New Aspects of Stochastic Phenomena Related to Nuclear Physics

C. H. Lewenkopf
Department of Physics, FM-15, *University* of Washington
Seattle, WA 98195, *U.S.A.*

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Recent experimental results on systems where the underlying dynamics is dominated by a stochastic Hamiltonian are examined using the Random Matrix Theory (RMT). This theory is shown to be a powerful tool in describing statistical features of quantum systems with few degrees of freedom, whose classical limit is chaotic, as well as for new facets of many-body systems at energies far above the ground state.

I. Introduction

In recent years, in many interesting papers, the expression "quantum chaos" has been used to discriminate a very broad variety of phenomena in different branches of science, such as atomic, molecular, nuclear and condensed matter physics and chemistry. In this contribution we shall present some recent examples of phenomena related to chaos and fluctuations in nuclear physics, or with direct interest for this field.

Since "quantum chaos" is in order, let us summarize the spirit of the present approach trying to avoid as much as possible a discussion based on semantics. As is it extensively discussed in the literature, chaos in Hamiltonian systems is only properly defined in classical mechanics (see for example Ref. [1]). There, its manifestation is essentially related to time evolution of trajectories in phase space. One fascinating aspect of chaotic dynamics is that, even if the equations ruling the evolution of a system are known, under the presente of chaos it is impossible to compute accurately a trajectory for an arbitrary long time. As far *as* quantum mechanics is concern, this is never the case. Once a state is prepared in an autonornous quantum system, its time evolution is trivial, and can be obtained with arbitrary precision. This is one of the reasons why no real consensus about the exact meaning of "quantum chaos" is yet achieved.

One current approach to "quantum chaos" is concerned with the quantum counterpart of classical chaotic systems with few degrees of freedom. Here research has been done developing a semiclassical formalism for these systems, which serve to study new aspects of the correspondence principle. Another aspect of these systems has its origin in 1984, when Bohigas and collaborators[2] postulated a correlation between the statistical fluctuations of the spectrum of a quantum system with a classical chaotic counterpart and those of the Gaussian Orthogonal Ensemble (GOE). This in distinction to the integrable case, where the quantum spectrum is characterized by Poissonian fluctuations[1]. In recent years many numerical studies and also few analytical works[3] came to support this conjecture. In the mean time, however, some simple counter examples[4] showed that Bohigas conjecture has its limitations. The extent and limitations of this universal behaviour are currently under investigation, particularly, extensions to scattering as well as to driven system are still open questions.

Another approach is concerned with systems with many degrees of freedom, where a classical counterpart is not defined and/or not attainable. In many cases, sucli as in nuclei, not even the Hamiltonian is precisely known. Here the GOE was introduced by Wigner many years ago[5] to model the lack of information[6]. This statistical modelling is very successful in explaining different aspects of nuclear reactions[7], molecular and atomic spectra[5] and electronic transport in disordered media[8]. Its connection to "quantum chaos" is basically: (a) these ensembles can be defined by few intrinsic synimetries of the system to be modelled and by minimizing the information[6]; *(b)* the rnodelled systems are toc "complex" (many coupled degrees of freedom, very large density of states, etc.) for a microscopic description[7]; (c) Bohigas conjecture[2].

In what follows both lines will be addressed. In sections 11 and III, RMT is used to obtain qualitative and quantitative understanding of nuclear physics phenomena. In sections IV and V, recent experimental results on systems with few degrees of freedom will be discussed in terms of RMT. A short summary is present in section VI.

II. Damping of multi-phonon giant resonances

In this section a basic review of the philosophy of stochastic modelling of complex systems is presented. In addition, its application for the description of the fragmentation of highly excited collective states is discussed.

A generic system can be described in terms of elementary excitations (quasi-particles). Their energy spectrum and effective interaction determine the observables of the system and its response to external perturbationii. In the mean field picture, the quasi-particles are stationary states with infinite lifetime. This is the niost convenient basis to represent the excited states. In Fermi systems, such as nuclei, the basis states $|k\rangle$ are shell-model-type configurations of excitons which will be called simple states.

Any state $|\alpha\rangle$ is a (normalized, $\sum_k |C_k^\alpha|^2 = 1$) superposition

$$|\alpha\rangle = \sum_k C_k^\alpha |k\rangle \,. \tag{1}$$

The number N_α of simple configurations $|k\rangle$ contributing *significan* ily to the combination (1) is the localization length of the state $|\alpha\rangle$ in the mean field basis. The average weight of the individual simple components is then $\overline{|C_k^\alpha|^2} = N_\alpha^{-1}$. Similar to Refs. [9,10] we use such estimiites for qualitative as well as quantitative conclusions.

The interaction between the quasi-particles is crucial in many aspects. Its coherent part singles out special superpositions (1) with enhanced response to specific operators Q which are "simple" in the sense of the exciton structure: they act within the given exciton class or between the neighboring classes. The random phase approximation (RPA) is an example of a microscopic theory which allows one to find such superpositions. Coherence of the state $|\alpha\rangle$ with respect to the operator Q means the following: let q_k be the matrix element of Q for a simple excitation $|k\rangle$, then the amplitude of the excitation $|\alpha\rangle$ is $\sum_k q_k C_k^\alpha$; this sum is coherent if q_k and C_k^α are correlated so that the sum can be estimated as $N_\alpha \bar{q} N_\alpha^{-1/2} = N_\alpha^{1/2}\bar{q}$, *i.e.* the transition is enhanced by factor $\sqrt{N_\alpha}$ in amplitude and N_α in probability as compared to typical values for simple states. For $N_\alpha \gg 1$, such a state $|\alpha\rangle$ corresponds to collective synchronous motion of N_α elementary excitations and it will be referred to as a Giant Resonance (GR) although this definition covers low-lying shape vibrations as well. Contrary to that, for a generic state (1) or an arbitrary operator Q the sum consists of a large number of uncorrelated terms so that the resulting amplitude as a rule will be less than in the coherent case by factor $\sqrt{N_\alpha}$.

