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Thermalization in Relativistic Heavy Ion Collisions

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We review some recent results on the theoretical approach to the process of thermal equilibrium formation in heavy ion physics. Several works using different (and to some extent complementary) methods come to the conclusion that thermalization may well occur in heavy ion collisions at RHIC energies. Estimates of thermalization times are presented.

I. Pntroduction

The future collider experiments on nucleus-nucleus (A + A) collisions at highly relativistic energies ($\sqrt{s} \geq$ 200A GeV) at the BNL Relativistic Heavy Ion Collider (RHIC) and the CERN Lepton Hadron Collider (LHC) offer an opportunity to study ultradense matter in the laboratory and to learn about the new physics that hopefully becomes observable in these reactions. In particular, among the main goals is the search for evidence of the predicted phase transition to a quark-gluon plasma or to a phase where chiral symmetry is restored. These phenomena are supposed to happen in hot and dense matter, i.e., matter in thermal equilibrium. Therefore thermalization of nuclear matter is a sine-qua-non condition for all those exotic phenomena to take place. Once thermal equilibrium is established one can use thermodynamical and hydrodynamical methods to describe the evolution and decay of this highly excited matter. The study of thermalization can be divided into two complementary approaches: i) the analysis of the initial conditions of high energy reactions and ii) the comparison of hydrodynamical models with experimental data. The first approach is more theoretical whereas the second is more phenomenological. The first one tries to "prove" (from first principles) that thermalization is possible and the second tries to describe concrete experimental data. Clearly both are necessary and one does not work without the other. In the next sections we are going to comment recent works which follow the first approach. Reports on progresses on the second approach, which will be called the phenomenological approach, can be found elsewhere.

II. Perturbative QCD

The standard formalism to study the possible formation of thermal equilibrium is provided by relativistic kinetic theory. In principle, once we know the initial one-body phase space distribution $F_a(x_\mu, p_\mu)$ for particles of type a and the relevant matrix elements which describe the collisions between the involved hadrons (protons, neutrons, mesons and resonances) or sub-nucleonic constituents (quarks, gluons) it is possible to solve the relativistic transport equations

$$p^\mu \, \partial_\mu \, F_a\,(x_\mu, p_\mu) = \sum_k I_a^k, \tag{1}$$

with

$$I_a^k = \int (\text{phase space}) \sum |M_{ab\to cd}|^2\,, \tag{2}$$

and find F_a as a function of time. With the distribution functions we can calculate the entropy density which is given by

$$s^\mu = -\sum_B \gamma_B \int \frac{d^3p}{(2\pi)^3}\frac{p^\mu}{p_0}\{F_B \ln F_B - (1+F_B)\ln(1+F_B)\} - \sum_F \gamma_F \int \frac{d^3p}{(2\pi)^3}\frac{p^\mu}{p_0}\{F_F \ln F_F + (1-F_F)\ln(1-F_F)\}, \quad (3)$$

where the labels B and F stand for boson and fermion respectively. During the collision, the shape of F will be changing and the entropy will increase. In terms of these quantities equilibrium is reached when F assumes a stable shape which has the form

$$F(x,p) = \frac{1}{\exp[\beta_\nu (p^\nu - \lambda^\nu)] \pm 1} \quad . \quad (4)$$

Here p^ν is the four-momentum of the particle the upper sign refers to fermions and the lower sign to bosons. λ_ν and β_ν are space-time dependent four vectors constructed from the local flow velocity u_ν:

$$\begin{aligned} \lambda^\nu &= \mu\, u^\nu, \\ \beta^\nu &= \beta\, u^\nu. \end{aligned} \quad (5)$$

μ and T $(= 1/\beta)$ are then interpreted as chemical potential and temperature respectively. The projection of F on the transverse momentum plane will then have the familiar rxponential form $F \sim \exp\{-p_T/T\}$. Simultaneously with the stabilization of F equilibration implies that the entropy will saturate. In principle, given the initial conditions of a collision, given the relevant degrees of freedom, hadrons or quarks and gluons, the theory or model which describes the interactions among them (QHD-I, σ model, etc. or QCD) and given the initial distribution F we are then able, with the above described formalism, to solve the differential equations and find out whether or not thermalization will be reached and how long it will take for it.

