Symmetries of Heavy Ion Transition Amplitudes

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The symmetries of approximate heavy-ion transition amplitudes are discussed. Their consequences on polarization observables and on the magnetic substate population in inelastic heavy-ion scattering are illustrated with examples. The limits of validity of the approximations are discussed with possible experiments to test them.

I. Introduction

During recent years, it has become well recognized that the fusion of heavy-ions at energies near and below the Coulomb barrier are strongly influenced by the internal degrees of freedom of the colliding nuclei, such as their rotational or vibrational excitations[1]. At the same time, it is now well established that the strong coupling of the inelastic and transfer channels affect the elastic scsttering at energies close to the Coulomb barrier. The elastic optical potential exhibits this by a strong loczlized energy dependence, a phenomenon which has been termed the "threshold anomaly"[2].

Even though a few comprehensive coupled channels calculations have been performed in a few cases to undestand the cause of the threshold anomaly[3], it is usually difficult to perform these calculations due to the large number of channels that need to be included. In view of this, quite often, approximations have been invoked which considerably simplify the coupled-channels calculations either by reducing the number of effective channels or else by completely decoupling them[4].

The exact transition amplitudes have symmetries dictated by the invariances of the underlying Hamiltonian. The approximate transition amplitudes, sometimes, exhibit additional symmetries. One of the most dramatic of such "approximate symmetries" which was pointed out by Gomez-Camacho and Johnson[5] was the conservation of the projection of the target spin along the bisecting direction of the initial and final momentum vectors. This symmetry, which has been referred to as the "tiditl symmetry" makes clear predictions on the behaviour of the polarization observables[6] and on the population of the magnetic substates in the case of the inelastic scattering of heavy ions[7]. It is the symmetries of the approximate transition amplitudes which forms the subject of this talk. We shall also discuss the range of validity of the approximations and possible experiments to test the validity.

II. Symmetries of the heavy-ion transition amplitudes

(a) Scattering observable

The amplitude for transition from a given initial state with spin **I** and z-component M to a final state with spin **I'** and z-component M', will be denoted by $A_{I'M',IM}(\theta,\Phi)$ where θ and Φ are the polar and azimuthal scattering angles. (We assume the projectile to be a spin zero, inert nucleus). One of the observables on the scattering is the differential cross section.

$$\frac{d\sigma}{d\Omega} = \frac{1}{2J+1}\Sigma_{MM'}|A_{MM'}(\theta,\Phi)|^2 \qquad \text{(II.1)}$$

This is an "incoherent" sum of squares of the transition amplitude in the case of the scattering of an unpolarised projectile by an unpolarized target nucleus.

Other observables which are more sensitive to the phase and M- dependence of the transition amplitudes are (i) the polarization of the scattered projectile, (ii) the analyzing powers in the case of the scattering of polarized projectiles and (iii) the population of the magnetic substates of the excited target in the case of inelastic scattering. The above quantities are not all independent but can all be expressed in terms of tlie polarization tensors[8]

$$t^{\rm out}_{kq}(I') = \frac{\Sigma_{MM'}(-1)^{I'-M'} < I' - M'I'M' + qk_q > A_{I'M',IM}A^{*}_{I'M'+q,IM}}{\Sigma_{MM'}|A_{A'M',IM}|^2} \qquad \text{(II.2)}$$

where $t^{\rm out}_{kq}(I')$ is a measure of the polarization of the outgoing particle of rank k and z-component q. A similar expression describes the analyzing powers in the scattering of a polarized projectile[8]). The magnetic substate population is defined by

$$P_{M'} = \frac{\Sigma_M|A_{A'M',IM}|^2}{\Sigma_{MM'}|A_{I'M',IM}|^2} \qquad \text{(II.3)}$$

normalized to unity, i.e;

$$\Sigma_{M'}P_{M'} = 1 \qquad \text{(II.4)}$$

$P_{M'}$ can be expressed in terms of $q = 0$ components of the even rank polarization tensors, t_{kq} (eq II.2).

Another observable which is also sensitive to the phase of the transition amplitude is the particle-gamma correlation function[9]

$$W(\theta, \Phi, \theta_\gamma, \Phi_\gamma) = \Sigma_{kq}t_{kq}(I \cdot I')\rho_{kq}(I', I) \qquad \text{(II.5)}$$

where $\rho_{kq}(I', \mathbf{I})$ describes the statistical tensor for the de-excitation of the state **I'** by gamma emission.

