Quark Degrees of Freedom in Nuclei*

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The problem of treating quark degrees of freedom in the nuclear many-body problem is discussed in terms of the Fock-Tani method. This method was originally developed for applications in atomic and molecular physics to investigate physical problems where the internal degrees of freedom of atoms and molecules cannot be neglected. Given a microscopic Hamiltonian in terms of quark degrees of freedom, it is possible to derive an effective Hamiltonian in terms of nucleon and meson degrees of freedom. To explain the formalism, I use a simple quark model where quarks interact by means of two-body forces and the nucleons are described as bound states of three quarks. Perspectives for realistic applications of the formalism in low-energy hadronic physics are discussed.

H. Pntroduction

The traditional approach to the nuclear many-body problem consists in solving the Schrodinger equation for structureless nucleons interacting through two-body potentials[1]. Since the original suggestion by Yukawa[2], the nucleon-nucleon potential has been thought as being due the exchange of mesons. The range of the potential is proportional to the inverse of the mass of the exchanged meson; the long-range part is attributed to the pion and the short-range part is attributed to the omega meson. Much has been progressed on this picture since Yukawa's suggestion, and meson exchange models have achieved a high degree of sophistication. Presently, these models provide a reasonably precise phenomenological description of the nuclear force.

However, whereas the long-range part of the interaction is well established in terms of one-pion exchange, the short-range part has to be regulated phenomenologically. The regularization is made by introducing form factors or by pure adjustment to data. On the other hand, since nucleons and mesons are not elementary particles, one might hope to circumvent this difficulty by invoking the quark and gluon degrees of freedom. In the case of bound nucleons, there is an additional difficulty with the meson exchange picture. The average distance between nucleons in a large nucleus is known to be of the order of $d \sim \rho_0^{-1/3} = 1.8$ fm, where $\rho_0 = 0.17\ fm^{-3}$ is the normal nuclear density. The radius of a nucleon is of the order of $\simeq 0.8\ fm$. From this, one has that two nucleons in a nucleus have great probability of superposition, and, therefore, a literal attribution of an exchange of an extended meson in such a situation seems to be quite artificial.

It seems, therefore, plausible that quark and gluon degrees of freedom will be invoked for a consistent description of short-range phenomena in nuclear physics. In this sense, it would be very desirable to have a new theoretical approach, where one can deal with the simultaneous presence of hadronic and quark and gluon degrees of freedom in nuclei. In other areas of physics there are examples of many-body systems where the interna1 degrees of freedom of composite particles cannot be neglected. Such examples include the superconductivity in metals, partially ionized plasmas, and atomic collisions. However, there is a fundamental difference between the constituents of these systems and the quarks and gluons of hadrons: the confinement of color, which implies that quarks and gluons are confined to a region of the order of the volume of the hadron. Moreover, since the overall picture of a nucleus is of

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a system of hadrons, the dynamics of the color degree of freedom ir nuclei is certainly limited to very short distance phenomena[3]. This means that the new approach, which considers explicitely quarks and gluons, should deviate minimally from, and contain in some limit, the traditional one, where the nucleus is treated as a system cf hadrons.

Several methods have been developed in the past for dealing with problems where constituents and composites are sinultaneously present. In this talk, I will consider a method which was invented independently by Girardeau[4] and Vorob'ev and Khomkin[5]. The method has been continuously improving in the last two decades, and has been used with success by Girardeau and collaborators[6,7] in several areas of atomic physics. The basic idea of the method is a generalization of a transformation employed by S. Tani[8] in 1960 to study single-particle scattering by a potential with a bound state. The method also has some similarities to the approaches of Dyson[9] to treat the Heisenberg model and to the method of Bohm and Pines[10] to treat collective modes in nuclei. Girardeau coined the name Fock-Tani method for the approach.

In the Foc c-Tani approach, one starts with the Fock representatior of the system, where creation and annihilation operators of elementary constituents satisfy canonical conimutation relations. "Ideal" composite creation and annihilation operators acting on an enlarged Fock space are introduced. The enlarged Fock space is a direct product of the original Fock space and an ideal state space. These ideal operators have the same quentum numbers as the composite particles of the system; however, by definition, they satisfy canonical conimutation relations. Next, a given unitary transforrnation, which transforms the composite operators intc the ideal operators, is introduced. The application of this unitary operator on the microscopic Hamiltonian, and on other operators expressed in terms of the elementary constituent operators, gives equivalent operators which contain the ideal composite operators. When acting on the image of the part of the enlarged Fock space which contains no ideal composites, such trarsformed operators explicitly express the interactions of composites and constituents.

In the followingsections, I discuss the Fock-Tani formalism in the context of a specific model for the quark structure of the nucleons. Although the results derived are valid in the context of the model, the method can be extended to other quark models. The aim is to present the basic formalism, and discuss its applicability to the nuclear many-body problem. There will be no attempt to present numerical results of specific applications of the method; these will be presented elsewhere.