There are numerous processes of incoherent collision-like interactions. Their role grows immensely with the increasing excitation energy. As a result, simple exciton states cease to be stationary; they have a finite lifetime and the corresponding energy uncertainty $\Gamma^\downarrow$. This process being usually referred to as damping[11] results in formation of actual stationary wave functions which are extremely "complicated". The number N of significant components is huge (about 10^6 already at the nucleon separation energy in heavy nuclei) and the statistical description is unavoidable. Using the time evolution language, one can start with a pure configuration that will be mixed after a short time with "doorway states" of the nearest exciton classes which evolve in the same direction. Although in the case of many degrees of freedom the hierarchy of exceedingly complex excitations is in many aspects similar to a heat bath and the damping process is practically irreversible, the time dependence of the initial excitation does not need to be exponential. The energy profile (lineshape), correspondingly, is not necessarily of the Breit-Wigner or Lorentzian type.

With increasing excitation energy, the decay to the continuum becomes more and more important. The dynamics is driven by the competition between intrinsic mixing and emission of particles. The role of the escape contribution $\Gamma^{\uparrow}$ to the total width grows and new phenomena connected with the coupling of intrinsic states via comrnon decay channels[12] emerge.

Below we consider a collective mode of the RPA type of relatively high excitation energy as multipole GR. The excitation energy is assumed to be low enough to neglect the escape width. This approach is reasonable up to temperatures of order 2 MeV in heavy nuclei. The collective state is embedded into a background of complicated states generated from the original simple (non-collective) configurations in the mixing process. The question arises: Is it possible to draw any physical conclusions'from the very idea of completely mixed states?

Eet us consider a collective excitation at some energy, which can be taken as an origin of the energy scale, interacting with the background of the complicated states $|\nu\rangle$ with energies ϵ_ν. In the stochastic limit the latter are completely mixed so that each of them contains on average the same portion of the doorway states appropriate for the damping. The local fluctuations and correlations of the background states are presumably of the GOE type but their exact properties are not significant: we need only their average spacing D and the spreading width $2a$ or the effective number $N = 2a/D$ of contributing states.

The background states are coupled to the collective one by the matrix elements V_ν scaled in the stochastic limit as $V \sim v/\sqrt{N}$ where v stands for the average matrix element of the residual interaction between the collective and doorway states. Generally, the matrix elements v and a are of the same order of magnitude unless there is no specific selection rules.

The problem of a "bright" level $|0\rangle$ interacting with the background $\{|\nu\rangle\}$ can be easily solved[13]. The exact solutions $|f\rangle$ are

$$|f\rangle = C_0^f|0\rangle + \sum_\nu C_\nu^f|\nu\rangle. \tag{2}$$

Their energies E_f are the roots of the secular equation

$$F(E) \equiv E - \sum_\nu \frac{V_\nu^2}{E-\epsilon_\nu} = 0\,. \tag{3}$$

Making the following assumptions: (i) background energies form an equidistant infinite sequence (picket fence model) with the step D; (ii) all squared matrix elements, V_ν^2, are of the same order of magnitude and can be substituted by their common value $V^2 \equiv \overline{V_\nu^2}$; and (iii) $V^2 \gg D^2$, one obtains the "standard model" result[13] where the strength function takes the Breit-Wigner shape,

$$S(E) \equiv \sum_f |C_0^f|^2\,\delta(E-E_f) \approx \frac{1}{2\pi}\frac{\Gamma}{E^2+\Gamma^2/4}, \tag{4}$$

with the centroid at the unperturbed position of the resonance and the "standard" width

$$\Gamma = \Gamma_s \equiv 2\pi\frac{V^2}{D} \tag{5}$$

Kowever, the standard model does not take into account the intrinsic scale a associated with the equilibration of the background states. This model corresponds to the limit of $a \to \infty$ which implies the relation $v \ll a$ between the two types of matrix elements.

Assuming the standard model and N-scaling of matrix elements, we can present the spreading width (5) as[14]

$$\Gamma = 2\pi\frac{v^2}{ND} = \pi\frac{v^2}{a}. \tag{6}$$

As was discussed in[14], it means that in the stochastic limit characterized by the N-scaling of generic matrix elements the spreading width of the collective resonance is expressed in terms of the residual interaction and does not depend on the underlying level density and, therefore, on temperature. The same procedure for the resonance built, in the spirit of the Brink-Axel hypothesis, on an excited state, is equivalent, in the stochastic limit, to the shift of the resonance curve by the average energy of the intrinsic excitation with no increase of the damping width. The localization length N grows exponentially but the damping width is saturated because of the fall of the coupling matrix elements.

Recent experiments[15] indicate as well saturation of the spreading width of the giant dipole resonance (GDR) in hot nuclei. It confirms that at temperature about 2 MeV we have to deal with stochastic dynamics in nuclei. (This regime is reached presumably even at lower temperatures in heavy nuclei as follows from the level statistics[16].)

The standard model of the strength function becomes invalid[17] as far as the spreading width (5) for the collective mode becomes comparable to the intrinsic spreading width of complicated states. Then the response pattern is determined by the relationship between the corresponding parts v and *a* of the residual interaction.

