mu\nu} + (\Box \widetilde{B} + m_T^2) P_L^{\mu\nu} \right] G_\nu \;, \qquad (II.28)$$

and

$$\begin{aligned}
\mathcal{L} &= z^t \partial^\mu G^\nu (G^t_\mu \lambda_T G_\nu + \varepsilon_{\mu\nu\rho\sigma} G^\rho \lambda_T G^\sigma) + \\
&+ t^t_T \partial^\mu G^\nu (G^t_\mu \Lambda_T G_\nu) + \omega^t_T \partial_\mu G^\mu (G^t_\nu \Lambda_T G^\nu) + \\
&+ v^t_T \partial_\mu G^\mu (G^t_\nu \Theta_T G^\nu) + (G^{\mu t} \Sigma_t G_\mu)^2 + \\
&+ (G^{\mu t} \Theta_T G_\mu)(G^{\nu t} \Gamma_T G_\nu) + \\
&+ \varepsilon^{\mu\nu\rho\sigma} (G^t_\mu \lambda_T G_\nu)(G^t_\rho \lambda G_\sigma) \;,
\end{aligned} \qquad (II.29)$$

wliere each of the column inatrices $z_T, ..., v_T$ transforms like

$$z_T = \Omega^t_T z \;, \qquad (11.30)$$

and each of tlie row matrices $\lambda_T, ..., \Gamma_T$ as

$$\lambda_T = \Omega^t_T \lambda \Omega_T \;. \qquad (11.31)$$

Observe then the appearance of a $\Omega_T$ matrix controlling tlie field basis transformations. It depends on the initial Lagrangian coefficients

$$\Omega_T \equiv \Omega_T \left( d,\ \alpha_i,\ \beta_i,\ \rho_i, m^2_{ij} \right) \;. \qquad (11.32)$$

Thus any information can be transposed from a given set of fields to some cliosen set. For instance, tlie relationsliip between propagators is given by

$$< T(V_\mu V_\nu) >= \Omega < T(G_\mu G_\nu) > \Omega^t \;, \qquad (11.33)$$

where $\Omega$ is a general matrix.

Eq. (11.33) is tlie propagators covariance law. This means that wliile fields transform as matrices, propagators behave as tensors. Its importance is due to tlie fact that R expression does not depend on tlie momenta. This yields tlie property that propagators transformation (11.33) will preserve tlie pole structure (the influeiice coming from R matrix will affect only tlie residues). Anotlier conseqiience derived from (11.33) is that non-diagonal propagators are symmetric in any field basis. Considering tliat matrices in (11.10) are ortliogoiial from gauge symmetry, one gets

$$< V_\mu V_\nu >_L =< V\mu V_\nu >^t_L \;. \qquad (11.34)$$

Tlien substituting (11.34) in (II.33), it gives

$$< G_\mu G_\nu >_L =< G_\mu G_\nu >^t_L \;, \qquad (II.35)$$

wliere (11.35) verifies a expected result from functional formalism and from time order decomposition.

Depending on tlie type of investigation, a certain field parametrization can be more useful. For analysing the spectroscopy the $\{G_I\}$ basis is more direct. However for the renormalization analysis there is another basis denoted by $\{\widetilde{D}, X_i\}$ where

$$\widetilde{D}_\mu(x) \equiv D_\mu(x) + \frac{1}{d^2}\alpha_i X^i_\mu(x) \ , \qquad (II.36)$$

which is mcre useful (because it avoids mixing propagators in tlie transverse sector). Consequently, there exists a large variety of fields parametrizations to be explored. In this way, we feel obliged to explore in a next chapter the properties of a general R-matrix which controlls any transformation between these field parametrizations.

## III. R matrix

The introduction of more fields in a same group develops different possibilities for a given physics to be observed[13]. One can read off the symmetry instructions in these extended gauge models involving bosons, fermions, potentials, and other fields, tlirough different parametrization systeins. Neutrino physics already shows cases where physics can be studied under distinct fermion parametrizations[14]. Consequently, from this possibility of liaving different bases for physics to be analyzed, a iiew theoretical structure called $\Omega$-matrix naturally emerges. It describes tlie transformation between two different sets of field parametrizations,

$$\varphi_s = \Omega\Phi_s \ , \qquad \text{(III.1)}$$

where $\varphi_s$ and $\Phi_s$ represent any set of fields with a same spin structure. Tlie generalized index $s$ represents the nature of the particle family. For instance, for the case of vector fields we have tlie following sets: $\{\varphi_s\} \equiv \{D, X_i\}$, $\{\Phi_s\} \equiv \{G_I\}$. Notation here is to associate $\Phi_s$ to tlie physical fields, which are defined as containing explicitly tlie physical masses as the poles of their corresponding two-point Green functions.

In such generalized model the relationsliip betweeii quanta and fields is not more in a one-by-one correspondente. The situation generated from Eq.(I.1) makes the correlation between fields and quanta to develop a new aspect where a given field can contain various quanta. This means tliat fields will work just as an auxiliary device for the dynamics to be observed, while quanta will emerge from this dynamics. A viewpoint is to make an analogy where the length of such new coordinates - the fields - would be determined by tlie degrees of freedom, and so, Eq. (111.1) represents possible clianges on "field coordinates" while the involved quanta are preserved. Therefore, the task will be to understand under which conditions tlie R matrix does not affect tlie physics.

The R matrix presence indicates that symmetry can be dressed with different field parametrizations. However tliis proposal witli field rotations must preserve physical structures such as, at least, S-matrix and the minimal action principle. Borscher theorem states tliat any local field redefinition will not affect S-matrix, and consequently a first condition for R matrix validity is to have any of its elements not depending on momentum[15]. This is easily verified because the R matrix elements are obtained from tlie free coefficients derived from tlie covariant fields strength written in (11.2). Being real scalars, the Lagrangian and the action are also invariant under Eq. (III.1),

$$\begin{aligned} \mathcal{L}_{[\varphi_s]} &= \mathcal{L}[\Omega\Phi_s] = \widetilde{\mathcal{L}}[\Phi_s] \\ S[\varphi_s] &= \widetilde{S}[\Phi_s] \end{aligned} \qquad \text{(III.2)}$$

which yield tlie following relationsliip between the minimal actions:

$$\frac{\delta S[\varphi_s]}{\delta\varphi_s} = \Omega^{-1}\frac{\delta\widetilde{S}[\Phi_s]}{\delta\Phi_s} = 0 \ . \qquad \text{(III.3)}$$

Thus Eq. (111.3) shows that whenever R has inverse, then tlie corresponding equations of motion in any set of field parametrization will preserve the on-shell information. Other two general aspects can work to compleinent the S-matrix and tlie minimal action principle invariances. Tliey are tlie conservation of the number of fields and tlie preservation of the spin structure under sucli R rotations. Heuristically, given that any reparametrization conserves the number of degrees of freedom one expects the number of fields involved will be preserved. Similarly, the fact that R is a Lorentz scalar makes Eq. (111.1) to preserve the spin structure, and so, tlie $\varphi_s$ and $\Phi_s$ families will belong to the same spin structure.

After the fundamental conditions are satisfied by Eq. (III.1) (for further analysis see Ref. [13]), we should investigate the characteristics which R develops. For this we will select two questions. The first would be whether $\Omega$ forms a group. Being a matrix, it contains the identity and associative properties, and satisfying the Borscher theorem, it contains the inverse. However the property that any two symmetry operations of a group performed in succession also corresponds to an operation in that group is not immediate. This means that closure property fails to hold in general, as soon we shall see. The second question, would be to understand the structure which $\Omega$ develops. This means to classify the expressions derived in this extended model relatively to $\Omega$, as the scalars and tensors generated, the similarity and covariant transformations obtained, and the relative and absolute quantities revealed. The present work will be interested in the study of the well known conservation laws respectively to Eq. (111.1).