In Section II, I present the quark model employed to define the nucleon structure. This is the necessary first step for the application of the method. Section III presents the formalism of the Fock-Tani method. In Section IV I derive an effective nucleon-nucleon interaction and discuss the many-body problem in context of the Fock-Tani formalism, and Section V presents the conclusions and perspectives.

II. Quark structure of the nucleons

The starting point of the Fock-Tani method is the specification of the microscopic Hamiltonian, expressed in terms of the field operators of the fundamental constituents (quarks and gluons), from which the hadronic bound states are derived. In principle, one would like to start with the fundamental Hamiltonian (or Lagrangian) of quantum chromodynamics (QCD). However, the problem of the confinement of the quarks and gluons has yet to be solved in QCD. For this reason, the use of models for the study of bound states in QCD is a practical necessity.

There is a variety of models designed to describe the low-energy properties of hadrons. In principle, the Fock-Tani method can be implemented in a straightforward way with any quark model that treats hadrons as bound states of a fixed number of particles. Nonrelativistic and relativistic potential and bag models are particularly suitable for the application of the method. In order to explain the method, I will consider nucleons only; the consideration of nucleons and mesons simultaneously is more involved, but can be done in a similar way. For simplicity, I will consider a quark model in which the nucleon is described as a bound-state of three quarks, gluon and antiquark degrees of freedom are neglected. A variety of popular models of the nucleon,

such as potential and bag models, are of this sort.

The model microscopic Hamiltonian I consider is of the general form

$$H = T^{\mu}\, q^{\dagger}_{\mu} q_{\mu} + V^{\mu\nu\sigma\rho} q^{\dagger}_{\mu} q^{\dagger}_{\nu} q_{\rho} q_{\sigma} \,, \qquad (1)$$

where $q^{\dagger}_{\mu}$ and q_{μ} are respectively the creation and annihilation operators of quarks with quantum numbers μ, where μ denotes the spatial, spin-flavor, and color quantum numbers $\mu = \{\vec{p}, \mathrm{a}, \mathrm{c}\}$. The quark operators satisfy canonical anticommutation relations,

$$\{q_{\mu}, q_{\nu}\} = 0 \,, \qquad \{q_{\mu}, q^{\dagger}_{\nu}\} = \delta_{\mu\nu} \,. \qquad (2)$$

A single-nucleon state, with center-of-mass momentum $\vec{P}$, spin projection M_S and isospin projection M_T, $|\vec{P}, M_S, M_T>$, can be written as

$$|\vec{P}, M_S, M_T> \equiv |\alpha> = B^{\dagger}_{\alpha}|0> \,, \qquad (3)$$

where $|0>$ is the vacuum state (no quarks), and $B^{\dagger}_{\alpha}$ is the nucleon creation operator

$$B^{\dagger}_{\alpha} = \frac{1}{\sqrt{3!}} \Phi^{\mu_1\mu_2\mu_3}_{\alpha} q^{\dagger}_{\mu_1} q^{\dagger}_{\mu_2} q^{\dagger}_{\mu_3} \,, \qquad (4)$$

where a sum over repeated indices is implied. $\Phi^{\mu_1\mu_2\mu_3}_{\alpha}$ is the nucleon wave-function. It is convenient to work with Φ normalized as

$$<\alpha|\beta> = \Phi^{*\mu_1\mu_2\mu_3}_{\alpha} \Phi^{\mu_1\mu_2\mu_3}_{\beta} = \delta_{\alpha\beta} \,. \qquad (5)$$

Using the quark anticommutation relations of Eq. (2), and the normalization condition above, one can easily show that the nucleon operators satisfy the following anticommutation relations:

$$\{B_{\alpha}, B_{\beta}\} = 0 \,, \quad \{B_{\alpha}, B^{\dagger}_{\beta}\} = \delta_{\alpha\beta} - \Delta_{\alpha\beta} \,, \qquad (6)$$

where

$$\begin{aligned} \Delta_{\alpha\beta} &= 3\Phi^{*\mu_1\mu_2\mu_3}_{\alpha} \Phi^{\mu_1\mu_2\nu_3}_{\beta} q^{\dagger}_{\nu_3} q_{\mu_3} \\ &- 3/2 \Phi^{*\mu_1\mu_2\mu_3}_{\alpha} \Phi^{\mu_1\nu_2\nu_3}_{\beta} q^{\dagger}_{\nu_3} q^{\dagger}_{\nu_2} q_{\mu_2} q_{\mu_3} \,. \end{aligned} \qquad (7)$$

The composite nature of the nucleons is expressed by the term $\Delta_{\alpha\beta}$ in Eq. (6). The presence of this term complicates enormously the probiem when one has hadrons and quarks present. The usual many-body techniques, such as the Green's function method, Wick's theorem, etc, apply to creation and annihilation operators that satisfy canonical relations. The nucleon operators $B^{\dagger}$ and B, are not convenient dynamical variables to be used. The idea of the Fock-Tani method is to change representation, such that the operators of composite particles are redescribed by operators which satisfy canonical commutation relations. Of course, the complications of the composite nature of the hadrons will appear somewhere else in the formalism.