In spite of the approximations contained in Eqs. (1) and (2), namely that particles follow straight line classical trajectories and that many-body correlations and many-body interactions are neglected, the eventual stabilization of F and saturation of the entropy would still be strong indication of thermalization. Recently, Klaus Geiger[1] presented a series of calculations in which the formalism described above is used together with perturbative QCD. His simulation is applied to nucleus-nucleus collisions at RHIC energies. The distribution functions F_a for quarks, antiquarks and gluons are computed at different time steps in a thin slab in the central region of the collision. They are found to reach the thermal shape and stabilize at a thermalization time $\tau_{\rm th} \cong 1$ fm for Au+Au collisions at $\sqrt{s} = 200$A GeV. At this time the entropy per particle reaches its final value ($\cong$ 3,6). The relaxation time (the time it takes to reduce the deviation from the equilibrium value by a factor of e) associated with the entropy production is 0.4 fm and it decreases with the energy $\sqrt{s}$, suggesting that at higher energies thermalization is even more likely to occur.

III. QED plasma methods

The theoretical approach to thermalization described above relies on the validity of perturbative QCD. It is possible however that even at very high energies non-perturbative effects play an important role. Therefore one must try to understand thermalization with a picture more sophisticated than the collection of pointlike particles behaving almost like an ideal gas, as implied in the perturbative approach. One serious and promising approach in the framework of non-perturbative QCD is based on plasma physics[2,3]. Here, thermalization is understood as a progressive equipartition of the energy of the system among the available degrees of freedom. It is well known that in electron-ion plasmas thermalization is not achieved by collisions. Instead, energy is distributed by the collective modes. This mechanism is more efficient if instabilities occur. It has been shown that for classical linearized QCD with highly anisotropic distribution functions the dispersion equation (derived from the QCD Maxwell equations), when solved, will give frequencies with positive imaginary parts. The corresponding collective oscillations will have exponentially increasing amplitudes. This kind of instability is called, in plasma language, filamentation instability. The initially homogeneous plasma streams get stratified in a direction transverse to the streaming direction by excitation of mixed (longitudinal + transverse) gluon fields. The analysis of the velocity fields in the presence of these instabilities shows that a collective deceleration of the mean streaming velocity takes place. This energy transfer from longitudinal to the transverse direction would certainly lead to a faster thermalization. However one has to be more quantitative and estimate the time scales for the development of instabilities. The most recent estimate was done as follows. The general dispersion equation of the (small) plasma oscillations reads

$$\det[\mathbf{k}^2\,\delta^{ij} - k^i\,k^j - \omega^2\,\varepsilon^{ij}(\omega,k)] = 0, \tag{6}$$

where k is the wave vector and w is the frequency. ε^{ij} is the chromoelectric permeability tensor, which in the collisionless limit is

$$\varepsilon^{ij}(\omega,k) = \delta^{ij} + \frac{2\pi\alpha_s}{\omega}\int\frac{d^3p}{(2\pi)^3}\,\frac{v^i}{\omega - \mathbf{k}\cdot\mathbf{v} + i0^+}\,\frac{\partial n}{\partial p^\ell}(p)\left[\left(1-\frac{\mathbf{k}\cdot\mathbf{v}}{\omega}\right)\delta^{\ell j} + \frac{k^\ell\,v^j}{\omega}\right], \tag{7}$$

where a, is the strong coupling constant, which is never assumed to be small, and

