Considerations About the Orbits of Trapped Particles in Low-Aspect-Ratio Tokamaks

M. Roberto
Faculdade de Engenharia, Universidade Estadual Paulista
12500, Guaratinguetá, SP, Brazil
and
Instituto Nacional de Pesquisas Espaciais
12101-970, São José dos Campos, SP, Brazil
and
R. M. O. Galvão
Instituto de Física, Universidade de São Paulo
C.P. 20516, 01498-990, SP, Brazil

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A study of the main quantities that depend on the topology of the orbits of trapped particles, viz, width of banana orbits, second adiabatic invariant, and bootstrap current, is carried out for low-aspect-ratio tokamaks. In order to investigate the variation of these quantities with only the geometry of the magnetic equilibrium configuration, an exact equilibrium model with fixed profiles, i.e.,the Soloviev equilibrium model, is used. For tokamaks with small aspect ratio ($R/a < 2$), the width of banana orbits is very narrow as compared to the values obtained for conventional tokamaks. Configurations with horizontal elongation can have a negative gradient of the second adiabatic invariant, giving rise to the possibility of stabilizing flute modes.It is shown that the bootstrap current depends on the geometrical parameters of the plasma column, that is, triangularity and elongation. The bootstrap current increases with the inverse aspect ratio only for elongated cross-sections of the plasma column.

I. Introduction

The discharges in low-aspect-ratio tokamaks are expected to present interesting characteristics that make them quite attractive for the study of the processes relevant for particle and energy transport. In thermonuclear plasmas. most of collisional and anomalous processes depend fundamentally on the topology of the orbits of trapped particles through quantities such as the width of banana orbits, the second adiabatic invariant J (important in tlie stabilization of flute modes), and the bootstrap current. The aspect ratio of the magnetic equilibrium configuration has a considerable influente on the length and width of the orbits of trapped and untrapped particles. In high aspect ratio configurations the banana orbits are usually short and large, whereas in low aspect ratio they are slimmer, for a given energy[1]. This occurs because, in small-aspect-ratio configurations, the flux surfaces become almost parallel to the B constant surfaces in the low-field side of the plasma column. For this reason, only the curvature effect is important on the drift away from the magnetic surface, leading to narrow orbits. The neoclassical transport may therefore be reduced, improving confinement[2].

The calculation of adiabatic invariant J in systems with axial symmetry is important because of its relation to the condition for the stabilization of trapped particles modes [3,4]. In the colisioness regime, this condition is given by $\nabla J.\nabla p > 0$[4], where p is the plasma pressure. Thus, one may in principle find magnetic field equilibrium configurations for which these modes are stable. Finally, the bootstrap current is quite relevant for the attainament of steady-state tokamaks reactors. This current may provide a large percentage of the total plasma current, without exceeding the Troyon stability criterium [5].

The effect of the value of the aspect ratio of the equilibrium configuration on the topology of the orbits of trapped particles has been previously investigated using numerically detesmined equilibria [1]. In this case it is quite hard to separate the effect of the geometry,i.e., values of the aspect ratio and of the elongation of the plasma column, from the effect of the profiles. As the aspect ratio is varied in a free-boundary equilibrium calculation , usually the two free profile functions have to be slightly changed to avoid steep gradients at the plasma boundary. Therefore, when comparing the width of banana orbits, for instance, for different values of the aspect ratio, one has to make sure that the equilibrium profiles have not been altered in the scan. For this reason we have decided to investigate the effect of the geometry on the topology of orbits of trapped particles using the exact equilibrium model of Soloviev with fixed equilibriurn profiles [6]. The pressure profile in the Soloviev model is parabolic, which is a good approximation for actual experimental profiles. The current profile, on the other hand, is almost flat, in disagreement with the experimentally observed ones. However, the geometry of the equilibrium flux surfaces depend mostly on the pressure profile, because the other contribution to the source term in Shafranov's equation is proportional to the plasma diamagnetism, which is usually small. Therefore, we expect that physical quantities which depend only on the geometry of the flux surface, as the shape of the banana orbits of trapped particles, are well characterized in this model. The magnitude of the bootstrap current calculated with this model is certainly much smaller than the maximum values obtained experimentally. This occurs because experiments intented to maximize the bootstrap current run at very high values of goloidal beta and, unfortunately, in the Soloviev model the value of poloidal beta is a parameter intrisically coupled with the geometry, which cannot be freely varied. For this reason, the values of the bootstrap current obtained in tliis work are rather low. Eventhough, relative variations of these values with the aspect ratio are correct.