Space - time charges and internal symmetries should be invariant under Eq. (111.1). The current density written in terms of components $\Phi_{sI}$, where I varies from 1 to N, is

$$\partial_\mu \tilde{J}^\mu[\Phi_s] = 0 \ ,$$

with

$$\begin{aligned} \tilde{J}^\mu[\Phi_s] &= -\tilde{T}^\mu_\nu[\Phi_s]\delta x^\nu + \frac{\partial\tilde{\mathcal{L}}(\Phi_s,\partial\Phi_s)}{\partial\partial_\mu\Phi^s_I}\delta\Phi^s_I \\ \tilde{T}^\mu_\nu[\Phi_s] &= \frac{\partial\tilde{\mathcal{L}}(\Phi_s,\partial\Phi_s)}{\partial\partial_\mu\Phi_{sI}}\partial_\nu\Phi_{sI} - g^\mu\nu\tilde{\mathcal{L}} \ . \end{aligned} \quad \text{(III.4)}$$

Then one gets that Eq. (111.4) is a scalar with respect to transformation (111.1):

$$\partial_\mu J^\mu[\varphi_s] = 0$$

with

$$\begin{aligned} J^\mu[\varphi_s] &= -T^\mu_\nu[\varphi_s]\delta x^\nu + \frac{\partial\mathcal{L}(\varphi_s,\partial\varphi_s)}{\partial\partial_\mu\varphi_{sI}}\delta\varphi_{sI} \\ T^\mu\nu[\varphi_s] &= \frac{\partial\mathcal{L}(\varphi_s,\partial\varphi_s)}{\partial\partial_\mu\varphi_{sI}}\partial_\nu\varphi_{sI} - g^\mu\nu\mathcal{L} \ . \end{aligned} \quad \text{(III.5)}$$

Concluding, the equivalence between eqs. (111.4) and (111.5) show that the Weyl group is preserved for different field parametrizations.

Further two invariances will be quoted here. They are the $\Omega$-invariance for the commutation rules and for the equations of motion. Given

$$[\varphi_{sI}(x,t), \Pi_{sJ}(x',t)] = i\delta_{IJ}\delta^3(x-x') \ ,$$

we have

$$[\Phi_{sI}(x,t), \Pi^\Phi_{sJ}(x',t)] = \delta_{IJ}\delta(x-x') \ . \quad \text{(III.6)}$$

Similarly,

$$\frac{\partial\mathcal{L}}{\partial\varphi^s_I} - \partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\varphi^s_I} = (\Omega^{-1})^t_{IJ}\left\{\frac{\partial\tilde{\mathcal{L}}}{\partial\Phi^s_J} - \frac{\partial\tilde{\mathcal{L}}}{\partial\partial_\mu\Phi^s_J}\right\} \ . \quad \text{(111.7)}$$

Nevertheless, the most profound insight from Eq. (111.1) is the possibility of modifying the simmetry shape without changing the substance of its instructions[13]. Consider the following phase transformation

$$\varphi^{'}_s = U(\omega)\varphi_s \ . \quad \text{(111.8)}$$

Substituting (111.1) in (III.8), one gets

$$\Phi^{'}_s = T(\omega)\Phi_s$$

where

$$T(\omega) = \Omega^{-1}U(\omega)\Omega \ . \quad \text{(III.9)}$$

Relation (111.9) shows that similarities which the symmetry refiects can change their shape according to the parametrization set. However it must preserve the physics, as well as the established conservation laws. One can note from Eq. (111.9) is that although the principle of local symmetry is preserved, the diagonal and non-diagonal versions of a same invariance are described through different matrices $U(\omega)$ and $T(\omega)$. Therefore a consequence from (11.10) is that a same symmetry can be accomplislied through isomorphic groups. For $T(\omega) = e^{i\ \omega^a\ H\ a}$, the corresponding change of basis is

$$\omega^a H_a = \Omega^{-1}(\omega^a t_a)\Omega \ . \quad \text{(III.10)}$$

For instance, restricting the group transformations to those with an unitary determinant, one notes as a consequence of the above similarity transformation that the traceless condition of their generations is preserved.

The intecference of R on $U(\omega)$ keeps the number of group parameters and the structure of the group algebra. Given

$$[t_a, t_b] = i f_{abc}\, t_c \ . \qquad \text{(III.11)}$$

one gets

$$[H_a, H_b] = i f_{abc}\, H_c \ . \qquad \text{(111.12)}$$

Thus expressions (111.12) and (111.13) show that a common structure constant $f_{abc}$ organizes a same Lie Algebra with distinct generators.

Classically, information derived from equations of motion, momentum-energy and angular momentum tensors, continous symmetries and their respective conserved currents, discrete symmetries, and current algebra are preserved under $\Omega$. On quantum grounds, the Wigner theorem ensures a unitary implementation for the inteinal symmetries. Thus although $T(\omega)$ is not unitary in the space representation where tlie fields transform as vectors, the Wigner theorem guarantees the existence of a corresponding unitary (or antiunitary) operator in the Hilbert space where the fields act as operators. This means that

$$\Phi'(x) = U(\omega)\Phi(x)U(\omega)^{-1} \ , \qquad \text{(111.13)}$$

$$\varphi'(x) = \Im(\omega)\varphi(x)\Im(\omega)^{-1} \ , \qquad \text{(III.14)}$$

where

$$U(\omega) = e^{i\,\omega^a\, Q_a[\varphi]} \ , \qquad \text{(111.15)}$$

$$\Im(\omega) = e^{i\,\omega^a\, \tilde{Q}_a[\Phi]} \ , \qquad \text{(111.16)}$$

and

$$Q_a[\varphi] = \int d^3x\, J_a[\varphi] = \tilde{Q}_a[\Phi] \ . \qquad \text{(111.17)}$$

Eqs. (111.13) - (111.17) show tliat quantum-mechanically the isomorphic correspondence between representations is trivially represented. The basic $U(\omega)$ and $\Im(\omega)$ matrices differ functionally, but algebraically are equal.

Up to now, this section has tried to understand R just abstractly. However, after this introduction, it turns now to be necessary to derive its expression. For this, a first clue is to consider the covariance of the equations of motion and conservation laws with respect to the fields parametrizations. As a result, one obtains two general matrix relationships, .

$$\Omega^t K \Omega = 11$$

$$\Omega^t M^2 R = m^2 \text{ (diagonal)} \qquad \text{(111.18)}$$

(for complex fields case, take $\Omega^+$ instead of $\Omega^t$).

The basic assumption for the invariance considered here is tliat any parametrization process must keep quanta invariance. Therefore any quantum number assigned to the quanta must be equally conserved by any proposed parametrization. Then taking in consideration mass invariance, one derives from (111.18) the following expression

$$\Omega^{-1}(K^{-1}M^2)\Omega = m^2 \ , \qquad \text{(III.19)}$$

which shows that physical masses are invariant under (111.1).

Tlius through Eqs. (111.18) and (111.19) one gets a first indication to determine $\Omega$. Nevertheless, only from quanta analysis is tliat R will be completely determined. For tliis one should rotate tlie initial general Lagrangian described by fields $\varphi$ into that one written in terms of physical fields. Then one needs to diagonalize the kinetic and mass matrices. It finally yields

$$\Omega^{-1} = R\tilde{K}^{1/2}S \ , \qquad \text{(III.20)}$$

where S and R are orthogonal (unitary) matrices which diagonalize $K$ and the subsequent mass term, respectively. The diagonal matrix $\tilde{K}$ is given by

$$\tilde{K} = SKS^t \ , \qquad \text{(III.21)}$$

Four consequences can be initially viewed from Eq. (111.20). First, it does not satisfy the closure relationship. Consequently it answers the initial question by saying that the set of rotations R does not form a group. Other aspects are that $\Omega$ is not necessarily orthogonal (unitary) and that the conditions for $\Omega$ be invertible will depend on $\tilde{K}$-matrix eigenvalues be non zero. At last, it verifies the (111.19) similarity relation.