$$n(p) = n_q(p) + \overline{n}_q(p) + 6n_g(p), \tag{8}$$

with n_q, $\overline{n}_q$ and n_g being the distribution function of quarks, antiquarks and gluons. The plasma is assumed initially colorless and homogeneous but not isotropic. In spite of the similarity to the electrodynamic formulae, Eqs. (6) and (7) take into account the essential non-Abelian effect i.e. the gluon-gluon coupling. The distribution function is chosen in the form

$$n(y,p_T,\phi) = \frac{1}{2Y}\,\theta(Y-y)\,\theta(Y+y)\,h(p_T)\,\frac{1}{p_T\,ch\,y}, \tag{9}$$

where y, p_T and ϕ denote parton rapidity, transverse momentum and azimuthal angle, respectively. Eq. (9) gives the parton number distribution which is flat in the rapidity interval (−Y, Y). When the momentum distribution is a monotonously decreasing function of p, as it is the case here, the longitudinal modes, those with the wave vector k parallel to the chromoelectric field E are stable. Thus we look for instabilities among transversal modes. When the instability occurs the kinetic energy of particles is converted into the field energy. Considering E = (0, 0, E), k = (k, 0, 0) (z is the beam direction) Eq. (6) becomes

$$H(\omega) \equiv k^2 - \omega^2\,\varepsilon^{zz}(\omega,k) = 0 \quad . \tag{10}$$

Substituting the distribution function (9) into the dielectric tensor (7) one can solve Eq. (10). Some of the solutions lead to unstable modes. For them the characteristic time of instability development is given by

$$\tau \cong \frac{1}{\mathrm{Im}\,w} \quad . \tag{11}$$

Using in the above expressions reasonable numbers τ can be estimated to be $\geq$ 0.70 fm/c (for RHIC experiments) and $\geq$ 0.26 fm/c (for LHC experiments) for Au+Au collisions.

The estimated time of instability development is shorter than the characteristic time of nucleus-nucleus interaction, which is at least a few fm/c. This means that a color fluctuation, which appears at the initial stage of the collision, with the chromoelectric field along the beam and the wave vector perpendicular to it has a chance to grow converting the longitudinal energy to the transverse one. It is known from the studies of the electron-ion plasma that such an energy transport is very effective. Thus, the appearance of the instability is expected to speed up equilibration of the system leading to a more isotropic momentum distribution.

IV. Lattice QCD

Another classical and nonperturbative approach to thermalization in QCD is due to Müller[4] and collaborators. They consider QCD with gauge fields only, located in a certain region of space and time which is discretized. With numerical tools they study the thermalization of gauge fields by direct simulation of their real-time evolution and show that nonabelian gauge fields far off equilibrium approach a thermally equilibrated state very rapidly. As in other studies, a thermally equilibrated state is the state in which the energy distribution over the microscopic degrees of freedom does not change with time and takes the thermal equilibrium form, which can be calculated from the canonical Gibbs ensemble.

Their first work along this line was based on the Hamiltonian formulation of lattice $SU(2)$ gauge theory governed by the Hamiltonian

$$H = \frac{g^2}{a}\left\{\sum_{\ell}\frac{1}{2}E_\ell^a E_\ell^a + \lambda\sum_{P}\left[1-\cos\left(\frac{1}{2}B_p\right)\right]\right\}, \tag{12}$$

where g is the coupling constant, $\lambda = 4/g^4$, and a denotes the lattice spacing. The electric and magnetic fields are expressed in terms of the link variables

$$U_\ell = \exp\left\{-\frac{1}{2}\tau^a A_\ell^a\right\}, \tag{13}$$

which are elements of the group $SU(2)$:

$$E_\ell^a = -\,\mathrm{i}\frac{a}{g^2}\,\mathrm{tr}\left[\tau^a \dot{U}_\ell U_\ell^{-1}\right],$$
$$\cos\left(\frac{1}{2}B_p\right) = \frac{1}{2}\,\mathrm{tr}\,U_p. \tag{14}$$

U_p is the product of all four link variables on an elementary plaquette, and the overdot denotes a time derivative.