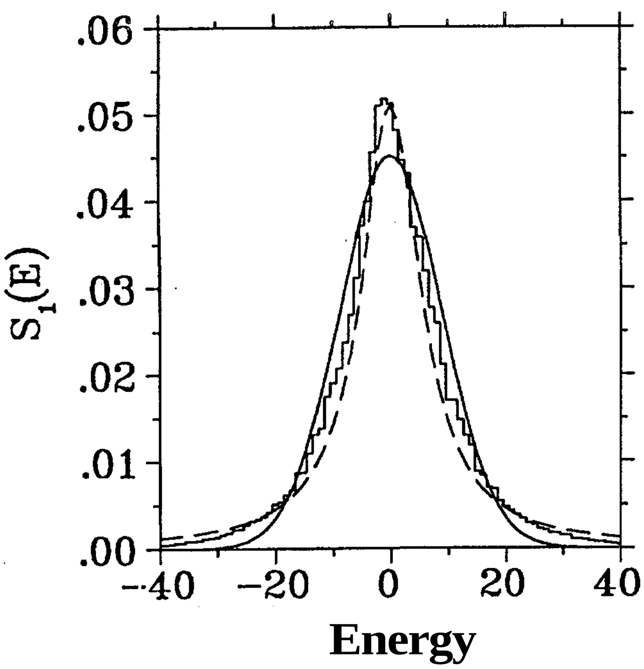

Figure 1: Ensemble average of the strength function for the single phsnon, modelled by a GOE background with V_ν given by **Eq.**(7), see text for parameters.

In order to simulate a more realistic pattern, we introduce a natural smooth cut-off of the coupling matrix elements associated with the intrinsic spreading width. This was done by the ensemble of the GOE spectra representing the intrinsic levels E, and by the random coupling matrix elements V, The latter were supposed to be Gausrian distributed with the variance decreasing with the increasing distance from the collective resonance,

$$\langle V_\nu^2\rangle^{1/2} = V_0 \exp(-\epsilon_\nu^2/2\Delta^2). \qquad (7)$$

The width A of the distribution (7) can be related to our scale parameter as a $= \sqrt{2\log 2}\Delta = 1.18\Delta$.

The strength distribution obtained from the GOE ensemble of 1000 matrices N = 200 with $V_0 = \sqrt{2}$ is shown on Fig.1 for A = 25. For comparison, the dashed line shows the Breit-Wigner curve for the constant matrix elements equal to %, i.e. for v = $V_0\sqrt{N}$ % 14. At these values of the parameters the "empirical" distribution is intermediate between the Breit-Wigner function of the standard model and the Gaussian one given by the dashed line.

Whence, the standard model is applicable in the case of relatively weak coupling of the collective mode to the incoherent background. In the strong coupling case the shape of the strength function gets more compact. The scale for estimating the coupling intensity is determined by the spreading width of the intrinsic states.

Recent measurements of the excitation of double GDR[18,19] indicate that the width Γ_2 of these resonances is typically about 1.5 of the width Γ_1 of the single GDR. As mentioned above, this width is associated with the damping of the collective mode the contribution of the escape width still being rather small.

The problem of widths of multiphonon GR was discussed from the viewpoint of stochastic dynamics in[17]. Since the dominating excitation mechanism is associated with the sequential excitation of the single GR, and the anharmonic effects are weak, the resulting strength function $S_2(E)$ of the double GR can be found by the convolution of the corresponding functions S_1(E) for single resonances[20],

$$S_2(E) = \int dE_1 \int dE_2\, \delta(E - E_1 - E_2)\, S_1(E_1)\, S_1(E_2)\,. \qquad (8)$$

As well known, the convolution of Breit-Wigner (or Lorentzian) distributions S_1 with centroids e and e' and widths Γ and Γ' leads to the similar distribution S_2 with the centroid e + e' and the width $\Gamma + \Gamma'$. It would give the ratio r, $= \Gamma_n/\Gamma_1$ of the widths of the n- phonon and single resonances equal to the number of phonons n. **At** the same time, folding (8) of the Gaussian distributions gives r, $= \sqrt{n}$. The latter result is of general nature being valid for any distribution $S_1(E)$ with the finite second moment $(\Delta E)^2$: we expect that the widths are scaled in the same way as those variances and therefore add in quadratures which results in $r_2 = \sqrt{2}$. Thus, the wings of the strength functions due to the coupling of the collective state to the remote background states are crucial for the lineshape and the observable ratio r,.

From the microscopic point of view, the double GR decays predominantly to the states "one phonon built on the intrinsic state $|\nu\rangle$". The point is that the background states $|\nu\rangle$ here are the same as for the decay of the single phonon (Brink-Axel hypothesis). Hence, using the standard model (5), the level density is the same in both cases. The only difference is the Bose factor which makes squared matrix elements $\langle V_\nu \rangle^2$ twice as large in the case of the decay of the double phonon state. As a result, the standard model predicts r, = n. It is confirmed by the detailed calculations using specific models like the doorway model[18].

The common shortcoming of all such arguments goes with the fact that the intrinsic energy scale a associated with the spreading width of the background states is not taken into account properly. As we discussed, the standard model, with the uniform coupling of the collective mode to all complicated states, implies the limit a $\rightarrow \infty$, or the inequality v $\ll$ a between the two parts of the residual interaction which leads also, according to Eq.(5), to $\Gamma_s/a \sim (v/a)^2 \ll 1$. The Breit-Wigner shape and the ratio r, = n correspond to the irreversible (exponential in time) independent decay of n phonons to this continuum.

We have seen already that in the opposite limit v $\gg$ a the strength covers the region increasing linearly with v and therefore growing as $\sqrt{n}$ with the phonon number. It turns out that the deviations from the standard model and the transition from the Breit-Wigner shape with r, = n to the Gaussian shape with r, $= \sqrt{n}$ occur already in the region v $\simeq$ a which seems to be appropriate for the GDR. One can say that, due to the finite intrinsic fragmentation width, phonons do not decay independently: they are coupled via common decay "channels".

To demonstrate the transition between two dynamical regimes leading to different predictions regarding the damping widths, we come back to the solution of the "bright level problem" for the background with the GOE spectrum and the coupling to the bright level with the natural cutoff (7). Even for the intermediate case of Fig.1, the folding (8) developes the properties of the distribution very distinctly so that the resulting distribution for the double phonon resonance is already close to the Gaussian (Fig.2).