(b) Symmetries of tlie exact transition amplitude

The symmetries of the exact transition amplitudes follow from tlie invariances of the underlying Hamiltonian. The consequences of the assumption of rotational invariance, invariance under parity inversion and time reversal invariance of the Hamiltonian on the transition amplitudes have been discussed in detail by Satchlerl[10]). These lead to the well known reciprocity theorem which relates the cross sections of inverse reactions, the polarization-asymmetry theorem which relates the polarization of scattered particles to the analyzing powers in the inverse reaction with polarized projectiles etc. In all reactions dominated by strong interactions tliese symmetries have been observed to remain exactly valid.

(c) Some experimental observations

There have been a few experiments during recent years which hint at other possible symmetries of the transition amplitudes.

(i) The study of the scattering of polarized projectiles by Fick and his collaborators[6]) has shown that, at energies close to the Coulomb barrier, the odd rank analyzing powers are negligibly small whereas different components of the even rank analyzing powers are related to each other. To be specific, in the scattering of polarized ^{7}Li on a target nucleus, the rank 1 and 3 analyzing powers were observed to be negligible, while the three components of rank 2 analyzing power could be simply expressed in terms of one underlying component. This observation was termed tlie "shape effect" by Fick[6], who suggested a simple geometric interpretation of this plienomenon.

It should be stressed that the shape-effect relations do not follow from the exact symmetries of the transition amplitudes discussed in (b).

(ii) More recently, tlie magnetic substate populations in the inelastic excitation of ^{92}Zr by ^{16}O ions (at a laboratory energy of 56 MeV) to the first excited 2^+ state of ^{92}Zr, were analyzed by Takagui et al.[11]). They observed that the inelastic cross section could not be described as a vibrational excitation with the use of a macroscopic dynamic deformation model. The large angle inelastic cross section could be fit either by introducing large reorientation coupling or by drastically modifying the transition form factor from traditional derivative form. At the same time, it was observed that the changes to the coupling form factors had little effect on the predicted magnetic substrate population.

III. Appraximate transition amplitude

In order to discuss the approximations invoked to simplify the coupled equations in the collision of heavy-ions, we shall consider the problem of the scattering of nuclei involving a rotor. Let us first consider the case of an "inert" zero spin projectile incident on an axially symmetric rotor. The exact coupled equations are of the form

$$(K_l + U_0 + V_c + \epsilon_I - E)u^J_{LI}(R) + \Sigma_{L'I'} V^J_{LI;L'I'}(R) u^J_{L'I'}(R) = 0 \qquad \text{(III.1)}$$

where K_L is the kinetic energy operator for relative motion with relative orbital angular momentum L, U_0 is the monopole nucleus-nucleus potential, V_c is the monopole Coulomb potential and ϵ_I is the energy of the state of the target with spin **I**. The relative wave function $u_{LI}(R)$ is characterized by the relative orbital angular momentum L, the spin of the target state **I**, and the total angular momentum **J**, $(\vec{J} = \vec{L} + \vec{I})$ which is conserved in the reaction. The matrix elememt of the multipole nucleus-nucleus interaction is expressed by $V^J_{LI;L'I'}(R)$.

It is customary to assume that in the case of collective excitation (rotational or vibrational excitation), the multiple interaction can be approximated by

$$V_{\text{coupl}} = \delta_2 \frac{dU_0(R)}{dR} P_2(\cos\theta) \;, \qquad \text{(III.2)}$$

where $\delta_2 (= \beta_2 R)$ is the deformation length with β_2 representing the deformation of the nucleus and θ the angle between the relative separation vector $\vec{R}$ and the orientation of the symmetry axis of the rotor.

It is this particular form eq (III.2), of the coupling interaction which allows for approximations to the coupled equations. There are two different approximations possible depending upon the kinematics of the reaction.

(i) The Sudden approximation

If the target nucleus is strongly deformed, it may be reasonable to neglect the energies of the excited states of the target in comparison to the projectile energy. Thus, one sets $\epsilon_1 = 0$. This is the case where the rotor moment of inertia is large. In such a case, it is possible to reduce the dimension of the coupled equations by transforming to the body-fixed system, i.e., defining new functions

$$\nu^{JK}_L = \Sigma_I < L K J - K | I0 > u^J_{LI}(R) \qquad \text{(111.3)}$$

one obtains the system of coupled equations

$$(K_L U_0 + V_c E) v^{JK}_L(R) + \Sigma_{L'} V^K_{LL'}(R) v^{JK}_{L'}(R) = 0 \qquad \text{(111.4)}$$

where the new coupling matrix elements $V^K_{LL'}(R)$ are given by[12]

$$V^K_{LL'}(R) = \delta_2 \frac{dU_0(R)}{dR} \frac{\hat{L}\hat{L}'}{\sqrt{5}} (-1)^k < LOL'O|20 >< LKL' - K|20 > \qquad \text{(III.5)}$$

where $\hat{L} = (2L + 1)^{1/2}$.