III. The Fock-Tani transformation

In this section the Fock-Tani transformation is presented in the context of the nucleon model presented in the previous section. The discussion follows closely Refs. [4], [6], where the Fock-Tani method is discussed in the context of atomic physics problems.

The main features of the Fock-Tani transformation are:

1) The transformation is performed by an unitary transformation U:

$$|\Omega> \rightarrow |\Omega) = U^{-1}|\Omega> , \qquad O \rightarrow O_{\mathrm{FT}} = U^{-1} O U \,. \qquad (8)$$

$|\Omega>$ is an arbitrary state vector and O an arbitrary operator, both expressed in terms of the quark operators $q, q^{\dagger}$ of the original Fock representation. $|\Omega)$ and O_{FT} are the corresponding quantities in the new representation[1]. Since U is unitary, scalar products and matrix elements are preserved under the change of representation

$$<\Omega|\Omega> = (\Omega|\Omega) , \qquad <\Omega|O|\Omega> = (\Omega|O_{\mathrm{FT}}|\Omega) \,. \qquad (9)$$

2) If $|\alpha>$ is a single-nucleon state, it is redescribed by an ("ideal") elementary nucleon state under the transformation:

$$|\alpha> = B^{\dagger}_{\alpha}|0> \rightarrow U^{-1}|\alpha> = (\mathrm{a}) = b^{\dagger}_{\alpha}|0) \,, \qquad (10)$$

[1] Note that in the new representation states are represented by rounded, instead of angular, bras and kets.

where, by definition, ideal nucleon operators $b^\dagger_\alpha$ and b, satisfy canonical anticommutation relations

$$\{b_\alpha, b_\beta\} = 0\,, \qquad \{b_\alpha, b^\dagger_\beta\} = \delta_{\alpha\beta}\,. \tag{11}$$

The state $|0\rangle$ is the vacuum of both q and b degrees of freedom in the new representation:

$$q_\mu|0\rangle = b_\alpha|0\rangle = 0\,. \tag{12}$$

In addition, in the new representation the quark operators $q^\dagger$ and q are formally independent of the $b^\dagger_\alpha$ and b,

$$\{q_\mu, b_\alpha\} = \{q_\mu, b^\dagger_\alpha\} = 0\,. \tag{13}$$

3) A n-nucleons state $|\alpha_1, \cdots, \alpha_n>$ constructed from mutually nonoverlapping and well-separated wave packets is transformed into a n-ideal-nucleons state $|\alpha_1, \cdots, \alpha_n)$:

$$|\alpha_1, \cdots, \alpha_n> \to U^{-1}|\alpha_1, \cdots, \alpha_n> = |\alpha_1, \cdots, \alpha_n) = b^\dagger_{\alpha_1}\cdots b^\dagger_{\alpha_n}|0)\,. \tag{14}$$

This is particularly important for nucleon-nucleon scattering processes, where asymptotic states are non-overlapping. However, this is not the case for a n-nucleons state of a nucleus, where considerable overlap among the nucleons is in principle possible.

4) The transformed Hamiltonian operator can generally be written as

$$H \to H_{FT} = U^{-1}HU \equiv H^{(o)}_{FT} + V_{FT}\,. \tag{15}$$

$H^{(o)}_{FT}$ is the non-interacting part; it contains the single-quark part of the original Hamiltonian, and a single-nucleon part which describes the free propagation of the composite-nucleons. V_{FT} is the interacting part, responsible for all possible interactions among the composites and the quarks. It describes only the "true" interaction processes, the binding of the quarks into nucleons is included in $H^{(o)}_{FT}$.

The unitary operator U is constructed as follows. Eet the physical Fock space, constructed from all linear combinations of the quark operators $q^\dagger$ satisfying the canonical anticommutation relations of Eq. (2), be denoted by $\mathcal{F}$. Define an independent Hilbert space $\mathcal{N}$, the "ideal nucleon space" (Dyson approach[9]), as the space constructed from all linear combinations of the ideal nucleon operators $b^\dagger$ which satisfy the anticommutation relations of Eq. (11). Next, define the "ideal state space" as the direct product of $\mathfrak{B}$ and $\mathcal{N}$, $\mathcal{I} = \mathfrak{B} \times \mathcal{N}$. The quark and ideal nucleon operators are kinematically independent and therefore satisfy Eq. (13) on Z. The anticommutation relations of Eqs. (2,6), initially defined on $\mathfrak{B}$, as well as those of Eq. (11), initially defined on $\mathcal{N}$, are also valid on Z. The idea is now to establish an one-to-one correspondence between the composite states in $\mathfrak{B}$ and the ideal states of a subspace of $\mathcal{I}$.