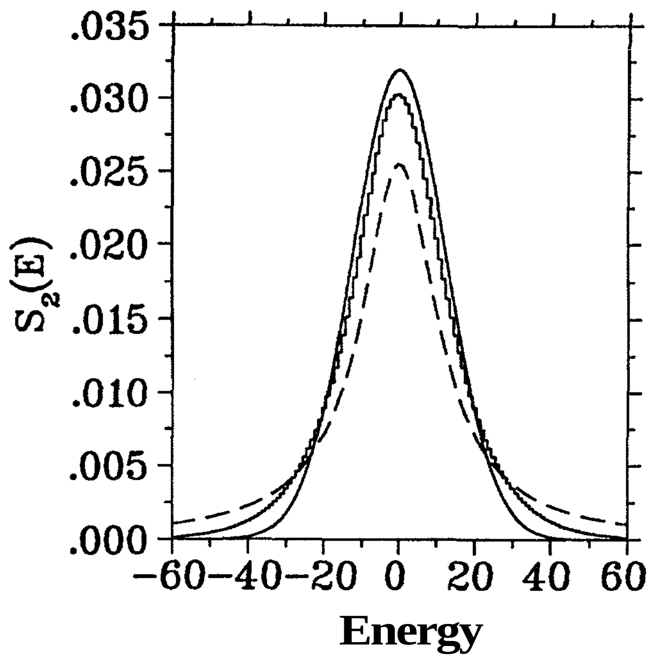

Figure 2: Folded double phonon strength function from the ensemble of Fig.1. The solid line is the Gaussian fit; the dashed line is the Breit Wigner curve, and the histogram represents the numerical simulation.

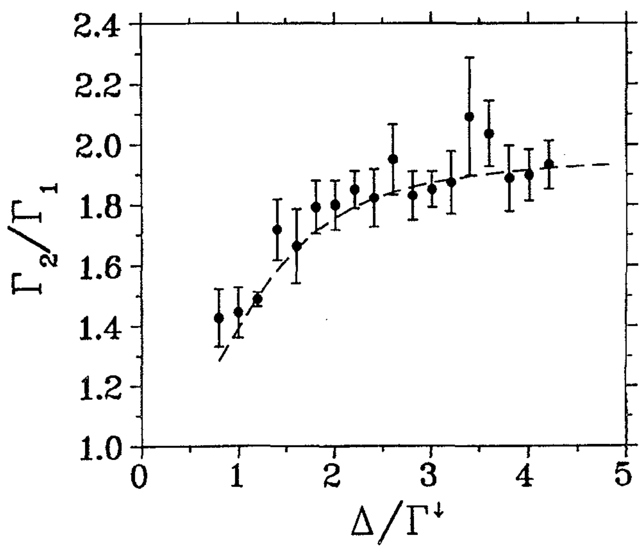

Figure 3: Double phonon - single phonon width ratio, as a function of A, points stand for numerical simulation for a GOE background (ensembles of 500 matrices), and dashed line is the exact solution for a picket fence.

The ratio r_2 of the widths extracted from such simulations is shown on Fig.3 as function of $\Delta/\Gamma_s = 0.27a^2/v^2$. The transition frorn $r_2 = \sqrt{2}$ at $\Delta \simeq \Gamma_s$ (or v $\approx a/2$) to $r_2 \approx 2$ (or v $\approx a/4$) is seen clearly. Such ratios presuinably correspond to a realistic physical situation which makes it possible to get an information on mixing interactions from the damping widths of multiphonon GR Reliable data for r, at n > 2 as well as systematics for r_2 for various mass numbers would help to make quantitative conclusions.

We believe that the discussed scenario is common for quantum many-body systems. Near the ground state there is an energy range where Landau's picture of weakly interacting elementary excitations is valid so that the response of the system to an external perturbation can be expressed in terms of the long-lived quasiparticles. Beyond this range, the fragmentation of simple modes becomes important and their damping width increases reaching saturation in the region of stochastic dynamics. In the same region one should expect to encounter the generic signatures of chaos in the local level statistics as well as in the statistical properties of wave functions and matrix elements. The saturation will persist uiitil the escape processes became dominant preventing the system from reaching the complete equilibrium.

III. Parity violation in compound nucleus reactions

Compound Nucleus (CN) reactions are believed to be a useful tool to study some fundamental processes in nature. In particular, one of the most exciting recent experimental proposals (TRIPLE collaboration) is to study time-reversal symmetry breaking investigating the limits of validity of detailed balance. Such an experiment seems to be competitive with the other present set-ups to parameterize the Kobayashi-Mazkawa model[21].

The very hasic pedestrian motivation for this idea relies on the Fermi golden rule: w $\approx 2\pi v^2\rho$, where even if one has a very small matrix element v (for a strongly suppressed process), a very large level density ρ can eventually enhance the cross-section sufficiently to make a hindered effect sizeable. The CN is definitely a good candidate for that.

The first step in this project is to understand how parity violation, where fewer uncertainties are involved, is enhanced in CN reactions. This step has a long history and started with the observation of large parity-violation effects in the transmission of epithermal polarized neutrons by heavy nuclei[22] that has led to systematic experimental investigations on several target nuclei. Most unexpectedly, the data on ^{232}Th[23] have not shown randomness in sign. The observed asymmetry does not correspond to predictions of the statistical model.

This sign correlation effect stays as a puzzle in the literature. Its understanding is important for the application of random matrix modelling, since a precise knowledge of the smooth behaviour of a system is necessary for the understanding of its fluctuations. And also: if one has problems understanding parity violation, how can one hope to learn about time-reversal symmetry breaking using CN reactions? It is nowadays believed that the origin of the sign correlation is related to the presence of a direct reaction. However, although several explanations have been offered for this phenomenon[24,25,26,27] none of them is able to conciliate the experimental data, a "natural" direct reaction mechanism and reasonable matrix element values. In what follows the observables are shortly reviewed, and a direct reaction mechanism[26], following strictly from formal scattering theory without further external elements, is discussed.