The new feature of the approximate radial wave functions $v^{JK}_L(R)$ is that a new conserved quantity, K, is obtained. This is the projection of the relative orbital angular momentum L on the axis of symmetry of the rotor.

(ii) Isocentrifugal approximation[4,5,13]

In the case of "nuclear excitation" of collective states in heavy-ion collisions, it may be justifiable to assume that one can neglect the change of the centrifugal potentials corresponding to a given total conserved angular momentum J. This is equivalent to assuming that the inertia parameter for relative motion (μR^2), where μ is the reduced mass and R is a characteristic distance of closest approach, is large. We can then replace the kinetic energy operator K_L by K_J.

Once again, as in eqn (3.3), we can reduce the dimensionality of the coupled systems by a transformation

$$w_I^J(R) = \Sigma < IOJO|LO > u_{LI}^J(R) \qquad \text{(III.6)}$$

with the functions $w_I^J(R)$ becoming solutions of the equations

$$(K_J + U_0 + V_c + \epsilon - E)w_I^J(R) + \Sigma_{I'} V_{II'}(R) W_{I'}^j(R) = 0 \qquad \text{(111.7)}$$

where

$$V_{II'} = \delta \frac{dU_0(R)}{dR} \frac{\hat{I}\hat{I}'}{\sqrt{5}} < IOI'O|20 >^2 \qquad \text{(III.8)}$$

The dimension of the new set of coupled equations is that of the number of target states one wishes to include. This is a drastic reduction of the dimensionality from the original system of equations eqn (111.1). The transformation, eqn (III.6), in this case rotates the system from the laboratory system to that of the "moving coordinate system". At this stage, the projection of the target spin on the bisector of the initial and final momenta is conserved.

IV. Symmetries of approximate transition amplitudes

(a) Sudden approximation

The symmetries in this case have been discussed by several authors[14]. If one chooses the z-axis along the recoil direction, $\vec{Q} = \vec{k}_i - \vec{k}_f$, then

$$A_{IM}^{\text{sudden}}(\vec{k}_f, \vec{k}_i) = (-)^M A_{IM}^{\text{sudden}}(\vec{k}_f, \vec{k}_i) \qquad \text{(IV.1)}$$

Thus only the states with even M will be populated. For a 0^+ to 2^+ transition, for instance, in the coordinate system with the z-axis along the recoil direction, the only non-vanishing polarization tensors (and analyzing powers) are

$$t_{kq}^{\text{sudden}}(\vec{k}_f, \vec{k}_i) = 0 \quad \text{unless} \quad q = \text{even} \qquad \text{(IV.2)}$$

(b) Isocentrifugal approximation

It was shown by Gomez-Camacho and Johnson[5] (see also Andres et al[13]) that, in this approximation, one finds

$$t_{kq}^{is}(\vec{k}_f, \vec{k}_i) = 0 \quad \text{unless} \quad k = \text{even}, \quad q = 0 \qquad \text{(IV.3)}$$

in the coordinate system with the z-axis along the recoil vector Q.

In the case of an odd A target (or projectile), assumed to be deformed, the amplitude for excitation from the initial state IM to a final state $I'M'$ becomes, in the isocentrifugal approximation

$$A_{I'M',IM(\theta,\Phi)} = \Sigma_K D_{MK}^I(\vec{Q}) f_K^{II'}(\theta) D_{M'K}^{I'*}(\vec{Q}) \qquad \text{(IV.4)}$$

This leads to the results

$$\frac{d\sigma}{d\Omega}_{I\to I''} = \Sigma_K |f_K^{II'}(\theta)|^2 \qquad \text{(IV.5)}$$

and for the analyzing power

$$T_{LM} = \frac{\Sigma_K D_{MO}^L(\vec{Q}) |f_K(\theta)|^2 I < IKI-K|LO >}{\Sigma_k |f_K(\theta)|^2} \qquad \text{(IV.6)}$$

If one chooses $\vec{Q}$ as the z-axis, the analyzing power becomes

$$T_{LM} = \delta_{MO} \frac{\Sigma_K < IKI-K|LO > |f_K(\theta)|^2}{\Sigma_k |f_K(\theta)|^2} \qquad \text{(IV.7)}$$

Eqn (IV.7) leads to the shape effect relations.

The conditions under which the results, eqn (4.4) follow have been discussed by Andres et al[13]. These are

i) The reaction must be nearside dominated. (The scattering should be Fresnel Scattering).
ii) The partial waves contributing to the reaction cross section must be large, L, J $\simeq L_{\text{grazing}}$
iii) The coupling interaction should be momentum and spin-independent.