II. Topology of banana orbits

The conventional calculation of banana orbits in tokamaks is based upon the integration of the equation for the velocity of the guiding center, using an expansion in terms of the inverse aspect ratio, $\epsilon = a/R << 1$. The equation for the orbit can be obtained as well as the width of the orbit. For low aspect ratios ($R/a < 2$) such expansion is not justificable. In this work we follow the scheme developed by Rome and Peng [1], which uses an exact representation of the orbits in the B $- \psi$ space, where B is the total magnetic field and ψ the poloidal flux function. In this case, the total equilibrium field and the geometry of the magnetic configuration are taken into account without approximations. To represent the orbits in this space, it is more convenient to use the following constants of motion: i)$,, that represents the position of the trapped particle with maximum parallel velocity on the equatorial plane; ii) the absolute value of particle velocity v; iii) $\zeta = (\vec{v}.\vec{J}_{\|})/(vJ_{\|}) |_{\psi m}$, which is the cosine of the angle between the particle velocity and the parallel component of the plasma current density.

The equation for the orbits of the trapped particles can be obtained from the equations for conservation of the toroidal component of the canonical angular momentum μ and energy. From theses equations, one obtains an expression for the absolute value of the magnetic field as a function of the flux function ψ along the orbit of a particle [1]; namely

$$B = \frac{B_m}{(\frac{1-\zeta^2}{2}) + \sqrt{(\frac{1-\zeta^2}{2})^2 + (\frac{I_m}{I})^2[\zeta - G_m(1 - \frac{\psi}{\psi_m}\sqrt{\frac{c^2}{v^2} - 1})^2]}}, \quad (1)$$

where $G_m := (ZeB_m\psi_m)/(mcI_m)$ is a dimensionless parameter, Z is the atomic number, e is the eletronic charge, B_m is the magnetic field for $\psi = \psi_m$, m is the mass of the trapped particle, c is the speed of light, $I_m = R_m B_{\varphi m}$, and $B_{\varphi m}$ is the toroidal field at the point (R_m, 0). B and B_m are normalized with respect to B_o, the value of the magnetic field at the geometric center of the cross section of the plasma column.

The flux surfaces obtained from the Soloviev equilibrium model [6] are given by

$$\psi(R,Z) = Z^2(R^2 - I_1) + \frac{(p_1-1)}{4}(R^2 - C^2)^2, \quad (2)$$

where all quantities are normalized with respect to their values at the geometric center of the plasma cross section, denoted by the subscript o. Space coordinates are normalized to R_0 and the normalized width and height of the plasma cross-section are represented by d and b, respectively. The plasma pressure p and total current I are normalized to p_0 and I_0, respectively. The flux function ψ is normalized to $\mu_0 I_0 R_0$. The free equilibrium profiles are specified as $p(\psi) = 1 - (4p_1/\alpha^2\beta_0)\psi$ and $(RB_\varphi)^2 = 1 + (4I_1/\alpha^2)\psi$, where p_1 and I_1 are constants determined by the boundary conditions, $\alpha = R_0B_0/\mu_0 I_0$ and $\beta_0 = 2\mu_0 p_0/B_0^2$. It follows that in this model the position of the magnetic axis is simply given by $C = \sqrt{1+d^2}$.

The orbit reflection point is calculated from the condition $\partial B/\partial\psi = 0$, which gives

$$\psi_r = (1 - \frac{\zeta}{G_m\sqrt{\frac{c^2}{v^2}-1}})\psi_m. \quad (3)$$

The equatorial $Z = 0$ plane is represented by a continuous curve in the B vs ψ plane; let us denote this curve by $(B-\psi)_{eq}$. Since the particle orbits move monotonically in ψ, they will intersect the $(B-\psi)_{eq}$. twice and their trajectories will lie inside the bounded region shown in Fig.1 . These curves refer to trapped protons with energy E = 8.82Kev, $\zeta = 0.60, 0.70$ and -0.70, for a circular plasmacross-section with d = 0.60. One can see the orbit becomes more elongated as ζ increases. A negative value of ζ means that particles move in the direction countrary to the plasma current. From Eq. **(3)** it can be shown that the orbit becomes broader as the energy of the particle increases, as already well-known [7].