It is customary to present the data in terms of the quantity

$$P(E) = \frac{\sigma_p^+(E) - \sigma_p^-(E)}{\sigma_p^+(E) + \sigma_p^-(E)}. \tag{9}$$

Here, $\sigma_p^{\pm}(E)$ is the total p-wave cross-section for neutrons with helicity $\pm$ transmitted by an unpolarized target at energy E. This quantity indicates parity violation: We have $P(E) = 0$ if parity is conserved. Sizeable values of P are found only at neutron energies E $= E_\mu; \mu = 1, \cdots, N_p$ corresponding to the N_p p-wave neutron resonances identified in a given experiment while $|P| \leq 10^{-5}$ outside these resonances. So far, it is not known experimentally whether a given p-wave resonance has spin 1/2 or spin 3/2. Only spin 1/2 resonances can mix via parity violation with $s_{1/2}$ resonances; only here sizeable values for $P(E_\mu)$ are expected. (The $p_{3/2}$ resonances can mix with $d_{3/2}$ resonances. However, penetration effects suppress this reaction so strongly that no observable parity violation is expected there).

Using first-order perturbation theory for V^{PV}, the parity-violating weak nucleon-nucleon interaction Sushkov and Flambaum[28] first predicted that parity violation could bring P to a few percent level. The prediction of Ref. [28] was experimentally confirmed by Alfimenkov et al. on a ^{139}La target[22]. This experimental work generated several more quantitative theoretical works[29] than the qualitative prediction of Ref. [28]. The standard expression[29] for $P(E_\mu)$, obtained by mixing the $p_{1/2}$-resonance $|\mu\rangle$ at E = E, with the neighbouring $s_{1/2}$-resonances $|\nu\rangle$ located at energies $E_\nu, \nu = 1, \cdots, N_s$ is

$$P(E_\mu) = \frac{2}{\gamma_\mu^p}\left\{\sum_{\nu=1}^{N_s}\frac{\langle\mu|V^{PV}|\nu\rangle}{E_\mu - E_\nu}\gamma_\nu^s\right\}, \tag{10}$$

where γ_μ^p (γ_ν^s) are the partial-width amplitudes for neutron decay of the $p_{1/2}$ (the $s_{1/2}$) resonances, respectively. According to the statistical model, the quantities $\gamma_\mu^p, \gamma_\nu^s$ and $\langle\mu|V^{PV}|$ v) are expected to be uncorrelated Gaussian distributed random variables with zero mean value. Therefore, it was expected that the average polarization asymmetry,

$$\overline{P} = \frac{1}{N_p}\sum_{\mu=1}^{N_p} P(E_\mu), \tag{11}$$

would vanish for $N_p \to \infty$. But the $N_p = 23$ pwave resonances in ^{232}Th contradict this expectation[23]. Therefore, the analysis of these data was redone[24], with the following form for $P(E_\mu)$:

$$P(E_\mu) = \frac{2}{\gamma_\mu^p}\left\{\sum_{\nu=1}^{N_s}\frac{\langle\mu|V^{PV}|\nu\rangle}{E_\mu - E_\nu}\gamma_\nu^s\right\} + B\sqrt{\frac{1\text{eV}}{E_\mu}} \ . \tag{12}$$

The first term on the *r.h.s.* takes account of close-lying s-wave resonances; it is expected that this term does satisfy the predictions of the statistical model. The constant B in Eq.(12) accounts for the observed deviation from the statistical model. The square-root factor accounts for p-wave penetration effects; it has the same dependence on E, as the ratio $\gamma_\nu^s/\gamma_\mu^p$. In the framework of Eq.(12), we have obviously $\overline{P} = (B/N_p)\sum_{\mu=1}^{N_p}\sqrt{1\text{eV}/E_\mu}$. (It is customary in neutron physics to refer to the energy 1 eV, although the *p*-wave resonances of interest here typically rather have energies in the 10 eV region). A maximum-likelihood method was used to determine B, the *r.m.s.* value M of $\langle\mu|V^{PV}|$ v), and q from the data. Here, q is the fraction of *p*-wave resonances carrying spin 1/2. On statistical grounds, one expects q = 1/3. Using random *matrix modelling* Bowman and collaborators obtained[24] q = 0.44, M = 1.2 $\left(^{+0.5}_{-0.4}\right)$ meV and $B = 8\left(^{+6.2}_{-6.0}\right)$%.

A careful theoretical examination of all intrinsic on resonance reaction mechanisms using Feshbach projection formalism leads to a non-negligible term $P^{dir}(E_\mu)$ to be added to Eq.(10). $P^{dir}(E_\mu)$ can be written in a compact way by introducing $\gamma_\mu^{s\,PV}$ as

$$P^{dir}(E_\mu) = \frac{2}{\gamma_\mu^p}\mathcal{P}\int dE'\,\langle\mu|H|\chi^p(E')\rangle\,\frac{1}{E^+ - E'}\,\langle\chi^p(E')|V^{PV}|\chi^s(E)\rangle\ , \tag{13}$$

where E = E, and H is the many-body Hamiltonian. The integral in Eq.(13) is completely dominated by contributions arising from E' $\gg$ E. This is because both matrix elements in Eq.(13) carry a pwave penetration factor which is tiny at E' = E, and approaches unity at about 1 MeV. This justifies the approximation $E_\mu \cong 0$ which yields

$$P^{dir}(E_\mu) \approx -\frac{2}{\gamma_\mu^p}\int_0^\infty \frac{dE'}{E'}\,\langle\mu|H|\chi^p(E')\rangle\,\langle\chi^p(E')|V^{PV}|\chi^s(E)\rangle \ . \tag{14}$$

Eqs.(13) and (14) demonstrate the existence of an enhancement: While the on-shell contribution (defined by replacing the principal-value integral in Eq.(13) by the imaginary part of the same integral) to P^{dir} would be tiny becense of penetration effects, the contribution arising from E' $\gg$ E in Eq.(14) is sizeable.