The equations of motion derived from the Hamiltonian (12) can be numerically integrated and one obtains the fields $U_\ell(t)$ as a function of time. The following gauge-invariant metric is then introduced

$$D\left(U_\ell, U_\ell'\right) = \frac{1}{2N_p}\sum_p\left[\mathrm{tr}\,U_p - \mathrm{tr}\,U_p'\right]. \tag{15}$$

D is proportional to the absolute local difference in the magnetic energy of two different gauge fields. The evolution of $D(t)$ for initially neighboring configurations on a 20^3 lattice was calculated. One observes that $D(t)$ grows exponentially as $D(t) = D_0\exp(ht)$ and then saturates because the magnetic energy per plaquette is bounded. The growth rate h is the largest of the Lyapunov exponents λ_i ($i = 1,\ldots,6N^3$). Extensive studies have shown that h is a universal function of the average energy per plaquette E such that

$$ha \cong \frac{1}{6}g^2 E a \quad . \tag{16}$$

This scaling property has been verified numerically over a wide range of values for g and a. It is interesting to note that h is independent of the lattice spacing a in the semiclassical limit, where g does not run with a. These calculations indicate that classical dynamics of the Yang-Mills field on a lattice is characterized by deterministic chaos. The divergence of nearby trajectories, as measured by the largest positive Lyapunov exponent $h(E)$, increases with growing excitation energy. Moreover it implies that the entropy S of an ensemble of gauge fields grows linearly with time as given by

$$S(t) = S_0 + \sum_i \lambda_i t\ , \tag{17}$$

until the available microcanonical phase space is filled and the system is equilibrated. The thermalization time is in this context given by the characteristic entropy growth time

$$\tau_{\mathrm{th}} = h^{-1} \cong 6/g^2\,\mathrm{E}. \tag{18}$$

Using reasonable values for g and E an estimate shows that $\tau_{\mathrm{th}} \cong 0.5$ fm. In the $SU(3)$ gauge theory, the corresponding gluon thermalization time is typically less than 0.4 fm. This promises the rapid formation of a locally thermalized gas plasma at sufficiently high energies.

Similarly to what was described in the first section, it is very important to follow the evolution of some distribution function in time. One can numerically integrate the equations of motion for given initial conditions and compute the energy distribution functions $P(E_M,t)$. One observes that after some time, compatible with the inverse of the maximal Lyapunov exponent, no noticeable change is seen. Moreover, the obtained equilibrated distributions $P(E_M)$ can be almost perfectly fit by

$$P(E_M) = Nf(E_M)\exp(-E_M/T_s), \qquad (19)$$

where N is a normalization constant and T_s is a "temperature" parameter. f(E) is the single plaquette phase factor.

V. Hadronic scenario

At higher energies quarks and gluons are expected to be the relevant degrees of freedom. However even at such energies hadrons might play an important role specially if non-perturbative QCD effects are strong. It is not known exactly at which point they stop to be relevant. Therefore it is interesting to use the kinetic theory described in section II for hadrons and verify whether thermal equilibrium can be reached already at the hadronic level. So far no exact (numeric) solution of the transport Eq. (1) for this purpose is available. Recently this problem was treated by Prakash and collaborators[5]. Instead of solving the Eq. (1), they make use of some approximations and calculate the transport coefficients and associated length and/or time scales for a mixture of pions, kaons and nucleons. The equilibrium concentration of these particles is chosen to closely resemble those expected in the mid-rapidity interval of CERN and RHIC experiments. Interactions between these particles are taken from experiment in the form of measured phase shifts. The main novel feature of these calculations with respect to the previous ones is that they go beyond the estimate of collision times and address persistence ratios and transport relaxation times as well.