In order to take some example with Eqs. (111.1) - (III.20), for simplicity one can choose a generalized scalar model. Consider the following Lagrangian involving N-scalar fields with a commom global transformation

$$\mathcal{L} = \varphi^t(K\Box + M^2)\varphi \ , \qquad \text{(III.22)}$$

where $\varphi^t \equiv (\varphi_1, \ \ldots \ , \ \varphi_N)$. The corresponding equations of motion are

$$(K\Box + M^2)\varphi = 0 \ . \qquad \text{(III.23)}$$

Substituting (111.18) and (III.22), one gets

$$\tilde{\mathcal{L}} = \Phi^t(\Box + m^2)\Phi \ . \qquad \text{(III.24)}$$

Then working out from (111.24) the corresponding N-Klein Gordon equations and comparing with (111.23) through (111.18) one gets a kind of closure.

## IV. A consistent spectroscopical analysis

An abundance of degrees of freedom is obtained from this generalized gauge model based on the presente of different connections associated to a single principal fibre bundle. Thus we should now investigate the physical excitations that **Eq.** (1.1) generates. From Borsher theorem, one gets that different field parametrizations sliould be available for describing the physics contained in a given Lagrangian. By consistent spectroscopical analysis we mean the physics invariance under R matrix rotations. Tlierefore any quantum number necessary to classify the spectroscopy must contain the property of being invariant under these possible choices of parametrizations.

Spectroscopy analysis means to classify entities such as spin, mass, internal symmetries, residues of the propagators and discrete symmetries for each involved particle. However the task here is not only to derive from a given Lagrangian the named spectroscopical entities but also to include a test of consistency, i.e., to sliow that they are invariant under transformations (111.1).

For the spin case, as the $\Omega$ matrix is a Lorentz scalar, one already lias the heuristic information that spin will be unaffected. However this fact can be also proved. Taking the classical transformation[16]

$$\left[S^k[\varphi], \ \varphi(x)\right] = \delta\varphi(x) \ , \qquad \text{(IV.1)}$$

where $S^b$ is the spin operator and $\delta\varphi(x) = \omega^{\mu\nu}\Sigma_{\mu\nu}\varphi(x)$ measures tlie spin rotation. Substituting (111.1) in (IV.1), one gets

$$\left[\tilde{S}^k[\Phi], \ \Phi(x)\right] = \delta\Phi(x) \ . \qquad \text{(IV.2)}$$

Tlien, comparing (IV.1) with (IV.2) we have tliat spin rotations do not depend on $\Omega$ matrix.

For the mass case, Eq. (111.19) shows the invariance under different field basis for tlie physical masses.

For internal symmetries, from Eqs. (111.4) and (III.5), one reads off that any associated Noether charge will be an invariant, although changing its functional shape. Thus for proving quanta invariance we still need to work out considerations ahout tlie residue signs and the discrete symmetries.

Tlie physical contribution from the residues is on the relative value of their signs. They will reflect or not the presence of ghosts. However for. taking such consideration just the diagonal propagators are relevant (tlie spectral function which determine each of the 1-particle state norm is only associated to them). Thus, considering a certain pole at $k^2 = m^2$, one derives that its corresponding residue matrix, $R_{\Phi\Phi}(k^2 = m^2)$, will be related to tlie residues in another basis $< T\varphi\varphi >$ through Eq. (111.1):

$$R_{\varphi_s\varphi_s}(k^2 = m^2) = \Omega^{-1}R_{\Phi_s\Phi_s}(k^2 = m^2)\Omega^{-1t} \ . \qquad \text{(IV.3)}$$

Thus imposing tlie following diagonalized basis,

$$R^{jk}_{\Phi_s\Phi_s}(k^2 = m^2) = \delta^{jk} \ , \qquad \text{(IV.4)}$$

we have

$$R^{ii}_{\varphi_s\varphi_s}(k^2 = m^2) = \sum_j(\Omega^{-1}_{ij})^2 > 0 \ , \qquad \text{(IV.5)}$$

and

$$R^{ij}_{\varphi_s\varphi_s}(k^2=m^2)=\sum_l \Omega^{-1}_{il}\Omega_{jl}\ . \qquad \text{(IV.6)}$$

Consequently from Eq. (IV.5) one gets that any residue matrix corresponding to a given pole will preserve their diagonal residue signs. For off-diagonal terms, Eq. (IV.6) is showing an undetermined sign. However this information is not necessary to justify the intended consistent spectroscopy. Concluding, Eq. (IV.5) is showing that whether there is any ghost in the basis $\Phi_s$ it will also be detected in the basis $\varphi_s$.

Ref. [13] has analysed the invariance under the three discrete symmetries P, C, and T. The proof has also shown how these separate discrete invariances are independent on the field frameworks that Section III has contemplated.

After concluding the discussion about quanta invariance, we have now to study properly the spectroscopy analysis. This means to derive the phenomenology contained in this extended gauge model. The starting point should be the Lorentz and Poincaré groups. They already indicate that Lagrangian (11.1) contains two families with spir 1 and spin 0 respectively, and also with components carrying different masses. Then, taking QED as a boundary condition, one is able to predict for the transversal sector the presence of N poles with spin 1, where at least one of them is massless. Similarly for tlie longitudinal sector, one expects the presence of a spurious massless pole together with other poles sliifted with radiative corrections.

A consequence from the quantum numbers invariance is that different channels of fields parametrizations appear offering the opportunity of choice for calculations be done. In this way, one has to get the feeling on what would be the best indication for analysing tlie spectroscopy. The main quantum tool for spectroscopy be analysed ir tlie propagator. From its poles one can read off the physical masses and from its residues the probabilities. Therefore our choice will be to work with a set of fields $\{G\}$, which diagonalizes the transverse sector.

Thus taking (11.28) one gets the following transverse and longitudinal propagators,

$$< G_{\mu I}G_{\nu I} >_T = \frac{\delta_{IJ}}{\Box+m_T^2}P^T_{\mu\nu}\ , \qquad \text{(IV.7)}$$

and

$$< G_{\mu I}G_{\nu J} >_L = \left(\frac{1}{\Box+\tilde{B}^{-1}m_T^2}\right)\tilde{B}^{-1}P^L_{\mu\nu}\ , \qquad \text{(IV.8)}$$

where

$$\tilde{B}=\Omega^t_T(K_L+G_F)\Omega^t_T\ , \qquad \text{(IV.9)}$$

and

$$m_T^2=\Omega_T^{-1}(K_T^{-1}m^2)\Omega_T\ . \qquad \text{(IV.10)}$$

Analysing tlie sector-T, physical masses are read in the diagonalized matrix $m_T^2$. It contains a zero and tlie others elements are depending on the free coefficients written in the initial Lagrangian. This means that tachyons can be avoided by controlling such coefficients. Analysing the sector-L, the mass spectroscopy analysis is less immediate. The particles that it embodies ("scalar photons") display masses that are eigenvalues of the matrix $(\tilde{B}^{-1}m_T^2)$. However, sectors T and L are not completely independent. There is a relashionship between the masses in both sectors. It is given by

$$m^2_{11,T}\ \cdots\ m^2_{NN,T}=(det\tilde{B})\ det(\tilde{B}^{-1}m_T^2)\ , \qquad \text{(IV.11)}$$

where (IV.10) is showing that the presence of any null mass in sector-T will correspond to a massless quantum in sector-L.