To evaluate the integral in Eq.(14) approximately, one can assume that H is surface-peaked, which yields to good results in nuclear spectroscopy. After a few straightforward manipulations one obtains

$$P^{dir}(E_\mu) = -2\left\langle\chi^p(E_\mu)\left|V^{PV}\right|\chi^s(E_\mu)\right\rangle \int_0^\infty \frac{dE'}{E'}\frac{\varphi_p^2(k'R)}{\varphi_p^2(kR)} \quad . \tag{15}$$

It is gratifying to see that $P^{dir}(E_\mu)$ does not depend on γ_μ^p and therefore it is independent of μ (save for a smooth dependence on E,) showing that P^{dir} contributes only to the constant B in Eq.(12), and not to the stochastic piece.

To estimate the integral in Eq.(15) $\varphi_p(kR)$ can be taken as the solution of the one-body scattering problem (this approximation is possible because $\chi^c(E)$ is the antisymmetrized product of the target nucleus wavefunction times a neutron wave function in the channel c) in a Woods-Saxon potential (without absorption), parameterized in the usual way[27]. The matrix element $\left\langle\chi^p(E)\left|V^{PV}\right|\chi^s(E)\right\rangle$ can be evaluated in the one-body scenario[31], using the parameterization

$$V^{PV}_{\text{one-body}} = \frac{1}{2}\epsilon\left\{f(r), \vec{\sigma}\cdot\vec{p}c\right\} \quad , \tag{16}$$

where $f(r) = (1+e^{(r-R)/a})^{-1}$, $\vec{\sigma} = 2\vec{s}$, $\vec{p} = -i\hbar\vec{\nabla}$ is the momentum operator and the weak coupling constant ϵ should be of the order of 10^{-7}.

Converting these results into the parameterization of Eq.(12), we obtained B $= 3.0\cdots1.0\%$, for $\epsilon = 4\times10^{-7}$. The range reflects various choices of the depth of the Woods Saxon potential and reveals the presence of a "sharp" single-particle shaper p-wave resonance near the threshold[26]. Unfortunately these results are not simultaneously consistent with the experiment, proposed reaction mechanisms and theoretical estimates foi ϵ.

As it was stated before, this fact raises many doubts over our present knowledge of stochastic phenomena in nuclear systems. It is important to stress again that although the presented discussion does not directly address RMT, it is crucial to understand exactly the average energy behavior of a system in order to extract and interpret its fluctuations.

The present avaliable experimental results support the conjecture that smooth and fluctuating processes are equally important for P. If for other heavy nuclei this large sign correlation is confirmed the theory needs badly to be improved. Understanding will be only achieved, once the extraction of the typical two-body matrix element from the fluctuating part of P and the one-body term from the smooth one turn to be consistent with each other[30] and with estimates coming from a microscopic theory[31].

IV. Cavity experiments

In this section, let us focus on applications of RMT for systems with few degrees of freedom, illustrating it by recent experimental results obtained using a superconducting microwave cavity[32]. Two-dimensional billiards are among the most thoroughly studied models for classical chaos and its quantum manifestations[33]. The transition from integrable to non-integrable classical behaviour can be studied by changing the shape of the billiard. On the quantum level, this transition is accompanied by a crossover from Poisson statistics to GOE statistics in the semiclassical region. These systems can be studied experimentally: In sufficiently flat microwave resonators, Maxwell's equations reduce to the Schrödinger equation for the free particle, and the condition of classical chaos is realized by properly shaping the cavity[34].

The here presented results were obtained with a superconducting niobium cavity with Q $\approx 10^5 - 10^7$, which has the shape of a quarter of a Bunimovich stadium billiard with inner dimensions: circle radius r $=$ 20 cm, straight segment a $=$ 36 cm and height d $=$ 0.8 cm, corresponding to $\gamma = a/r = 1.8$. The geometry of a quarter of a stadium restricts the quantum problem to a single symmetry class[35]. The cavity has been put into one of the cryostats of the new superconducting Darmstadt electron linear accelerator

S-DALINAC, where it was cooled down to 2K. Three antennas were located in small holes (3 mm diameter) and to keep their influence on the field distributions negligibly small care was taken that they did not penetrate into the cavity. Three independent transmission spectra were taken by using a vector network analyzer and different combinations of the antennas. All spectra were checked against each other for consistency and finally combined into a single spectrum. It consists of $1.8 \cdot 10^6$ data points in the range from 0.75 to 18.75 GHz. The step width of the measurement was 10 kHz. The large signal-to-noise ratio (> 100) allows us to identify each resonance by taking many data points in the tails. This is why one is almost certain not to have missed any modes. Further support for this statement derives from the fact that the smallest observed spacing is 300 kHz and by semiclassical analysis. By cooling the cavity from 300 K to 2 K a striking resolution improvement of about three orders of magnitude was achieved. To ensure the two-dimensionality of the cavity the analysis of the spectra has been confined to $f < 17.5$ GHz[34]. Up to this frequency 1060 eigenmodes were counted (compared to 898 modes at room temperature). It is worth to recall that many years were necessary to accumulate such large set of resonances in compound nucleus reactions.