Starting from the transport Eq. (1) in the dilute limit (neglecting also medium effects), using the relaxation time approximation (the distribution functions are not far from their equilibrium values at a certain temperature) and considering elastic scattering only the relaxation times for the πkN mixture are calculated. The results are shown in Table I. These collisional relaxation times are chiefly a measure of the frequency of collisions and are strong functions of the cross sections. The mean relaxation time of a nucleon in the πkN mixture is comparable to that of a pion in the mixture. This is due to the large πN cross section. The result for kaons in the mixture, on the other hand, is considerably larger. From Table 1 alone one cannot conclude that nucleons are equilibrated to the same extent as the pions. Large cross sections do not necessarily imply equilibration. One must also examine whether energy and momentum are efficiently degraded in collisions. In encounters between heavy and light constituents, the heavy particle continues its path nearly undisturbed, while the light particle bounces off in a direction unrelated to that of its previous motion. This is clearly seen in Table 2 where mean momentum persistence ratios are shown. Table 3 shows the energy relaxation times for binary mixtures. The interpretation of these tables is that in collisions between heavy and light particles, energy transfer is inefficient and consequently inhomogeneities in the energy relax much more slowly for kaons and nucleons than for pions. These considerations together with the hydrodynamical estimate of the lifetime of a typical fireball containing this mixture ($\tau_h \cong$ 10–30 fm) led the authors to the conclusion that (since the transport relaxation times of kaons and nucleons are comparable to τ_h and that of pions is much smaller) pions may be in local equilibrium before the freeze-out of the system but this is not the case for kaons or nucleons.

Table 1. Mean relaxation times (fm/c) in a πKN mixture.

T (MeV)	Pion	Kaon	Nucleon
120	5.6	9.1	3.4
140	2.9	5.6	2.1
160	1.8	3.8	1.3
180	1.2	2.7	0.95

Table 2 Mean momentum persistence ratios.

$(1 \to 2)$	T (MeV) m_1/m_2	120 $\langle\omega_{12}^p\rangle$	180 $\langle\omega_{12}^p\rangle$
$(n \to N)$	0.15	0.27	0.37
$(n \to K)$	0.28	0.32	0.32
$(\pi \to \pi)$	1.00	0.23	0.28
$(K \to n)$	3.6	0.63	0.65
$(N \to \pi)$	6.8	0.83	0.83

Table 3. Energy relaxation times (fm/c).

T	πK		πN	
(MeV)	π	K	π	N
140	8.60	15.78	7.46	21.74
160	6.99	12.0	5.54	15.01
180	6.03	9.78	4.41	11.20

VI. Conclusions

In this survey we have presented recent results concerning the formation of thermal equilibrium in heavy-ion collisions. Basically four research programs have been discussed. Two of them (in sections 11 and V) have reached the stage of making predictions. They state, borned by some approximations used, that thermalization is likely to be achieved for typical situations at RHIC or LHC. The other two approaches (in sections III and IV) are more exact and profound but are still in their infancy. Their preliminary results indicate that non-perturbative QCD effects would rather help in distributing energy among the degrees of freedom of the considered systems.

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References

1) K. Geiger, Phys. Rev. D47, 133 (1993); Phys. Rev. **D46**, 4965 (1992); Phys. Rev. **D46,** 4986 (1992).
2) S. Mrówczyński, Phys. Lett. **B314**, 118 (1993); Phys. Lett. **B214**, 587 (1988); Phys. Rev. **D39**, 1940 (1989).
3) P. K. Kaw, Plasma Physics and Controlled Fusion 34, 1795 (1992); J. Bhatt, P. K. Kaw and J. C. Parikh, Phys. Rev. D39, 646 (1989).
4) B. Muller and A. Trayanov, Phys. Rev. Lett. 68, 3387 (1992); C. Gong, Phys. Lett. **B298**, 257 (1993); T. S. Biro, C. Gong, B. Muller and A. Trayanov, Duke University preprint DUKE-TH-92-40.
5) M. Prakash, M. Prakash, R. Venugopalan and G. Welke, Phys. Rep. 227, 321 (1993); Phys. Rev. Lett. 70, 1228 (1993).