V. Tests of approximate symmetries

We shall illustrate the validity of the approximations by the study of a few representative reactions

i) ^{23}Na + ^{208}Pb collision

The first of these is the collision of 170 MeV polarised ^{23}Na ions with ^{208}Pb target. Coupled-channels calculations were performed for this system using double-folded potentials and including the first and second excited states of ^{23}Na ($5/2^+$ and $7/2^+$ states). Experimentally, the elastic and inelastic scattering could not be resolved[16] and the quasielastic cross sections and analyzing powers were measured. In the figs. 1(a) and 1(b), we compare the coupled-channels predictions for the quasielastic cross section and the analyzing powers. In order to test whether the isocentrifugal approximation is vitlid, the transition amplitudes were transformed to the recoil coordinate system where the recoil vector $\vec{Q}$ is taken as the z-axis. If the isocentrifugal approximation is valid, these amplitudes should be diagonal in the projection of the target spin on the recoil axis. Figs. 2(a) and 2(b) show these amplitudes for the elastic and inelastic scattering to the first excited state. It is noticed that the diagonal contributions ($\Delta k = 0$) are larger tlian the off diagonal ones by an order of magnitude. In these calculations, the Coulomb excitation had not been included. The effect of Coulomb excitation on these amplitudes is shown in Figs. 3(a) and 3(b). One can see that the Coulomb excitation causes the non-diagonal contributions in the tidal spin (Ak = 0) tc enhance thereby suggesting that isocentrifugal approximation is not valid for Coulomb multipole forces. Since the quasielastic analyzing powers are dominated by the elastic amplitudes, they are seen to agree well with the shape-effect relation, as shown in Fig. 4.

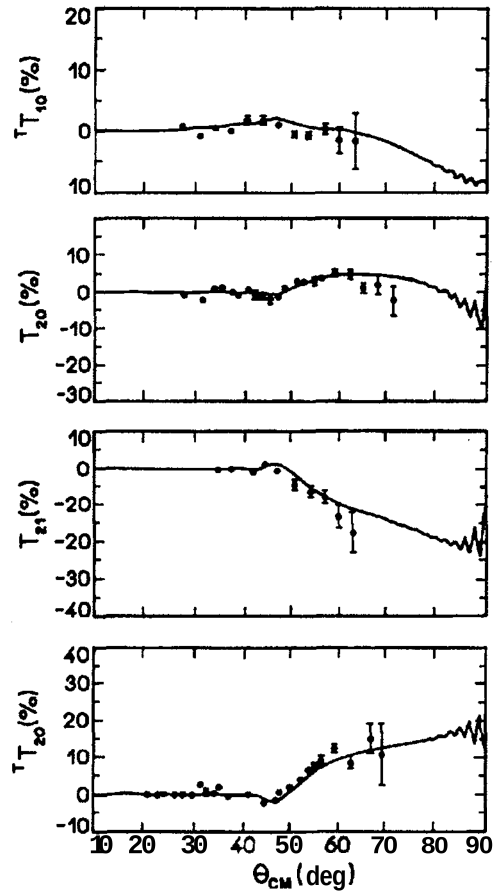


Figure 1(b): The analyzing powers for the quasi-elastic scattering predicted by the three-channel (solid line) calculations compared with the experimental data (16).

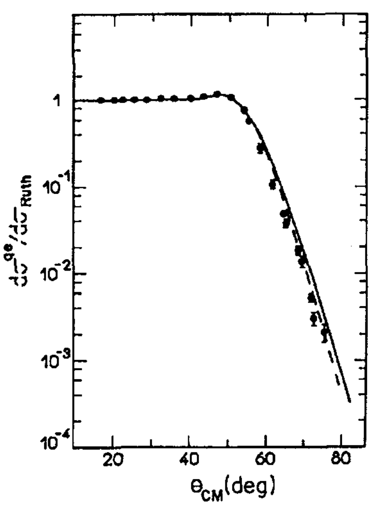


Figure 1(a): Summed cross sections of the elastic and inelastic scattering to the first $5/2^+$ state of ^{23}Na are shown as a ratio to the Rutherford cross section. The datapoints are from reference (16) for 170 MeV ^{23}Na incident on ^{208}Pb. The dashed curve is the prediction of two-channel and the full line curve is that for the three-channel calculation.