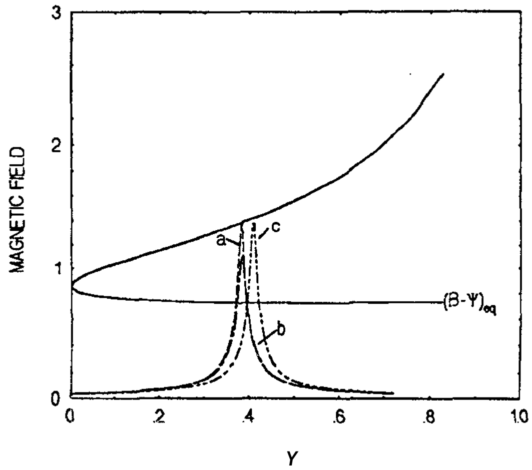

Figure *1:* Banana orbits for trapped protons (Z = 1), in $B-\psi$ space, for $b = d = 0.60, \delta = 0$, with $\zeta = 0.70$(a), 0.60(b) and -0.70(c), with energy $E = 8.82 Kev$.

Fig. 2 shows the variation of the width of the banana orbit with the inverse aspect ratio, for a given energy. Note that as the aspect ratio decreases the width of the orbit becomes narrower. This could in principle lead to reduced losses caused by trapped particles effects in low-aspect-ratio tokamaks.

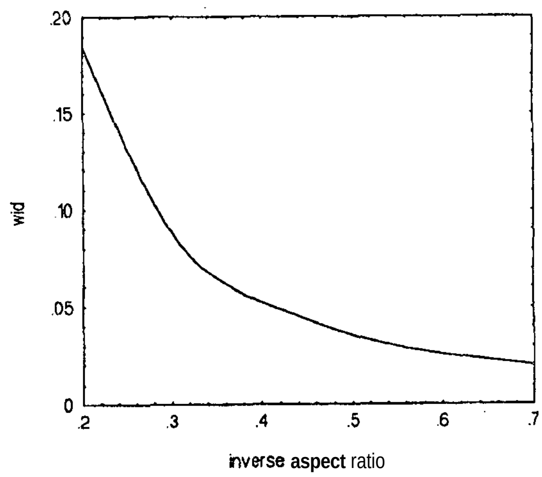

Figure *2:* Variation of the banana orbit width normalized to $\psi_b = \alpha^2\beta_0/4p_1$, with the inverse aspect ratio, for a proton with energy $E = 8.82 Kev$.

III. Adiabatic invariant J

The objective of calculating J is to verify whether the condition for drift reversal, $\nabla J.\nabla p > 0$[3], can occur in tokamaks as the aspect ratio decreases. Since ∇p is usually negative, this condition requires ∇J negative. The second adiabatic invariant J is defined as

$$J = \oint_\psi v_{\parallel} dl, \tag{4}$$

where $v_{\parallel}$ is the particle velocity parallel to the magnetic field and the integral is carried out along the closed banana orbit of a trapped particle[3].

For the Soloviev equilibrium model, the integration can be more conveniently carried out using the non-orthogonal coordinate system $(\rho, \vartheta, \varphi)$, where $\rho = \sqrt{\psi}, \vartheta = \arctan[\frac{2Z}{(R^2-C^2)}\sqrt{\frac{R^2-I_1}{p_1-1}}]$ and φ is the azimuthal angle[8].

Considering that $v_{\parallel} = \pm v\sqrt{1-(1-\zeta^2)B/B_m}$, J can be written as

$$J = 4\int_0^{\vartheta_r} \sqrt{1-(1-\zeta^2)\frac{B}{B_m}}dl, \tag{5}$$

where ϑ_r is the angle of the returning point. The length element is given by [8]

$$dl = \frac{1}{d}\sqrt{(\frac{-\rho\sin\vartheta}{R\sqrt{p_1-1}})^2 + \frac{\rho^2}{R^2-I_1}(\cos\vartheta + \frac{\rho\sin\vartheta^2}{\sqrt{p_1-1}(R^2-I_1)})^2 d\vartheta}. \tag{6}$$

Fig. 3 shows the curves of J vs (ψ_r/ψ_b), where ψ_r is the value of ψ at the reflection point and ψ_b is the value at the plasma boundary, for $\kappa = b/d = 0.45$ and 0.60, with $\delta = 0.5, 0.3$ and 0.0. The values the of safety factor at the magnetic axis q_m, are also shown in the figure. Note that the condition for drift reversal, $\nabla J < 0$, can be obtained only for a configuration with horizontal elongation.