To explore whether the degeneracy degree of tlie eigenvalues of the matrix $m_T^2$ is the same for $(\tilde{B}^{-1}m_T^2)$, consider a sector-T with M independent fields $G_{\mu I}(M\leq \text{N})$ with zero mass:

$$m_T^2G_{\mu 1}=0$$

$$m_T^2G_{\mu M}=0\ , \qquad \text{(IV.12)}$$

then multiplying by $\tilde{B}^{-1}$, one gets:

$$(\tilde{B}^{-1}m_T^2)G_{\mu I} = 0$$

$$(\tilde{B}^{-1}m_T^2)G_{\mu N} = 0 \qquad \text{(IV.13)}$$

Thus from (IV.11), (IV.12), (IV.13) one concludes the presence of a constraint where to each zero mass in sector-T will correspond a zero mass in sector-L.

A next aspect to investigate on the mass spectroscopy is its dependence on gauge-fixing parameters. Being the matrix $\tilde{B}^{-1}$ dependent on $G_F$, we have to study explicitly this question. For this, consider the general (N x N) matrices

$$K_L = \begin{pmatrix} 0 & 0 \\ 0 & s \end{pmatrix} \quad \text{and} \quad G_F = \frac{1}{\alpha}\begin{pmatrix} 1 & \sigma \\ \sigma^t & \sigma^t\sigma \end{pmatrix}, \qquad \text{(IV.14)}$$

where s and a are respectively (N-1) ⊗ (N-1) and 1 ⊗ (N-1) inatrices. Thus,

$$(K_{\rm L} + G_{\rm F})^{-1}\, M^2 = \begin{pmatrix} 0 & -\sigma s^{-1}M^2 \\ 0 & s^{-1}M^2 \end{pmatrix}, \qquad \text{(IV.15)}$$

which provides the following secular equation for the longitudinal masses, $m_{\rm L}^2$:

$$\lambda.\det(\lambda - s^{-1}M^2) = 0\ . \qquad \text{(IV.16)}$$

Thus (IV.15) finally shows that sector-L spectra contains a zero solution and (N-1) eigenvalues of the matrix $(s^{-1}M^2)$. Concluding, this result proves that the inasses are completely independent from the gauge parameters a and $\sigma_i$.

A further step for the understanding of the physics of the inodel is to notice the influence of tlie gauge fixing parameters, a, on tlie residues at the propagator poles. Physical entities such as the cross section or the norm of the physical states can not depend on a. For this, we have to examine at each order in perturbative theory the residue matrix corresponding to each pole. As we know, only the diagonal elements of every residue matrix should not depend on $\alpha$. However, for saving the cross section from any prejudice, we need to be sure that all elements in sucli matrices do not display such a dependence. By considering tlie {G} basis, tlie sector-T will work positively. Nevertheless for tlie sector-L, one needs to calculate explicitly the involved residues. For this we have to study the propagators. From (II.33), one gets tlie following matrices expression:

$$< G_\mu G_\nu >_{\rm L} = \Omega_{\rm T}^{-1}\left[\frac{1}{\Box + (K_{\rm L}+G_{\rm F})^{-1}M^2}(K_{\rm L}+G_{\rm F})^{-1}\right](\Omega_{\rm T}^{-1})P^{\rm L}_{\mu\nu}\ . \qquad \text{(IV.17)}$$

Calculating in pieces,

$$\left[\frac{1}{\Box + (K_{\rm L}+G_{\rm F})^{-1}M^2}(K_{\rm L}+G_{\rm F})^{-1}\right] = \begin{pmatrix} a & b \\ b^t & c \end{pmatrix},$$

where

$$\begin{aligned} a &= \frac{1}{\Box}\left[\alpha - \sigma s^{-1}M^2(\Box + s^{-1}M^2)^{-1}(s + \frac{1}{2}\sigma^t\sigma)^{-1}\sigma^t\right]. \\ &\quad .\left[1 - \frac{1}{2}\sigma(1 + \frac{1}{2}\sigma^t\sigma)^{-1}\sigma^t\right] \\ b &= -\frac{1}{\Box}\sigma s^{-1}\left[1 - M^2 s^{-1}(\Box + s^{-1}M^2)^{-1}\right] \\ c &= (\Box + s^{-1}M^2)^{-1}s^{-1}\ . \end{aligned} \qquad \text{(IV.18)}$$

Observe then the following dependencies: $a = a(k^2;\alpha,\sigma_i)$, $b = b(k^2;\sigma_i)$, and the C matrix will depend on powers of momenta related to the number of involved fields (it represents a sector without any gauge fixing parameters).

For the $\Omega$ matrix, one takes formally the following expression

$$\Omega_T^{-1} = \begin{pmatrix} x & y \\ \omega & Z \end{pmatrix} , \qquad (IV.19)$$

where x is a number; y a row matrix 1 $\otimes$ (N-1); w a column matrix (N-1) $\otimes$ 1, and Z a (N-1) $\otimes$ (N-1) matrix.

Substituting (IV.18), (IV.19) in (IV.17), it yields

$$< G\mu_1 G_{\nu 1} >_L = x^2 a(k^2;\alpha,\sigma_i) + x(by^t + yb^t) + yCy^t , \qquad (IV.20)$$

$$\begin{aligned} < G_{\mu 1} G_{\nu i} >_L &= x\omega_i a(k^2;\alpha,\sigma) \\ &+ x b_j (Z^t)_{ji} + y_j b_j^t \omega_i + y_j C_{jk} (Z^t)_{ki} , \end{aligned} \qquad (IV.21)$$

$$\begin{aligned} < G_{\mu i} G_{\nu j} >_L &= \omega_i \omega_j a(k^2;\alpha,\sigma) + \omega_i b_k (Z^t)_{kj} + \\ &+ Z_{ik} b_k \omega_j + Z_{ik} C_{kl} (Z^t)_{lj} , \end{aligned} \qquad (IV.22)$$

for $i$, $j$, k, $l$ = 1, .., N.

Then the question here is to isolate the residues at the poles ($k^2 = 0$ or $k^2 = s^{-1}M^2$) witch contain contributions frcm the gauge parameters. For (IV.20), one notes that such dependence is controlled through the element $(\Omega_T^{-1})_{11} \equiv x$. For (IV.21), this situation is less restrictive because there are two parameters able to make such cancellation: x and $\omega_i$. For (IV.22), it can be obtained through $\omega_i = 0$ or $\omega_j = 0$. Tliis would be the tree level analysis. Concluding, we would observe that the spectroscopy for a generalized gauge model was derived in this chapter. The emphasys here was that Eq. (11.1) is not only a simple bag with vector and scalar excitations. It also carries an internal consistency for to physics be played which is given by the quanta invariance under field parametrizations.