In $\mathcal{I}$, there is a subspace which is isomorphic to the original Fock space $\mathcal{F}$, namely the space $\mathcal{I}_0$ consisting of those states $|\Omega>$ with no ideal nucleons,

$$b_\alpha|\Omega> = 0\,, \quad \text{or} \quad N_b|\Omega> = 0\,, \tag{16}$$

where N_b is the ideal nucleon total number operator

$$N_b = \sum_\alpha b^\dagger_\alpha b_\alpha\,. \tag{17}$$

The meaning of this is that in $\mathcal{I}_0$ the ideal nucleons are "redundant modes" (Bohm and Pines approach[10]).

The change of representation is performed by the operator

$$U = \exp\left(\frac{\pi}{2}F\right)\,, \qquad F = \sum_\alpha (b^\dagger_\alpha B_\alpha - B^\dagger_\alpha b_\alpha)\,, \tag{18}$$

with $B^\dagger_\alpha$ given by Eq. (4). It is not difficult to prove[6] that the transformation implemented by such an U indeed has the characteristics 1) to 4) discussed above.

U acts on $\mathcal{I}$, and cannot be defined on $\mathbf{3}$. However, it is defined on $\mathcal{I}_0$, which is isomorphic to $\mathbf{3}$. The image $\mathcal{F}_{\rm FT} = U^{-1}\mathcal{I}_0$ of $\mathcal{I}_0$ is the subspace of $\mathcal{I}$ that consists of all states $|\Omega)$ in the new representation related to the states $|\Omega>$ of $\mathcal{I}_0$ by the transformation of Eq. (8) by

$$|\Omega) = U^{-1}|\Omega> \ . \tag{19}$$

Therefore, any calculation in the original Fock space $\mathbf{3}$ is equivalent to a calculation in the Fock-Tani space $\mathcal{F}_{\rm FT}$. For any two states $|\Omega>$ and $|\Omega'>$ and any observable O in $\mathcal{F}$, one has

$$<\Omega|O|\Omega'> = (\Omega|U^{-1}OU|\Omega') = (\Omega|O_{\rm FT}|\Omega') \ . \tag{20}$$

The great advantage of working in $\mathcal{F}_{\rm FT}$ is that all creation and annihilation operators satisfy canonical commutation relations. However, the complication of the composite nature of the nucleons will appear elsewhere. First of all, the transformed operators $O_{\rm FT} = U^{-1}OU$ give rise to an infinite series, which physically represents, in some sense, an expansion in the density of the system. The method will be efficient for practical calculations if a few terms in the series are sufficient for describing the physical effects of the overlap of the structures. This, of course, will depend on each system considered, and has to be analyzed case by case. The other potential complication is related to the constraint of Eq. (16), called the "subsidiary condition". The imposition of this constraint might make the solution of any practical calculation as complicated as that dealing with the original $B^\dagger$ and B operators. However, for scattering problems, which starts with the proper definition of the asymptotic states, Eq. (16) is trivially satisfied. For bound state problems, as in the case of the nuclear many-body problem, some caution in not violating this constraint has to be exercised.

IV. Effective nucleon-nucleon potential and the nuclear many-body problem

The nucleon-nucleon scattering matrix involves the computation of matrix elements involving the interaction Hamiltonian between asymptotic nucleon states. A general two-nucleon state is of the form

$$|\alpha,\beta> = B^\dagger_\alpha B^\dagger_\beta|0> \ , \tag{21}$$

where $B^\dagger_\alpha$ is defined in Eq. (3). Since the two nucleons have no overlap in the asymptotic region, one has trivially

$$\begin{aligned} U^{-1}|\alpha,\beta> &= U^{-1}B^\dagger_\alpha UU^{-1}B^\dagger_\beta UU^{-1}|0> \\ &= b^\dagger_\alpha b^\dagger_\beta|0) \ . \end{aligned} \tag{22}$$

In order to compute the scattering matrix in the new representation, one has to consider the Fock-Tani transformation of the quark Hamiltonian of Eq. (1):

$$\begin{aligned} H_{\rm FT} &= U^{-1}HU \\ &= T^\mu U^{-1}q^\dagger_\mu UU^{-1}q_\mu U + V^{\mu\nu\sigma\rho}\, U^{-1}q^\dagger_\mu UU^{-1}q^\dagger_\nu UU^{-1}q_\rho UU^{-1}q_\sigma U \ . \end{aligned} \tag{23}$$