A naive statistical analysis using the Weyl average level density semiclassical formula to extract (see [36]) the fluctuating part of the spectrum leads to strong deviations from the GOE prediction This interesting result points to the fact that we are dealing with a dynamical system and have to understand its special characteristics. The universal signatures of chaos ought to emerge only when all its "pathological" features have been taken into account. In the case of the Bunimovich billiard the phase space is fully chaotic except for a set (of measure zero) of non-isolated periodic orbits that bounce between the two straight boundaries[35]. These trajectories, called bouncing ball orbits, are only marginally unstable. In the spirit of the Gutzwiller formula, taking their contribution for the cumulative level density into account, one obtains:

$$N^{\mathrm{bb}}(k) = \frac{1}{(2\pi)^{3/2}} \frac{a}{r} (2kr)^{1/2} \sum_{m=1}^{\infty} m^{-3/2} \cos(2mkr - \frac{3\pi}{4}) \tag{17}$$

The inclusion of $N^{\mathrm{bb}}(k)$ in the unfolding scheme[36] changes the results quantitatively. For the unfolded spectrum the correlation between neighbouring level spacings is $C = -0.298 \pm 0.030$ in agreement with the GOE predicting $C = -0.271$ [36]. For the Nearest Neighbour spacing Distribution (NND) the effect of $N^{\mathrm{bb}}(k)$ is not very large (Fig.4 depicts NND without the inclusion of $N^{\mathrm{bb}}(k)$ in the unfolding). For the $\Delta_3(L)$ statistics the difference is striking. The presente of marginally stable periodic orbits dramatically changes the rigidity of the spectrum for large values of L (measured in terms of the mean level spacing). Proper handling of these orbits, brings the spectrum back to the expected GOE-like behaviour of classically chaotic systems. Moreover, the $\Delta_3(L)$ statistics very closely follows the GOE prediction up to $L = 20$, where it saturates, as predicted by Berry[3].

So far only the sharp resonance spectrum of the cavity was discussed. In the present case one is allowed to associate the resonance energies with the eigenmodes of the cavity since the mean resonance spacing d is much smaller than the average resonance width Γ. The experimental data provides one, however, with more information. Microwave experiments measure the full S-matrix (matrix elements with phases[37]). These experiments, provided the cavities are properly shaped, are in close analogy to CN reactions. Thus, new experimental information can in principle help one to illustrate and understand in a simple system some still open questions of statistical scattering theory. Quantum manifestations of chaotic scattering are also a very interesting subject per se, where little semiclassical work is done.

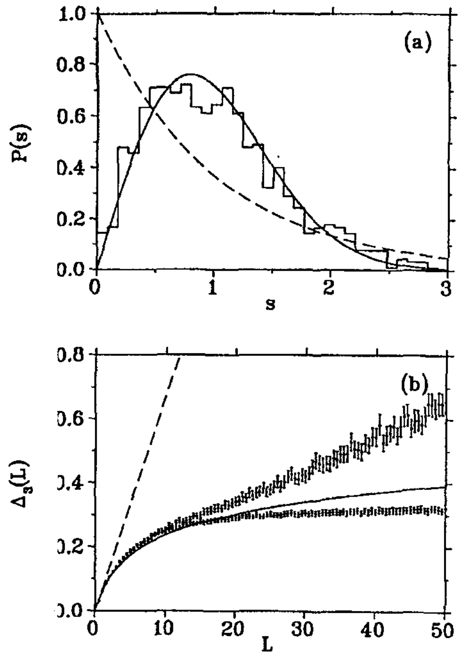

Figure 4: Nearest Neighbour spacing Distributions (upper part). The histogram corresponds to the data and the dashed lines to the theoretical predictions. Lower part: $\Delta_3(L)$-statistic of the experimental data set compared with theoretical predictions (dashed lines). The open circles (squares) derive from the unfolded spectrum with (without) the bouncing ball orbit contribution.

V. Quantum dots

In the past few years technological developments in the field of microstructures allow one to produce devices in the nanometer range. Early conductance g measurements[38] on micrometer long metallic devices drove attention to many novel questions related to quantum mechanical phase coherence over distances encompassing thousands of atoms and electronic transport theory. Surprising new physics came from such experiments: although the measured average $\langle g \rangle$ is a function of the length of the probe L, the fluctuations $\langle g^2 \rangle^{1/2}$ for different L's are universal and of order unity in units of e^2/h, provided that the electronic mean free path l is smaller than the localization length (ξ) and smaller than the electron-phonon mean free path (l_ϕ), in other words, low temperatures are required. This interesting new phenomenon, called Universal Conductance Fluctuations (UCF), motivated intensive theoretical efforts[8] and it is understood in terms of random matrix modelling, nowadays the basic tool to study electronic transport in the diffusive regime in disordered systems. Such results have a direct technological impact: at a time where so much effort is put in miniaturizing systems, one faces a limitation at an unexpected long scale, where even at zero temperature sizeable intrinsic fluctuations due to disorder are unavoidable.

Further new experiments[39] explored the possibility of producing very small probes, where L < l and thus study conductance fluctuations in the ballistic regime – no disorder. Such $GaAs/Al_xGa_{1-x}As$ quantum dots can be fabricated in any specific shape. Since the geometry of the experiment is naturally quasi 2-D, the correspondence to classical mechanics can be easily done and chaotic as well as integrable dynamics are achieved by properly shaping the dots. In this manner one is able to study the influence of the underlying classical dynamics on electronic transport phenomena.

The relevance of this measurement for nuclear physics is given by the fact that the conductance can be expressed by the Landauer formula. For a two leads device this formula reads

$$g(E_F) = 2\sum_{a,b} |S_{ab}^{LR}(E_F)|^2 \tag{18}$$

where $S_{ab}^{LR}(E_F)$ is the S-matrix element connecting the channel a (defined by the geometry of the lead) in the "entrance" lead (defined arbitrarily by the current) L to the channel b in the "exit" lead R. E_F is the Fermi energy and defines the number of allowed transverse modes in the leads, or open channels **A**. In the absence of direct processes the conductance is determined by the fluctuating part of the S-matrix. In analogy with the Bohigas conjecture, chaotic scattering is then properly modelled by assuming that the quasi-bound states, corresponding to the dynamics *in* the dot, are GOE-like[40]. Due to the nature of the modelling, here there is also a strong relation between these data and CN reactions cross section fluctuations. An important difference is that, based on the ergodicity hypothesis, the statistical treatment of the compound nucleus replaces energy averages by ensembles averages. The most difficult part of the referred experiments is to produce the probes, less difficult but also involving is to vary the gate voltages in the leads and in this manner vary E_F. So far the experiments study g as a function of the magnetic field B. The ergodicity hypothesis is based on the classical picture that for different B-fields electrons coming from the same point in phase space in the entrance lead will follow different paths in the dot and the chaotic dynamics guarantees different realizations of a ensemble, it is presently however unclear how to construct S-matrix ensembles as a function of B.