## V. Longitudinal diagonalisation

Another possibility for deriving the spectrum should be to diagonalize the longitudinal part. It is defined by

$$V_\mu = \Omega_L L_\mu , \qquad (V.1)$$

$$\Omega_L = S_L^t \widetilde{B}_L^{-1/2} R_L^t , \qquad (V.2)$$

where L, are the physical fields in sector-L, $S_L$ and $R_L$ rnatrices are defined similary with G case, and $\widetilde{B}_L$ is

$$\widetilde{B}_L = S_L B S_L^t \qquad \text{(diagonal)} . \qquad (V.3)$$

Longitudinal physics masses are now determined by

$$m_L^2 = \Omega_L^t M^2 \Omega_L . \qquad (V.4)$$

Substituing the above expressions in (II.10), one rewrites

$$r = \frac{1}{2} L_\mu^t \left( \Box \widetilde{K} + \widetilde{M}^2 \right) P_T^{\mu\nu} L_\nu + \frac{1}{2} L_\mu^t (\Box + m_L^2) P_L^{\mu\nu} L_\nu , \qquad (V.5)$$

where

$$\begin{aligned} \widetilde{K}_T &\equiv \Omega_L^t K_T \Omega_L \\ \widetilde{M}^2 &\equiv \Omega_L^t M^2 \Omega_L . \end{aligned} \qquad (V.6)$$

Here, the sector-T becomes more complicated. Propagators and residue matrices are given by

$$< L_\mu L_\nu >_T = \Omega_L^{-1} \left( \frac{1}{\Box + K_T^{-1} M^2} K_T^{-1} \right) \Omega_L^{-1t} , \qquad (V.7)$$

and

$$(R_T^{LL}) = \sum_{jk} (\Omega_L^{-1})_{ij} (R_T^{VV})_{jk} (\Omega_L^{-1})_{ik} . \qquad (V.8)$$

Thus, from

$$\widetilde{K}_T^{-1} M^2 = \Omega_L^{-1} \left( K_T^{-1} M^2 \right) \Omega_L , \qquad (V.9)$$

we get the invariance for transversal physical masses, $m_T^2$. However, the information about ghosts is not preserved. Given a positive-definitive matrix $K_T$, from Eq. (V.6) one notes that the corresponding matrix $\widetilde{K}_T$ will be not connected through a similarity transformation. Eq. (V.8) is also indicating such information. It shows that only the sign of the residue determinant is preserved. Consequently the two equations are not enough for controlling the ghosts presence. Therefore, we will need to calculate explicitly the propagator and residues in sector T.

From (V.7) a first step is to calculate $\Omega_L^{-1}$. Here $S_L$ and $R_L$ are orthogonal matrices defined through (V.3) and through the physical masses in the sector-L

$$m_L^2 = R_L\left(\widetilde{B}_L^{\frac{-1}{2}} S_L M^2 S_L^t \widetilde{B}_L^{\frac{-1}{2}}\right) R_L^t \qquad \text{(diagonal)}, \tag{V.10}$$

where

$$B = \begin{bmatrix} -\frac{1}{\alpha} & -\frac{1}{\alpha}\sigma \\ -\frac{1}{\alpha}\sigma^t & s - \frac{1}{\alpha}\sigma^t\sigma \end{bmatrix}$$

and

$$\widetilde{B}_L \equiv \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_N \end{pmatrix}$$

Observe that $\lambda_1$, ..., $\lambda_N$ are B eigenvalues which depend on $a$. Calculating the matrix $S_L$, one gets

$$S_L = \begin{pmatrix} 1 & \frac{1}{\alpha}s_1 \\ 1 & \\ \vdots & \Sigma \\ 1 & \end{pmatrix}$$

where s and C, respectively a line and a matrix, are given by

$$s_1 = \frac{\sigma}{\alpha}\left(s - \frac{1}{\alpha}\sigma^t\sigma - \lambda_1\right) \tag{V.11}$$

and

$$\Sigma = \begin{bmatrix} \frac{\sigma}{\alpha}\left(s - \frac{1}{\alpha}\sigma^t\sigma - \lambda_2\right) \\ \vdots \\ \frac{\sigma}{\alpha}\left(s - \frac{1}{\alpha}\sigma^t\sigma - \lambda_N\right) \end{bmatrix} \tag{V.12}$$

The matrix $R_L$ can be formally written as

$$R_L = \begin{bmatrix} r & r_1 \\ r_2 & r_3 \end{bmatrix}, \tag{V.13}$$

where each element can depend on $\alpha$. Then, we finally get

$$\Omega_L^{-1} = \left[\begin{pmatrix} r\lambda_1^{1/2} + r_1\Lambda & \frac{r}{\alpha}\lambda_1^{1/2}s_1 + \frac{1}{\alpha}r_1\widetilde{b}^{1/2}\Sigma \\ \lambda_1^{1/2} + r_3\Lambda & \frac{1}{\alpha}\lambda_1^{1/2}r_2 s_1 + \frac{1}{\alpha}r_3\widetilde{b}^{\frac{1}{2}}\Sigma \end{pmatrix}\right]. \tag{V.14}$$

Note that **Eq.** (V.14) shows an explicit dependence on $\alpha$. This result is very subtle. It means that for a certain value of $a$ there will appear a degenerate $\Omega_L$ matrix, which implies a non-well-defined transformation (V.l). This fact can probably exclude the L„ parametrization. In future work, with more experience, we expect to understand whether a given extended gauge theory would not give a preference for a type of parametrization which diagonalizes the highest, spin sector contained in the involved fields.

Calculating formally the propagators, one obtains

$$< L_{\mu 1}L_{\nu 1} >_T \;=\; \left(r\lambda_1^{\frac{1}{2}} + r_1\Lambda\right)^2 P_{11} +$$

$$+ \frac{1}{\alpha}\left(r\lambda_1^{1/2}+r_1\Lambda\right)\left(r\lambda_1^{1/2}Ps_1^t+P\Sigma^t\widetilde{b}r_1^t\right)+$$
$$+ \frac{1}{\alpha}\left(r\lambda_1^{1/2}s_1+r_1\widetilde{b}^{1/2}\Sigma\right)\left(r\lambda_1^{1/2}P^t+P^tr_1\Lambda\right)+$$
$$+ \frac{1}{\alpha^2}\left(r\lambda_1^{1/2}s_1+r_1\widetilde{b}^{1/2}\Sigma\right)\left(r\lambda_1^{1/2}Qs_1^t+Q\Sigma^t\widetilde{b}^{1/2}r_1^t\right), \quad \text{(V.15)}$$

$$\begin{aligned} < L_{\mu 1}L_{\nu i} >_{\rm T} \; = \; & \left(r\lambda_1^{\frac{1}{2}}+r_1\Lambda\right)\left(\lambda_1^{1/2}r_2^t+\Lambda^t r_2^t\right)P_{11}+ \\ + & \frac{1}{\alpha}\left(r\lambda_1^{1/2}+r_1\Lambda\right)\left(\lambda_1^{1/2}Ps_1^tr_2^t+P\Sigma^t\widetilde{b}^{1/2}r_3^t\right)+ \\ + & \frac{1}{\alpha}\left(r\lambda_1^{1/2}s_1+r_1\widetilde{b}^{1/2}\Sigma\right)\left(\lambda_1^{1/2}P^tr_2^t+P^t\Lambda^tr_3^t\right)+ \\ + & \frac{1}{\alpha^2}\left(r\lambda_1^{1/2}s_1+r_1\widetilde{b}^{1/2}\Sigma\right)\left(\lambda_1^{1/2}p^tr_2^t+P^t\Lambda^tr_3^t\right)+ \\ + & \frac{1}{\alpha^2}\left(r\lambda_1^{1/2}s_1+r_1\widetilde{b}^{1/2}\Sigma\right)\left(\lambda_1^{1/2}Qs_1^tr_2^t+Q\Sigma^t\widetilde{b}^{1/2}r_3^t\right), \end{aligned} \quad \text{(V.16)}$$

$$\begin{aligned} < L_{\mu i}L_{\nu j} >_{\rm T} \; = \; & \left(\lambda_1^{1/2}r_2+r_3\Lambda\right)\left(\lambda_1^{1/2}r_2^t+\Lambda^t r_3^t\right)P_{11}+ \\ + & \frac{1}{\alpha}\left(\lambda_1^{1/2}r_2+r_3\Lambda\right)\left(\lambda_1^{1/2}P^ts_1^tr_2^t+P\Sigma^t\widetilde{b}^{1/2}r_3^t\right)+ \\ + & \frac{1}{\alpha}\left(\lambda_1^{1/2}r_2s_1+r_3\widetilde{b}^{1/2}\Sigma\right)\left(\lambda_1^{1/2}P^tr_2^t+P^t\Lambda^tr_3^t\right)+ \\ + & \frac{1}{\alpha^2}\left(\lambda_1^{1/2}r_2s_1+r_3\widetilde{b}^{1/2}\Sigma\right)\left(\lambda_1^{1/2}Qs_1^tr_2^t+Q\Sigma^t\widetilde{b}^{1/2}r_3^t\right), \end{aligned} \quad \text{(V.17)}$$

where

$$\frac{1}{\Box+K_{\rm T}^{-1}M^2}K_{\rm T}^{-1}\equiv\begin{pmatrix}P_{11} & P\\ P^t & Q\end{pmatrix}. \quad \text{(V.18)}$$