This, in turn, requires the transformation of the quark operators:

$$U^{-1}q_\mu U = e^{-\frac{\pi}{2}F}q_\mu e^{\frac{\pi}{2}F} \ . \tag{24}$$

The evaluation of this expression is very complicated, since it involves the consideration of multiple commutators of F and q. A practical way to evaluate the multiple commutators is provided by the "equations of motion" method. This consists in the following. For any operator O, one defines

$$O(t) = \exp(-tF)\,O\exp(tF)\,. \tag{25}$$

Differentiatir g the above equation with respect to "time" t, one gets an equation of motion for O,

$$\frac{d}{dt}O(t) = [O(t), F]. \tag{26}$$

One has the "initial condition"

$$O(0) = O\,, \tag{27}$$

the physical value of t is $t = \pi/2$, and so

$$O(\pi/2) \equiv O_{FT} = U^{-1}OU\,. \tag{28}$$

The equation of motion for the quark operator q is given by:

$$\frac{d}{dt}q_\mu(t) = -\frac{3}{\sqrt{3!}}\delta_{\mu,\mu_3}\Phi_\alpha^{\mu_1,\mu_2,\mu_3}q^\dagger_{\mu_1}(t)q^\dagger_{\mu_2}(t)b_\alpha(t)\,. \tag{29}$$

Note that a sum over repeated indices is implied.

To solve this equation, one needs the equation for $b_\alpha(t)$:

$$\frac{d}{dt}b_\alpha(t) = B_\alpha(t)\,. \tag{30}$$

This equation requires the equation for $B_\alpha(t)$:

$$\frac{d}{dt}B_\alpha(t) = -b_\alpha(t) - \Delta_{\alpha\beta}(t)b_\beta(t)\,, \tag{31}$$

where $\Delta_{\alpha\beta}$ is given by Eq. (7).

Eqs. (29-31), together with their Hermitean conjugates, form a set of coupled nonlinear ordinary differential equations. Obviously, this cannot be solved in a closed form and an approximation method is required. A convenient and physically appealing way to solve the equations is an iteration procedure: starting from a "zero-order" approximation, where the overlap among the nucleons is neglected, one proceeds to higher order by collecting terms in the same degree in the bound state wave functions Φ_α and Φ^*_α. One then writes for each operator the expansions

$$q_\mu(t) = \sum_{i=1}^{\infty} q_\mu^{(i)}(t)\,, \qquad b_\alpha(t) = \sum_{i=1}^{\infty} b_\alpha^{(i)}(t)\,, \qquad B_\alpha(t) = \sum_{i=1}^{\infty} B_\alpha^{(i)}(t)\,. \tag{32}$$

The superscript i denotes the power in which Φ_α and Φ^*_α appear. For consistency with the anticommutation relation in Eq. (6), the implicit Φ_α and Φ^*_α in the definition of $B^\dagger_\alpha$ and B_α are not counted[4]. The expansion in powers of the bound-state wavefunction is essentially an expansion in powers of the density of the system.

The zero-order approximation is the one where the effects of the nucleon structure are neglected. This amounts to neglecting respectively the terms $\Delta_{\alpha\beta}$ and $\Phi_\alpha^{\mu_1\mu_2\mu_3}$ on the r.h.s. of Eq. (31) and Eq. (29):

$$\frac{d}{dt}B^0_\alpha(t) = -b^0_\alpha(t)\,, \qquad \frac{d}{dt}b^0_\alpha(t) = B^0_\alpha(t)\,, \qquad \frac{d}{dt}q^{(0)}_\mu(t) = 0\,. \tag{33}$$

Using the initial condition Eq. (27), one finds the zero-order solutions:

$$b^0_\alpha(t) = b_\alpha \cos t + B_\alpha \sin t\,, \tag{34}$$

$$B^0_\alpha(t) = B_\alpha \cos t - b_\alpha \sin t\,, \tag{35}$$

$$q^{(0)}_\mu(t) = q_\mu. \tag{36}$$

If one were to stop here, one would have the situation where all effects of quark interchange were neglected. The next iteration is obtained by collecting terms with one wavefunction:

$$\frac{d}{dt}B^0_\alpha(t) = -b^0_\alpha(t)\,, \qquad \frac{d}{dt}b^0_\alpha(t) = B^0_\alpha(t)\,, \tag{37}$$

$$\frac{d}{dt}q^{(1)}_\mu(t) = -\frac{3}{\sqrt{3!}}\delta_{\mu,\mu_3}\Phi^{\mu_1,\mu_2,\mu_3}_\alpha q^{(0)\dagger}_{\mu_1}(t)q^{(0)\dagger}_{\mu_2}(t)b^{(0)}_\alpha(t)\,. \tag{38}$$

Since the initial conditions were assigned to the zero-order terms, one has from **Eq.** (27) and **Eq.** (32) that

$$b^{(i)}_\alpha(t=0) = B^{(i)}_\alpha(t=0) = q^{(i)}_\mu(t=0) = 0\,, \quad \text{for} \quad i \geq 1\,. \tag{39}$$

These imply in

$$b^{(1)}_\alpha(t) = 0\,, \tag{40}$$

$$B^{(1)}_\alpha(t) = 0\,, \tag{41}$$

$$q^{(1)}_\mu(t) = -3\delta_{\mu\mu_3}\Phi^{\mu_1\mu_2\mu_3}_\alpha q^\dagger_{\mu_1}q^\dagger_{\mu_2}\left[b_\alpha \sin t + B_\alpha(1-\cos t)\right]\,. \tag{42}$$