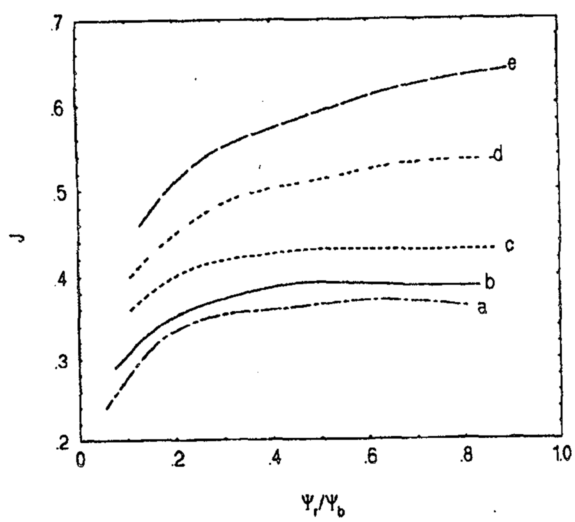

Figure 3: Curves of J vs (ψ_r/ψ_b), showing that inversion of the J profile occurs only for configurations highly elongated in the horizontal direction ($\kappa < 0.5$). The curves a, b, c, d and e correspond to $\kappa = 0.45, \delta = 0.5, q_m = 0.95, \kappa = 0.45, \delta = 0.3, q_m = 0.93, \kappa = 0.45, \delta = 0.0, q_m = 0.82, \kappa = 0.60, \delta = 0.3, q_m = 0.99, \kappa = 0.60, \delta = 0.0, q_m = 0.91$, respectively.

Stable configurations with elongation less than one and non vanishing triangularity have also been found by Miller et al[9], using a numerical equilibrium code. Galvão, Simpson and Wilner[7], using approximate solutions for circular cross-section, have shown that there are inversions of the J profiles in vertically elongated configurations. However, the correct treatment, without approximations, shows that it is not possible to obtain inversion of J profiles for the vertical elongations, biit only for the horizontal ones, confirming the results obtained by Miller et al.[9].

IV. The Bootstrap current

Recents results in TFTR[10], JET[11], and JT-60[12] indicate that a substantial fraction of the total plasma current may be due to the pressure- driven bootstrap current. This opens the possibility for steady-state tokamak reactors. In low-aspect-ratio tokamaks there is little space available for the main solenoid of the ohmic heating transformer, and this limits the maximum current that can be inductively driven. Therefore, the sucess of this concept depends substantially on the possibility of driving current by non-inductive methods. In particular, it is important to investigate the variation of the bootstrap current with aspect ratio. The bootstrap current is proportional to the fraction of trapped particles divided by the poloidal field. As the aspect ratio decreases, both these quantities increase if other relevant parameters are kept fixed. The net result depends on details of these variations. In this section we study the behaviour of the bootstrap current with aspect ratio, elongation and triangularity.

The expression for the bootstrap current in the low collisionality regime obtained for Hirshmann[13] will be utilized in this work. Supposing that the pressure and temperature profiles are the same, and with the expression for the $<\vec{j}.\vec{B}>$ normalized with respect to $B_0^2/\mu_0 R_0 \alpha$, the bootstrap current is given by the following equation[13,14]

$$<\vec{j}.\vec{B}>= -I(\psi)\frac{dp}{d\psi}(\frac{\alpha^2\beta_0}{2})x[A(x)+\frac{A(x)}{Z}+\frac{\Lambda(x)A(x)}{Z}-B(x)], \tag{7}$$