Observe then, the explicit $\alpha$ dependence from *(V.16)* to (V.18).

Thus although the longitudinal basis preserves the quanta invariance, it brings two aspects for discussion. They are that the $\Omega$ matrix and the propagator residues appear depending explicity on the gauge fixing parameter.

## VI. Stability

We are going to follow the algebraic renormalization technique. The method is based on BRS technique which combined with the Quantum Action Principle transforms the renormalizability as a cohomology problem[18].

From (II.1), one gets

$$\Sigma = -\frac{1}{4}\int d^4x Z_{\mu\nu}Z^{\mu\nu} + \int d^4x\left[b\partial.(D+\sigma X)+\xi\frac{b^2}{2}+\bar{c}\partial^2 c\right],$$

where $b, c, \bar{c}$ are respectively the Lagrangian multiplier, the ghost, the antighost and

$$Z_{\mu\nu}=dD_{\mu\nu}+\alpha X_{\mu\nu}+\beta\Sigma_{\mu\nu}+g_{\mu\nu}\delta(X.X)+\gamma X_\mu X_\nu\,. \quad \textit{(VI.1)}$$

The action C is invariant under the nilpotent BRS transformation:

$$sD_\mu=-\partial_\mu c,\ sX_\mu=0,\ sc=0,\ sb=0,\ s\bar{c}=b\,. \quad \textit{(VI.2)}$$

The canonical dimensions and the ghost number of the fields D,, *X,, b,* c, *C* are respectively *1, 1,* 2, *2, 0* and *0, 0, 0, 1, -1.*

The Slavnov identity corresponding to the $s$-invariance is

$$\int d^4x \left[c\partial_\mu \frac{\delta\Sigma}{\delta D_\mu} + b\frac{\delta\Sigma}{\delta\bar{c}}\right] = 0 \ . \qquad \text{(VI.3)}$$

The equations of motion are

$$\begin{aligned} \frac{\delta\Sigma}{\delta b} &= \partial.(D+\sigma X) + \xi b \\ \frac{\delta\Sigma}{\delta c} &= \partial^2\bar{c}, \ \frac{\delta\Sigma}{\delta\bar{c}} = \partial^2 c \ , \end{aligned} \qquad \text{(VI.4)}$$

For an effective quantic action, whose divergent part is local, one can define at each order $\hbar$ the following finite action

$$\Gamma = \Sigma + \epsilon\Gamma^c \ , \qquad \text{(VI.5)}$$

where $\Gamma^c$ express the counterterms. It is an integrated local functional, field polynomial with four dimension and zero ghost nuinber.

Quantically, one gets

$$\begin{aligned} &\partial_\mu \frac{\delta\Gamma^c(D,X)}{\delta D} = 0 \\ &\frac{\delta\Gamma^c}{\delta b} = 0, \ \frac{\delta\Gamma^c}{\delta c} \ , \ \frac{\delta\Gamma^c}{\delta\bar{c}} = 0 \end{aligned} \qquad \text{(VI.6)}$$

Therefore $\Gamma^c(D,X)$ has the expression

$$\begin{aligned} \Gamma^c(D,X) = &-f\int d^4x D_{\mu\nu}D^{\mu\nu} + j\int d^4x D_{\mu\nu}X^{\mu\nu} + \\ &+ \int d^4x \left[a(\partial.X)^2 + e\partial^\mu X_\nu \partial_\mu X^\nu + \mu(X.X)(\partial.X) + \right. \\ &\left. \nu X_\sigma X^\mu \partial^\sigma X_\mu + z(X.X)(X.X)\right] \ . \end{aligned} \qquad \text{(VI.7)}$$

Verifying the stability for $\Sigma(D,X,b,c,\bar{c})$ under radiative corrections

$$\Sigma + \epsilon\Gamma^c = \Sigma(D_0, X_0, c_0, \bar{c}_0, b_0, \alpha_0, \beta_0, \gamma_0, d_0, \rho_0, \sigma_0, \xi_0) + \sigma(\epsilon^2) \ , \qquad \text{(VI.8)}$$

the initial parameters are redefined as

$$D_0 = (1+\epsilon\ Z_D)D; \ \ldots ; \xi_0 = (1+\epsilon\ Z_\xi)\xi \ ,$$

where $Z_b$, $Z_\sigma$ and $Z_\xi$ depend on otlier renormalization terms; $Z_c$ and $Z_{\bar{c}}$ can be redefined freely; parameter d can be incorporated on the field-D, redefinition, i.e. $d = 1$, $Z_d = 0$ and $Z_D = \frac{f}{2}$; and finally

$$\begin{aligned} Z_\alpha &= -\frac{3}{8}\frac{(a-b)}{\alpha^2} - \frac{3}{8}\frac{(a+b)}{\beta^2} - \frac{7}{4}\frac{j}{\alpha} - \frac{7}{4}f + \\ &- \frac{3}{8}\frac{\nu}{\gamma\beta} - \frac{z}{8(4\rho^2+\gamma^2+2\rho\gamma)} + \frac{3(\mu-\nu)}{8\rho\beta} \quad ; \end{aligned}$$

$$\begin{aligned} Z_\beta &= -\frac{7}{8}\frac{(a-b)}{\alpha^2} - \frac{7}{8}\frac{(a+b)}{\beta^2} - \frac{7}{4}\frac{j}{\alpha} - \frac{7}{8}f + \\ &- \frac{3}{8}\frac{\nu}{\alpha\beta} - \frac{z}{8(4\rho^2+\gamma^2+2\rho\gamma)} + \frac{3(\mu-\nu)}{8\rho\beta} \quad ; \end{aligned}$$

$$\begin{aligned} Z_\gamma &= -\frac{(a-b)}{4\alpha^2} - \frac{(a+b)}{4\beta^2} - \frac{j}{2\alpha} - \frac{f}{4} + \\ &+ \frac{\nu}{4\gamma\beta} - \frac{z}{4(4\rho^2+\gamma^2+2\rho\gamma)} + \frac{21(\mu-\nu)}{28\rho\beta} \quad ; \end{aligned}$$

$$\begin{aligned} Z_\rho &= -\frac{(a-b)}{4\alpha^2} - \frac{(a+b)}{4\beta^2} - \frac{j}{2\alpha} - \frac{f}{4} + \frac{3}{4}\frac{\nu}{\beta\gamma} + \\ &- \frac{Z}{4(4p^2+y^2+2\rho\gamma)} \quad , \end{aligned}$$

$$\begin{aligned} Z_X &= \frac{3}{8}\frac{(a-b)}{\alpha^2} + \frac{3}{8}\frac{(a+b)}{\beta^2} + \frac{3}{4}\frac{j}{\alpha} + \frac{3}{8}f + \\ &+ \frac{3}{8}\frac{\nu}{\gamma\beta} + \frac{z}{8(4\rho^2+\gamma^2+2\rho\gamma)} - \frac{3(\mu-\nu)}{8\rho\beta} \end{aligned} \qquad \text{(VI.9)}$$

Concluding, the classical action is stable under radiative corrections, i.e. $\int d^4x\ Z^0_{\mu\nu}\ Z^{\mu\nu 0}$ preserves the square shape and therefore is renornializable. A dependente on six renormalization parameters is observed.