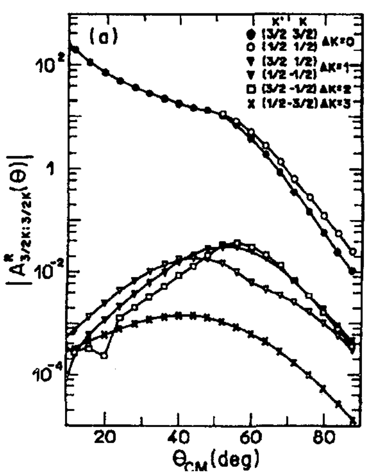


Figure 2(a): The elastic scattering amplitudes in the recoil frame, $A^R_{3/2K',3/2K}$, predicted by the three channel calculation is shown as a function of the scattering angle. The effect of Coulomb excitation is not included. The insets specify the values of (K', K).

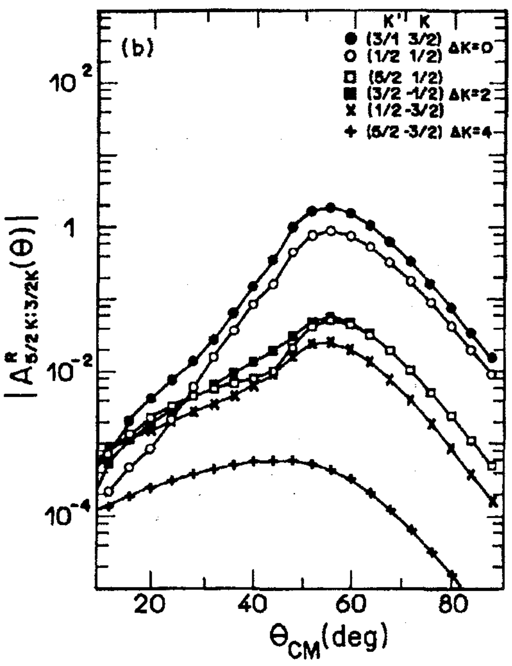


Figure 2(b): Same as (a) for the inelastic amplitudes, $A^R_{5/2K',3/2K}$. The figure shows only the components with $\Delta K = K' - K$ = even.

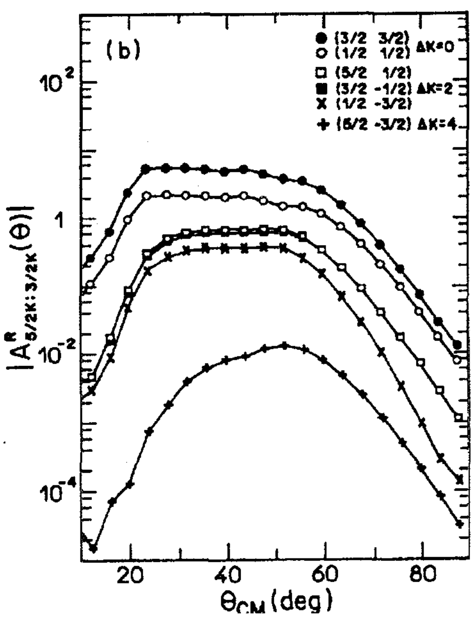


Figure 3(b): Same as 2 (b) with Coulomb excitation included.

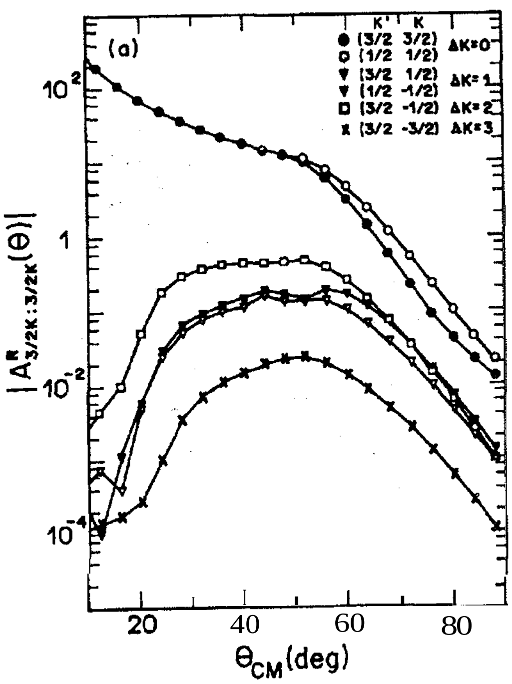


Figure 3a: (a) Same as 2 (a) with the inclusion of Coulomb excitation.