The second order terms introduce a slight complication. The complication is that at this order the solutions have secular terms, i.e., the "time" dependence does not appear in terms of trigonometric functions only, powers of t also appear. Among other things, the secular terms introduce post-prior discrepancies in the analysis of scattering problems. This problem was recently solved by Girardeau and Straton[11]. They generalized the **F** operator such that the secular terms cancel. The new **F** operator is constructed order by order. Here I will simply present the result for the second order operators obtained with the Girardeau-Straton approach. The details will be presented elsewhere. The second order operators are given by:

$$b^{(2)}_\alpha(t) = -3\Delta_{\alpha\beta}B_\beta \sin t\,, \tag{43}$$

$$B^{(2)}_\alpha(t) = -3\Delta_{\alpha\beta}b_\beta \sin t\,, \tag{44}$$

$$\begin{aligned} q^{(2)}_\mu(t) = {} & 3/26_{\mu\nu_3}\Phi^{*\mu_1\mu_2\mu_3}_\alpha\Phi^{\mu_1\mu_2\nu_3}_\beta\left[b^\dagger_\alpha q_{\mu_3}b_\beta \sin^2 t + 2B^\dagger_\alpha q_{\mu_3}b_\beta \sin t\right. \\ & -B^\dagger_\alpha q_{\mu_3}B_\beta \sin^2 t - b^\dagger_\alpha q_{\mu_3}B_\beta \sin t\cos t - B^\dagger_\alpha q_{\mu_3}b_\beta \sin t\cos t \\ & \left.+2B^\dagger_\alpha q_{\mu_3}B_\beta(1-\cos t)\right] - 3/2\delta_{\mu\nu_3}\Phi^{*\mu_1\mu_2\mu_3}_\alpha\Phi^{\mu_1\nu_2\nu_3}_\beta\left[b^\dagger_\alpha q^\dagger_{\nu_2}q_{\mu_2}q_{\mu_3}b_\beta \sin^2 t\right. \\ & +2B^\dagger_\alpha q^\dagger_{\nu_2}q_{\mu_2}q_{\mu_3}b_\beta \sin t - B^\dagger_\alpha q^\dagger_{\nu_2}q_{\mu_2}q_{\mu_3}B_\beta \sin^2 t - b^\dagger_\alpha q^\dagger_{\nu_2}q_{\mu_2}q_{\mu_3}B_\beta \sin t\cos t \\ & \left.-B^\dagger_\alpha q^\dagger_{\nu_2}q_{\mu_2}q_{\mu_3}b_\beta \sin t\cos t + 2B^\dagger_\alpha q^\dagger_{\nu_2}q_{\mu_2}q_{\mu_3}B_\beta(1-\cos t)\right]\,. \end{aligned} \tag{45}$$

The iteration process can be systematically continued to higher orders in a straightforward way. However, as one goes to higher orders, the complexity of the operators increases significantly. Since there is a systematic algorithm to be followed, it is probably feasible to program an algebraic manipulator on a computer to generate the more complex structures.

Stopping at the third order is already sufficient to discuss an effective nucleon-nucleon potential. Substituting the expressions for the $U^{-1}q_\mu U$, and the corresponding ones for $U^{-1}q^\dagger_\mu U$, to this order (the results for the third order operators will be presented elsewhere) in Eq. (23), one obtains the Fock-Tani Hamiltonian. Among other terms, the Fock-Tani Hamiltonian presents a term involving the ideal nucleon operators only. This term is an effective nucleon Hamiltonian, and has the general form:

$$H_{NN} = T_N^\alpha \, b_\alpha^\dagger b_\alpha + V_{NN}^{\alpha\beta\gamma\delta} \, b_\alpha^\dagger b_\beta^\dagger b_\delta b_\gamma \,, \qquad (46)$$

where T_N^α is the single-nucleon energy, corresponding to the nucleon rest mass and the nucleon c.m. kinetic energy

$$T_N^\alpha = M^\alpha + \frac{\vec{P}^2}{2M^\alpha} \,. \qquad (47)$$

The term $V_{NN}^{\alpha\beta\gamma\delta}$ is a sum of terms which involves explicitly the 'iound-state wavefunctions of the nucleon. One immediate consequence of this is that the effective potential is nonlocal in space. Note that all baryonic excitations can be taken into account in the scattering problem; although the asymptotic states are ground state nucleons, the sum over α in Eq. (18) can include all the excititctions. These excitations would contribute as intermediate states as in the usual approach using elementary liadrons.