## VII. Conclusion

The gauge method is enlarged by taking the various connections that a given group contains. Three general aspects emerge. They are the possibility of avoiding the Higgs particle, the fact that gauge particles are no longer defined strictly as those which intermediate interactions, and that nature diversity can be explained more freely (without requiring the usual mechanism which consider multiples with a soft symmetry breaking). This work has served as basis for these aspects. The spectroscopy of Eq. (I.1), that was extensively studied here, has shown a compulsory massless particle (associated to the photon) and other massive particles (associated to mediator and vector mesons plus scalar particles). Consequently, in principle, it was not necessary to adcpt the spontaneous symmetry breakdown process in order to develop a model with massive excitations, but note that a scalar sector is embodied. Another result from this spectroscopy was the variety of fields suffering gauge transformations that were generated. This proliferation shows tliat not only intermediate particles as the photon, $W^{\pm}$, $Z^{o}$ and others should be articulated by symmetry. Particles as pions, Kaons, D - and F - mesons, and many others are also now candidates for being described by the gauge approach. Finally, **Eq.** (1.1) shows that the differences between the particles does not only need to follow the guide of soft breaking expressions, as in the Gell Mann-Okubo formula. Quanta with different masses can be obtained from a not broken Abelian group.

Other two relevant features, altliough not original, that such extended models develop are the distinction between fields and quanta and the presence of non-diagonal propagators. The first aspect is observed through the transverse and longitudinal sectors containing realities qualitatively distinct. The other, similarly to the well known cases for chiral fermions, Weinberg-Glashow-Salam model, and Kobayslii-Mashawa matrix is sliowing that the quanta can be propagated througli non-diagonal two point Green's functions.

Thus, after initial considerations about general aspects tliat this generalized gauge model contains, one should now discuss the properties that it systematizes. The emphasys in this work was the property of working with different fiels parametrizations. Tests have been studied for guaranteeing such diverse viabilities for physics be read off. Thus fields make a basis under which the various quantum numbers whicli build up the quanta are proved to be invariant, while other physical entities transform as tensors. For instance, the beta function becomes a tensor in flavour space[13].

Following this field parametrizations property this work got reasons for the so-called spectroscopical consistency. Sections III - V complemented with Appendix A were devoted to understanding this internal argument tliat Eq. (I.1) promotes. Therefore it was emphasized in the introduction that before moving for renormalizability and unitarity aspects, one should first consider tlie spectroscopical analysis. Evidently, there are other consequences coming from the existence of such possibility of working with different field basis. One of them would be the opportunity of a strategy in the calculation. For instance, take the question about the necessity of a gauge fixing term in order to invert the propagator. {G} basis at Eq.(IV.8) does not answer it explicitly, but working on constructor basis one gets that without including a gauge fixing term, the propagator would not exist. Similarly, sometimes it is more useful to work with a basis that avoids mixing propagators between $D_\mu$ and $X_\mu{}^i$ fields. It is the set $\tilde{D}$ , X defined by Eq. (11.35) which is more adequate for observing the counter terms. In Section V it was also also observed that a gauge model has preference for the sets of fields wliich diagonalize the highest spin state carried by the involved fields.

Then, finally, let us now observe on the possible meanings that a generalized Abelian gauge model triggers. For this, we should first understand the context that it develops. Experimentally, a model with the freedom of including an arbitrary number of particles is faded to be a disaster. The common tradition in physical theories is the prediction aspect, and so, from such freedom of chosing the number of involved particles, one gets the indication that Eq.(I.1) *will* not develop this standard notion that a physical theory requires.

The traditional feeling of prediction will be lost. Then an answer of this basic inquisition must be given. We argue that the main aspect inserted in Eq.(I.1) is not to predict, but to organize. A model which proposes to organize a given number of particles would mean to describe tlieoretically their measured properties as mass, charge, lifetime, principal decays, and so on, but without requiring grand nnification principles. Eq.(I.1) is only be able of organizing a context and without the Higgs presence[18]. Thus such an extended gauge model might be able to unify distinct spin families, but with a systematization just embedded in the organizing context. The continency for the unification scheme dictated by Eq. (1.1) appears to be only to derive numbers which fit tlie experimental facts (cross sections, lifetime, etc). This means that this equation can be proposed only to solve half of the questions about particles - its function consists only in describing the coexistence between the particles without seeking for their roots.

A physical theory needs the prediction aspect. However Eq.(I.1) is limited by its organizative character. Therefore, something must complement it. A viewpoint is to consider that such expected thing are the quarks. The proposal is to consider that while quarks are the roots from which contact the predictions are made, Eq. (1.1) task will only be to organize tlie variety of facts that such roots develop. For example, consider the vectorial and pseudo-scalars nonet cases. There, from the quarks model one is able to predict the existence of particles with their wavefunctions, but their dynamics is maintained unknown. Thus tlie mission for Eq. (I.1) pluriformity viewpoint would be to complement the Gellman-Zweing approach by offering a possibility of Quantum Colourless Dynamics. Thus as a first real challenge for Eq. (I.1), one sees the investigation on the dynamics of the particles predicted in the nonet. For this, in further work we expect to introduce charged fields and interna1 global symmetries for trying some description.

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### Appendix A - A model involving two potential fields

In order to exemplify abont the spectroscopical properties analysed in the text we are going to take tlie simplest case: a model involving two potential fields. Consider that a, b, c, s, $M^2$ are free coeficients, $a$ fixes the gauge and $\sigma$ is another gauge parameter which tliis extended model allows to.

We have now to understand the presence of tachyons, ghosts, and tlie influence of the $a$ parameter at tree level. For performing these aspects we will initiate with $\{G_{\mu I}\}$ parametrization.

The non existence of ghosts in the transversal sector is given by tlie condition of matrix $\tilde{K}_T$, defined by Eq. (III.21), be positively defined. It gives

$$\begin{aligned} K_T &= \begin{pmatrix} a & c/2 \\ c/2 & b \end{pmatrix}, \quad M^2 = \begin{pmatrix} 0 & 0 \\ 0 & M^2 \end{pmatrix}, \\ K_L &= \begin{pmatrix} 0 & 0 \\ 0 & s \end{pmatrix}, G_F = \begin{pmatrix} 1 & \sigma \\ \sigma & \sigma^2 \end{pmatrix} \end{aligned} \quad \text{(A.1)}$$

$$\tilde{K}_T = \begin{pmatrix} \lambda_+ & 0 \\ 0 & \lambda_- \end{pmatrix}$$

where

$$\lambda_\pm = \frac{1}{2}\left[(a+b) \pm \sqrt{\Box}\right] \; ,$$

witli

$$\Box = (a-b)^2 + c^2 \; . \quad \text{(A.2)}$$

Thus for ghosts be avoided one gets from (A.2) the following restrictions in the free coefficients

$$a > 0, \qquad b > 0, \qquad ab > \frac{c^2}{4} \; . \quad \text{(A.3)}$$

A next step is to calculate the transformation matrix $\Omega_T$,

$$V_\mu \equiv \begin{pmatrix} D_\mu \\ X_\mu \end{pmatrix} = \Omega_T G_\mu \; , \quad \text{(A.4)}$$

where

$$\Omega_T^{-1} = \begin{pmatrix} x_T & y_T \\ 0 & z_T \end{pmatrix} ,$$

with

$$\begin{aligned} x_T &= \frac{1}{2\sqrt{\Box a}}\left[a(a-b+\sqrt{\Box}+\frac{c^2}{2}(1+a+b-\sqrt{\Box}\,)\right] \\ y_T &= c/2 \\ z_T &= -\sqrt{ab-\frac{c^2}{4}}\ . \end{aligned} \tag{A.5}$$