where $I(\psi) = RB_\varphi$, $A(x) = [0.754 + 2.212 + Z^2 + x(0.348 + 1.2432 + Z^2)]/D(x)$, $B(x) = (0.884 + 2.074Z)/D(x)$, $\Lambda(x) = -1.172/(1+0.465x)$ and $D(x) = 1.4142 + Z^2 + x(0.754 + 2.657Z + 2Z^2) + x^2(0.348 + 1.2432 + Z^2)$. Here x represents the ratio between the fractions of trapped and circulating particles in velocity space, thàt is, $x = f_t/f_c$, and the notation $<\chi>$ denotes the flux average operator ($\oint dlB_p/B_p^2$), where B_p is the poloidal magnetic field. An explict expression for f_c is[13]

$$f_c = \frac{3}{4}\frac{<B^2>}{B_{max}^2}\int_0^1 \frac{\lambda d\lambda}{<\sqrt{1-\frac{\lambda B}{B_{max}}}>}. \tag{8}$$

Figure 4 shows curves of the ratio between the bootstrap current and the plasma current for several values of aspect ratio, for $\kappa = 1.0$ and 1.5, $\delta = 0.0, 0.2$ and 0.5. It is shown that for zero triangularity and any elongation, there is a slight increase of the fraction of bootstrap current with the inverse aspect ratio. Mowever, for non vanishing triangularity and any elongation, the fraction of bootstrap current decreases for d $\geq$ 0.4, which represents the inverse aspect ratio in our notation In the limit of high aspect ratio, the bootstrap cuirent is proportional to $\sqrt{d}\beta_p$, where β_p is the poloidal beta[14]. For the Soloviev model β_p is approximately constant for $\delta = 0$ but decreases with d for $\delta \neq 0$. This i; the reason for the slight decrease of the fraction of the bootstrap current with the aspect ratio. To increase the fraction of the bootstrap current as the aspect ratio decreases it is necessary to elongate the cross-section of the plasma column. Actually, in a free-boundary equilibrium, the cross-section of the plasma column has a natural tendency to become elongated as the aspect ratio decreases[15].

The values obtained for the fraction of the bootstrap current are quite small (from 8 to 12 %) as compared to values usually quoted. This occurs because experiments intended to maximize the bootstrap current run at very high values of the poloidal beta and, unfortunately, in the Soloviev model the value of the poloidal beta cannot be freely varied. Nevertheless, the relative variation with the aspect ratio is meaningful and indicates that in the low-aspect-ratio tokamaks one can expect a fraction the bootstrap current as large as observed in current devices.

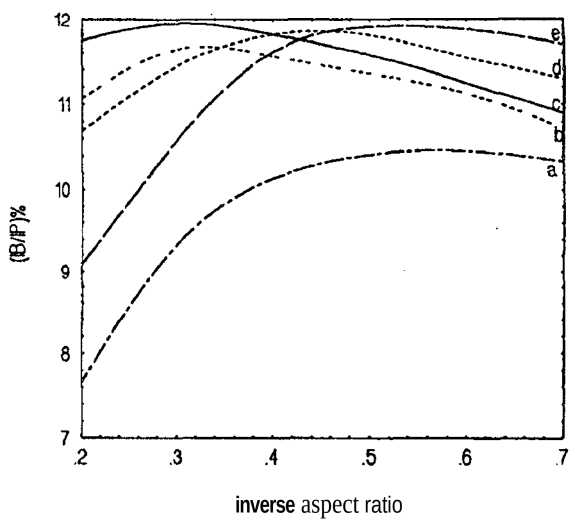

Figure 4: Ratio between the bootstrap and plasma current for the several values of the aspect ratio, for $\kappa = 1.0, \delta = 0$(a), $\kappa = 1.0, \delta = 0.2$(b), $\kappa = 1.5, \delta = 0.5$(c), $\kappa = 1.5, \delta = 0.2$(d) and $\kappa = 1.5, \delta = 0.0$(e).

V. Conclusions

The topology of banana orbits, the invariant adiabatic J, and bootstrap current have been studied, using an analytical equilibrium model. Low-aspect-ratio tokamaks have banana orbits more elongated and narrow in comparison with the ones in high-aspect-ratio devices. Only horizontal configurations have tlie possibility of operating in the condition of drift reversal. Similar results have been obtained by Miller et al for numerically computed equilibria. Therefore, stabilization of trapped particle modes is not favoured in small-aspect-ratio tokamaks with naturally elongated cross-section. To allow that a reasonable fraction of the total current is provided by tlie bootstrap current effect, it is necessary to elongate the plasma cross-section as the aspect ratio decreases.

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