In the following, I discuss three representative terms that contribiite to V_{NN}. One of such terms comes from the kinetic energy of the quarks:

$$T^\mu \, \Phi_\alpha^{*\mu_1\mu_2\mu} \, \Phi_\alpha^{*\nu_1\nu_2\nu_3} \, \Phi_\gamma^{\mu_1\nu_2\nu_3} \, \Phi_\delta^{\nu_1\mu_2\mu} \,. \qquad (48)$$

Note that the quark indices are all contracted; the nucleon labels contract with the nucleon operators $b^\dagger$ and b (see Eq. (46)). This contribution can be represented graphicaily, as shown in Fig. (1) below.

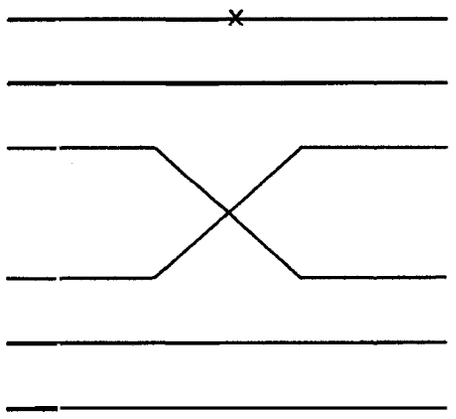

Figure 1: Diagrammatical representation of Eq. (48).

Next, I present typical terms which arise from the quark-quark interaction. One of such terms is of the form

$$V^{\mu\nu\sigma\rho} \, \Phi_\alpha^{*\mu_1\mu_2\mu} \, \Phi_\alpha^{*\nu_1\nu_2\nu} \, \Phi_\gamma^{\nu_1\nu_2\rho} \, \Phi_\delta^{\mu_1\mu_2\sigma} \,. \qquad (49)$$

The diagrammatical representation of this term is presented in Fig. (2).

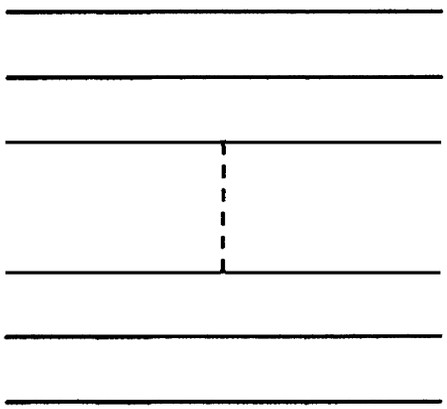

Figure 2: Diagrammatical representation of Eq. (49).

Note that this term does not involve quark exchange between the two nucleons. Such a term gives no contribution to V_{NN} when the quark-quark interaction carries color, which is the case of an one-gluon exchange. However, it is a commonfeature in several quark models that quarks interact via colorless bosonic fields, such as pions and scalar solitons. For such quark interactions, the diagram of Fig. (2) gives a nonzero contribution. Note that such an interaction term corresponds to the usual nucleon-nucleon interaction described in the language of elementary nucleons.

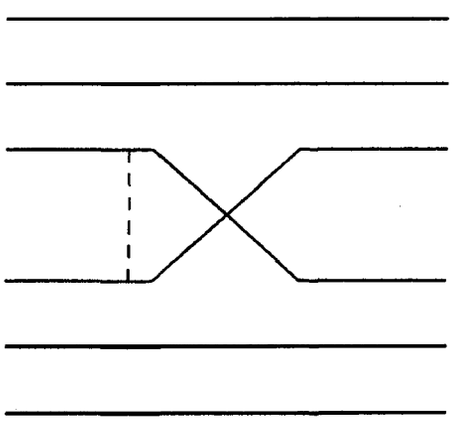

Figure 3: Diagrammatical representation of Eq. (50).

There are several terms in Eq. (46) which involve an interaction between two quarks belonging to different nucleons and a simultaneous quark interchange. Fig. **(3)** presents a typical graph of this sort. In terms of the bound-state wavefunctions, it is given by

$$V^{\mu\nu\sigma\rho}\,\Phi_{\alpha}^{*\mu_1\mu_2\mu}\,\Phi_{\alpha}^{*\nu_1\nu_2\nu}\,\Phi_{\gamma}^{\nu_1\nu_2\sigma}\,\Phi_{\delta}^{\mu_1\mu_2\rho}\,. \qquad (50)$$

Other graphs similar to the one of Fig. **(3)** involve different quark interchanges, with different quark-quark interactions.

There is an extensive literature on the study of the nucleon-nucleon interaction in terms of the underlying quark structure of the hadrons. Since the pioneering work of Liberman[12] and DeTar[13], who employed respectively a quark potential and the MIT bag model, different approaches employing different models have been used for more than a decade. An important piece of work on the subject employs the resonating group method. The work of Ribeiro[14], Oka and Yazaki[15], and Warke and Shanker[16] were the first ones in this line. Variations on the details of resonating group methods, as well as the use of different quark models, characterize basically all the work on the subject in the last decade.

Maltman and Isgur[17] employed variational techniques to study the ground state properties of the deuteron in the context of the potential quark model. They obtained an excellent agreement with experiment. The variational method was also employed by Weinstein[18] to study hadronic molecules.