Thus we get

$$\Omega_T^{-1}\ \mathbf{E}\ \Omega_T^{-1}(a;\ b;\ c)\ . \tag{A.6}$$

In sector-T, physical masses can be obtained through Eq. (III.18). As expected, one is zero and the other depends on the initial theory parameters of the initial tteory:

$$m_{T,1}^2 = 0\ , \tag{A.7}$$

$$m_{T,2}^2 = \left(\frac{s_{12}^2}{\lambda_+}+\frac{s_{22}^2}{\lambda_-}\right)M^2\ , \tag{A.8}$$

where $s_{12}$ and $s_{22}$ are s-matrix elements **(Eq.** 111.21) given by

$$\begin{aligned} s_{12} &= -\frac{2\,|\,2\,|}{c}\frac{(a-\lambda_+)}{\sqrt{4(a-\lambda_+)^2+c^2}} \\ s_{22} &= -\frac{2\,|\,2\,|}{c}\frac{(a-\lambda_-)}{\sqrt{4(a-\lambda_-)^2+c^2}}\ . \end{aligned} \tag{A.9}$$

The residue matrices corresponding to these poles are

$$Res\ (k^2=0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} ,$$

and

$$Res\ (k^2-m_2^2) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\ . \tag{A.10}$$

The physical longitudinal propagators are

$$\begin{aligned} < G_{\mu 1}\, G_{\nu 1} >_L &= -\frac{\alpha x_T^2}{\Box}P_{\mu\nu}^L + \frac{\sigma x_T}{s}(x_T\sigma - 2y_Y + y_T^2)\left(\Box + \frac{M^2}{s}^{-1}\right)P_{\mu\nu}^L \\ < G_{\mu 1}\, G_{\nu 2} >_L &= \frac{z_T}{s}\frac{(-\sigma x_T + y_T)}{(\Box + \frac{m^2}{s})}P_{\mu\nu}^L \\ < G_{\mu 2}\, G_{\nu 2} >_L &= \frac{z_T^2}{s\,(\Box + \frac{m^2}{s})}P_{\mu\nu}^L\ . \end{aligned} \tag{A.11}$$

Then taking $x_T = 0$, there is no dependence on the gauge fixing parameter $a$ (and neither on a).

The residue matrices at the physical masses of the sector-L physical masses, $\Box = 0$ and $\Box = \frac{M^2}{s^2}$, are

$$R_G^L\ (k^2=0) = \begin{pmatrix} -\alpha x_T^2 & 0 \\ 0 & 1 \end{pmatrix} ,$$

and

$$R_G^L\ (k^2 = \frac{M^2}{s}) = \begin{pmatrix} \sigma x_T(x_t\sigma - 2y_T + y_T^2) & z_T(y_T - x_T\sigma) \\ z_T(y_T - x_T\sigma) & z_T^2 \end{pmatrix}\ . \tag{A.12}$$

Knowing the existence of a consistent spectroscopy, one would expect to obtain the same physical information working with a parametrization system which diagonalizes the longitudinal sector. The transformation matrix, $\Omega_L$, whicli relates the constructor and longitudinal basis is

$$\Omega_L \equiv \begin{pmatrix} x_L & y_L \\ 0 & z_L \end{pmatrix} ,$$

where

$$\begin{aligned} x_L &= -\sqrt{-\frac{1}{\alpha}\frac{\delta_2}{\sqrt{\delta_1}}} \\ y_L &= -\frac{2\alpha}{\alpha} \\ z_L &= -\sqrt{s} \end{aligned}$$

with

$$\begin{aligned} \delta_1 &= (1+\alpha s-\alpha^2)^2+4\sigma^2 \\ \delta_2 &= \frac{1}{2}\left(1+\alpha s-\alpha^2+\alpha\sqrt{\delta_3}\right)+\frac{\sigma^2}{\alpha}\left(\alpha-1+\alpha s-\sigma^2-\alpha\sqrt{\delta_3}\right) \\ \delta_3 &= \left(-\frac{1}{\alpha}-s+\frac{\sigma^2}{\alpha}\right)^2+\frac{4\sigma^2}{\sigma^2} . \end{aligned} \tag{A.13}$$

Consequently (A.13) exihibits an explicit $a$ dependence

$$\Omega_L^{-1} = \Omega_L^{-1}(\alpha;\ \sigma;\ c) . \tag{A.14}$$

Calculating the propagators

$$\begin{aligned} < L_{\mu 1}\ L_{\nu 1} >_T &= \frac{1}{\alpha}\left[\frac{f(\alpha,\sigma;\alpha s)}{k^2}+\frac{g(\alpha,\sigma;\alpha s)}{k^2-m_T^2}\right] \\ < L_{\mu 2}\ L_{\nu 2} >_T &= \frac{1}{\alpha}\frac{1}{2a(b^2-\frac{c^2}{4a})}\frac{h(\alpha,\sigma;s)}{k^2-m_T^2} \\ < L_{\mu 1}\ L_{\nu 2} >_T &= -\frac{s}{b^2-\frac{c^2}{4a}}\frac{1}{k^2-m_T^2} \end{aligned} \tag{A.15}$$

where

$$\begin{aligned} f(\alpha,\sigma;\alpha s) &= \frac{\delta_2}{a\delta_1} \\ g(\alpha,\sigma;\alpha s) &= \frac{c^2}{a\delta_1}\delta_2^2-\frac{\sigma}{\alpha}\frac{1}{4ab-c^2}\left[\frac{c\sqrt{-\alpha}}{\sqrt{\delta 1}}\delta_2+2a\sigma\right] \\ h(\alpha,\sigma;\alpha s) &= \left(\frac{-\alpha s}{\delta 1}\right)^( 1/2)[\delta_2+4a\sigma] \end{aligned} \tag{A.16}$$

and

$$\begin{aligned} < L_{\mu 1}\ L_{\nu 1} >_T &= \frac{1}{\Box}P_{\mu\nu}^L \\ < L_{\mu 2}\ L_{\nu 2} >_T &= \frac{1}{\Box+\frac{m^2}{s}}P_{\mu\nu}^L . \end{aligned} \tag{A.17}$$

The corresponding residue rnatrices for $\Box = 0$ and $\Box = \frac{M^2}{s^2}$ are

$$R_L^T(k^2=0) = \frac{1}{\alpha}\begin{pmatrix} f(\alpha,\sigma;\alpha s) & 0 \\ 0 & 0 \end{pmatrix}$$

and

$$R_L^T(k^2 = \frac{M^2}{s}) = \begin{pmatrix} \frac{1}{\alpha} g(\alpha,\sigma;\alpha s) & \frac{1}{\alpha}\frac{h(\alpha,\sigma;\alpha s)}{a(b^2-\frac{c^2}{4a})} \\ \frac{1}{\alpha}\frac{h(\alpha,\sigma;\alpha s)}{a(b^2-\frac{c^2}{4a})} & \frac{s}{a(b^2-\frac{c^2}{4a})} \end{pmatrix} . \quad (A.18)$$

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