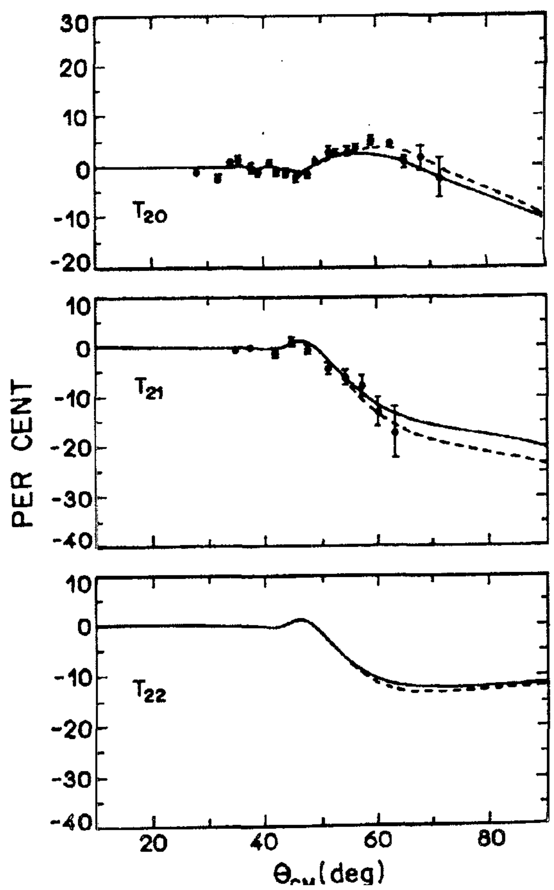
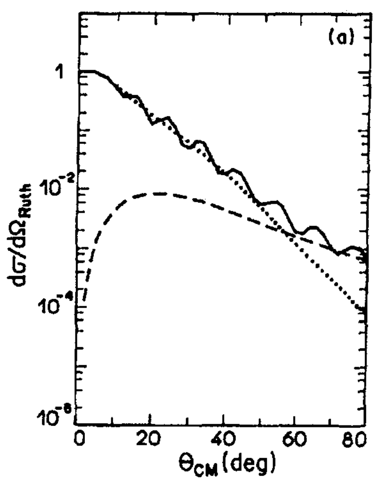


Figure 4: Three coupled-channels predictions of the rank 2 quasielastic analyzing powers for 170 MeV $^{23}Na+^{208}Pb$ system are compared with the predictions from the shape-effect relations. The experimental data are (16).

These ca.culations suggest that (a) at the enegies where the elastic scattering is dominated by Rutherford scattering (Fresnel region), the isocentrifugal approximation is valid for nuclear excitation, (b) the isocentrifugal apprɔximation is not suitable for (long range) Coulomb excitation.

ii) ^{7}Li + ^{26}Mg collision

As a seco1d example, we consider the scattering of 44MeV polarized ^{7}Li ions by ^{26}Mg target nuclei. The projectile energy is high relative to the Coulomb barrier resulting in Fraunhofer scattering. This is shown in Figs 5(a) and 5(b) for elastic scattering and inelastic scattering to ,he first excited ($1/2^-$) state of ^{7}Li. These figures also show the near and far-side contibutions to the cross sec;ions. The near and far-side amplitudes are comparable in magnitude resulting in strong oscillations observed in the cross sections. The predicted rank 2 analyzing powers for the elastic and inelastic scattering are shown in Figs 6(a) and 6(b). The exact coupled-cliannels predictions are shown by the solid lines, the nea:: side contribution by the dotted line and the far-side contributions by the dashed line. Since the coordinate system has the recoil vector as the z-axis, only the T_{20} component is non-zero for the near side contribution. It is clearly seen that far-side amplitude violates the predictions of the isocentrifugal approximation.



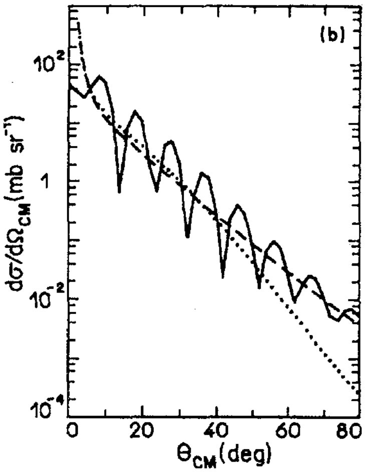


Figure 5: The differential cross sections (solid curves), the nearside contributions (dotted curves) and the farside contributions (broken curves) evaluated by coupled-channels calculations of 44 MeV ^{7}Li scattering on ^{26}Mg are shown for (a) elastic scattering ($I' = 3/2^-$), and (b) inelastic scattering to the 1/2 state of ^{7}Li at 478 keV.