A different approach to treat hadronic interactions in terms of quarks appeared in the last year, the so-called "quark Born diagram" formalism[19]. The formalism is based on the assumption that (nonresonant) hadron scattering is dominated by perturbative one-gluon-exchange, followed by quark interchange. The non-perturbative aspects of QCD enter via the hadron bound-state wave functions. This approach has achieved a reasonable accurate description of a variety of hadronic processes[20].

An interesting feature of the Fock-Tani approach is that it contains both the resonating group and quark Born diagram methods. Solving the two-nucleon scattering Schrodinger equation for the Fock-Tani Hamiltonian of is equivalent to the resonating group calculation. Evaluation of the scattering matrix for the effective nucleon Hamiltonian in Born approximation is equivalent to the quark Born diagram calculation. Perhaps the most attractive feature of the Fock-Tani method is that the traditional field theoretical methods can be employed in a straightforward way, since all operators satisfy canonical commutation relations.

In connection to nuclear structure studies, application of the Fock-Tani approach can be very interesting. Once one has an effective nucleon-nucleon interaction, the traditional approach can be applied to the many-body problem. In the following I discuss one of such applications.

Suppose one takes the zero order ground-state wavefunction as being a Fermi gas of A ideal nucleons:

$$|A) = b^{\dagger}_{\alpha_1} b^{\dagger}_{\alpha_2} \cdots b^{\dagger}_{\alpha_A} |0)\,. \qquad (51)$$

The Hartree-Fock energy of the system, calculated with the effective nucleon Hamiltonian is given by:

$$E_{HF} = \sum_{\alpha}^{A} T_N^{\alpha} + \sum_{\alpha\beta}^{k_F} \left(V_{NN}^{\alpha\beta\alpha\beta} - V_{NN}^{\alpha\beta\beta\alpha} \right)\,, \qquad (52)$$

where T_N^{α} and $V_{NN}^{\alpha\beta\alpha\beta}$ are defined in Eq. (46), and k_F is the Fermi momentum. In the case that there is no overlap among the (real) nucleons in the system, the state of Eq. (51) transforms under U to the usual Fermi-gas of real nucleons, and the Hartree-Fock energy calculated as above is equivalent to the traditional calculation.

Now, the H_{FT} Hamiltonian includes all possible processes involving quarks and hadrons compatible with the microscopic quark Hamiltonian: in addition to an effective nucleon-nucleon term, there are terms where two nucleons collide and one or the two nucleons are ionized into quarks. Such terms do not contribute (in principle) asymptotically due to the confinement of the quarks. However, they may contribute as intermediate processes, and will produce deviations from the zero-order Fermi-gas state of Eq. (51). In real nuclei, these deviations certainly are limited to short distances, and will manifest themselves as short-range correlations. Detection of signals of such effects would be very interesting, since one would have the unique opportunity to study interesting low-energy QCD physics which is unaccessible in a two-nucleon scattering in free space.

V. Conclusions and perspectives

The subject of quark degrees of freedom in nuclei is an interesting and fundamental problem in nuclear physics. Although there are no experimental evidence of quark effects in nuclei, it is very likely that quarks will be required for a consistent description of the short-range part of the nucleon-nucleon interaction, and short-range correlations in nuclei. The Fock-Tani method may provide a convenient tool for treating the nuclear many-body problem with quark degrees of freedom present.

Although the discussion of the Fock-Tani formalism has been explained in terms of a specific quark model, its applicability is not limited to the model discussed in this talk. The method can be extended to nucleon models which include a cloud of $q\bar{q}$ pairs. The techniques developed by Gilbert[6] would be adequate for this purpose. Another possibility is to use the Cloudy Bag Model[21], where elementary pions are coupled to a three quark state. This would be a simpler calculation as compared to the one with a $q\bar{q}$ cloud.

Before I conclude, I would like to stress some of the limitations of the Fock-Tani method. The most serious one refers to the fact that it is applicable only to models where the composites are bound states of a fixed number of particles. The extension of the method to composites described by relativistic quantum field models, where bound states are composed by an infinite number of particles and antiparticles, has yet to be made. However, there is no problem in applying the method to models based on the Dirac equation, such as bag and relativistic potential models. Another limitation is that the method is of practical use for systems where the degree of superposition of the composites in the medium is not very high, in the sense that the composites do not become completely dissolved into constituents. This is because the change of representation is effectuated by an iterative srocedure, and as the superposition of the composites increases, the number of iterations has to be increased prohibitly. The method, as presently formulated, is also inadequate to treat phase transitions problems, involving the coexistence of the hadronic and quark-gluon phases. This sort of limitation was recently discussed by Gilbert[22].

Concluding, I think that the Fock-Tani formalism has the potentiality of becoming an important tool to investigate quark effects in nuclear matter. In applications in atomic and molecular physics, this formalism has been used with success. There is no a priori reason why such a formalism should not be tried in realistic applications of low-energy hadronic physics.

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