From the considerations above the statistical approach predicts that $\overline{g}$ can be expressed by a Hauser-Feshbach-like formula

$$\overline{g} = 2(\sum_a T_a^L)\, \Pi^{LR} (\sum_b T_b^R) \tag{19}$$

where T_c^L (T_c^R) is the transmission coefficient corresponding to the channel c in the lead L (R) and $(\Pi^{LR})^{-1} = \sum_c (T_c^R + T_c^L)$. As it is known from CN reactions, this formula is an approximation and correct only in first order in powers of Π^{LR}. This implies that Eq.(19) works best in cases where many channels are open.

The interesting quantity here is $\overline{\delta g^2}$ defined as $\overline{g^2} - \overline{g}^2$. Since Eq.(18) contains a summation over all channels L and R, a calculation of $\overline{\delta g^2}$ demands more than the knowledge of the leading term of S_{ab}^{LR} as above[41]. Despite of the technical difficulties this problem was solved in the context of UCF, by considering higher terms[8]. There, Π^{LR} is a good expansion parameter since in a typical experimental geometry one has of the order of 10^3 open channels. For quantum dots this is no longer the case. The present experiments deal with one up few open channels per lead. A quantitative understanding of the conductance fluctuations for the stochastic theory can then be acquired by numerical simulation. On the other hand, ballistic electrons moving in a given geometry are suitable for an exact treatment, since one can solve an one-body scattering problem for a given potential[42]. Although this calculation gives very detailed information about the process, it is very important to extract from it universal and non-universal features. Moreover, one should not forget that the one-body scenario as well as the modelled boundary conditions are approximations to the real physical problem. One of the aims of this investigation is to learn some general features of quantum dots, even when they do not have an ideal shape, there is where the random matrix modelling can help.

The results presented in this section are originated from a numerical simulation on a fully stochastic scatterer: the resonance part of the S-matrix is constructed as in[40], each point corresponds to an average over 10 matrices each containing 200 poles. Here the only input parameter is the number of open channels A (each channel c with $T_c = 1$, a feasible experimental situation). The motivation is to have a hint how one evolves from the few-channel case to the regime were one can use results from UCF. In this regime (A $\gg$ 1), the Hauser-Feshbach formula (19) predicts $\overline{g}$ = A/2 and $\overline{g^2} = 1/2$. Indeed, this is the observed behavior in Fig.5. It is, however, surprising how fast the semiclassical limit sets in, confirming results from detailed calculations[43].

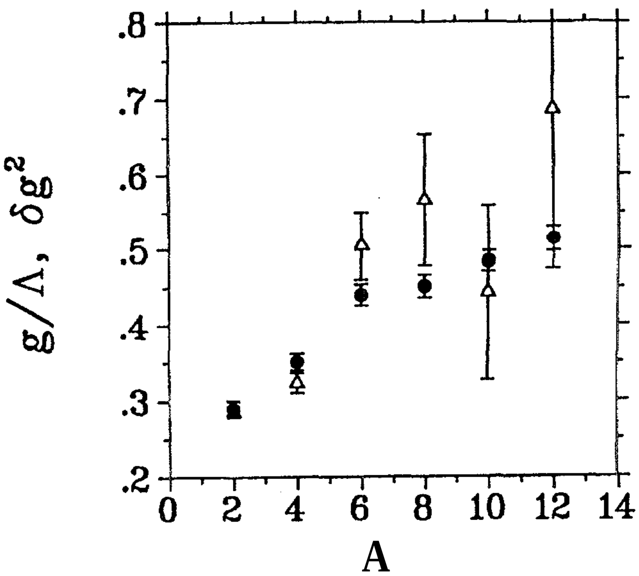

Figure 5: Averaged conductance ($\overline{g}$) A in units of e^2/h scaled by the number of open channels • and its variance ($\overline{\delta g^2}$) as a function of A. For A $\gg 1$ asymptotic expansions predict $1/2$ for both quantities.

These results correspond to a variation in the Fermi energy and fixed magnetic field. Further theoretical investigation is necessary on averaging over the B field and fixed E_F. Here one would like to understand if the conductance-conductance fluctuations $\langle g(B)g(B+\delta B)\rangle$ can be related to Ericson fluctuations and how should the ergotic hypothesis for the ensemble defined for $S(E_F)$ be modified for $S(B)$. Another important feature of the quantum dot experiments is "dephasing". In condensed matter this effect is normally attributed to electron-phonon or electron-electron scattering. In considering the second, the analogy to precompound nucleus reactions is an appealing subject to be explored. These are questions we are trying to answer presently.

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VI. Outlook

In the four given examples it was showed that many aspects of "complexity" in many-body and one-body systems have to be modelled in terms of Random Matrix Theory, since very little can be learned from microscopic descriptions – in the case where they can be performed. Gaussian ensembles are very powerful tools for this purpose. However, for many specific situations refinements are necessary. In the case of multiple giant resonances one has to introduce a localization length, which would come naturally if one had taken banded matrices[44] and seems also to be a better modelling of the nuclear case. For parity non-conservation in CN reactions a better treatment of the smooth background is in order. If the magnitude of the sign correlation is experimentally confirmed[45], it crucial to understand the interplay between one-body and two-body matrix elements. Global hard chaos (pure hyperbolic dynamics) is as rare in physical systems as full integrability. Therefore situations where one faces chaos in the classical limit are in general globally non-universal. Here, one of the interests is to device methods to extract the universal behavior. The semiclassical theory is one possibility, and was applied in the microwave cavity example. Finally, with the development if new field of nanostructures very many experimental possibilities are open to test the validity of our concepts of stochastic modelling, and to the study of temperature effects and quantum coherence.

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