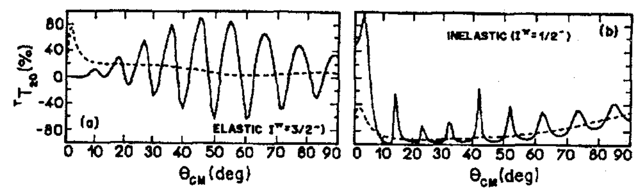


Figure 6: The predicted rank 2 analyzing powers $^RT_{20}$ shown on a function of the centre-of-mass scattering angle for (a) the elastic scattering and (b) inelastic scattering to the first excited state of ^{7}Li. The different curves are defined as in Fig. 5.

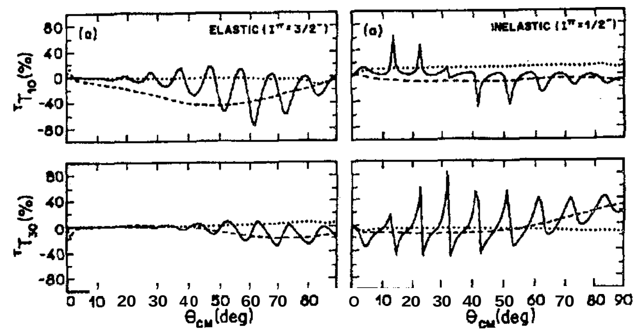


Figure 7: Same as Fig. 6 but for the transverse odd rank analyzing powers $^TT_{10}$ and $^TT_{30}$.

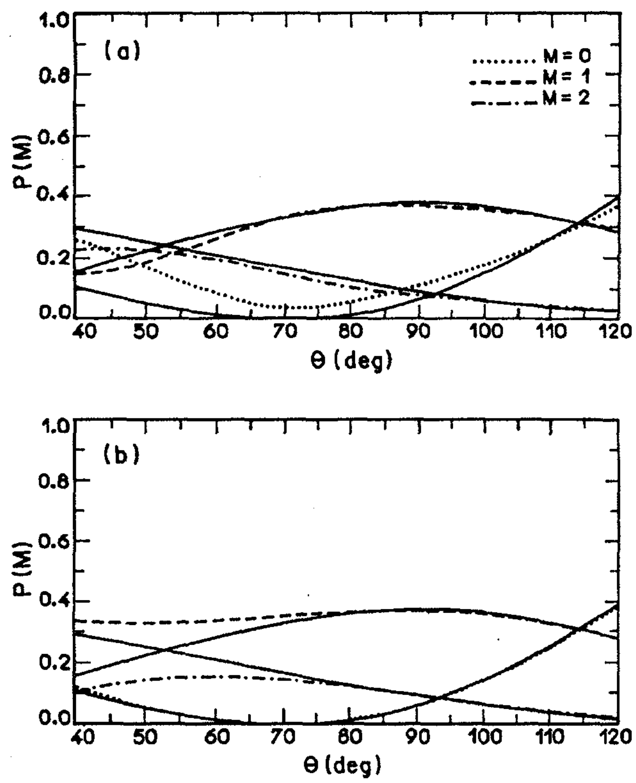


Figure 8: The predictions of the magnetic substate probability, $P(M)$, in the collision of 56 MeV 160 on ^{92}Zr leading to its first excited 2^+ state for the case of pure nuclear excitation. *(a)* The solid curves represent the predictions of the tidal symmetry model while the other three are coupled-channels predictions in the case of no reorientation coupling. (b) Similar to (a) but for the case with reorientataion coupling.

This becomes even more apparent from Figs 7(a) and 7(b) which show the predictions of the coupled channels calculations for the odd rank analyzing powers. If the isocentrifugal approximation were valid, these would be identically zero.

Thus, these results indicate that the isocentrifugal approximation would be unsuitable for reactions where far-side scattering is non-negligible.

iii) Magnetic substate population in ^{16}O + ^{92}Zr collision

As a final example, we consider the inelastic scattering of ^{92}Zr to its first excited 2^+ state by 56 MeV ^{16}O projectiles. The cross section and magnetic substate populations in this system were measured by Takagui et al[11]. It was observed that standard coupled channels-calculation assuming 2^+ state to be a vibrational state failed to describe the large angle cross section. The inelastic cross section could be fit either by introducing very large reorientation coupling or using transition form factors which differed drastically from the derivative form. However, these changes had little effect on the predicted magnetic substate population.

If isocentrifugal approximation is valid it follows that the transition amplitudes are symmetric around the recoil direction, i.e.,

$$A_{2M}(\theta, \Phi) = Y_{2M}(\vec{Q}) f_2(\theta) \qquad \text{(V.1)}$$

where $\vec{Q}$ is the recoil direction. The M dependence of the amplitude enters through the spherical harmonic $Y_{2M}(\vec{Q})$. Hence, this will predict for the magnetic substate population

$$P_M(\theta, \Phi) = \frac{4\pi}{5} |Y_{2M}(\vec{Q})|^2 \qquad \text{(V.2)}$$

In Figs 8(a) and 8(b) the predictions of coupled channels calculation of nuclear excitation on the magnetic substate calculations are shown, the first without reorientation coupling and the second with reorientation coupling. The solid lines in these represent the predictions of the isocentrifugal approximations eqn (V.2). It can be seen that the tidal symmetry predictions are well preserved and there is no effect of reorientation coupling.

We had already commented that Coulomb excitation violates tidal symmetry. This is shown in Fig 9 where we have chosen to calculate the magnetic substate population in the recoil coordinate system. The non vanishing of the M = 1 substate population is an indication of the violation of tidal symmetry.

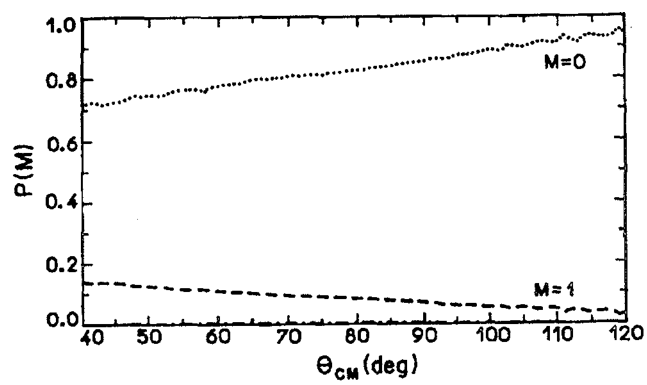


Figure 9: Same as Fig. 8 for the case of pure Coulomb excitation. Here the probabilities are shown in the recoil coordinate system.

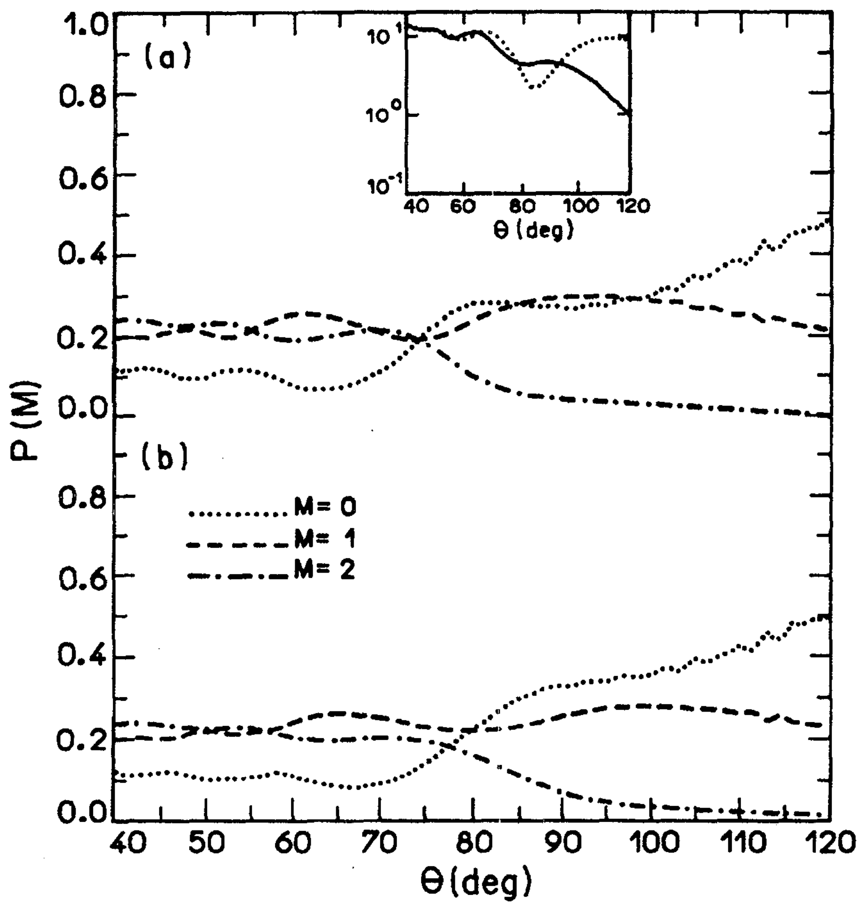


Figure 10: (a,)Same as Fig. 8(b) but the probabiities have been evaluated allowing for both nuclear and Coulomb excitations. The ineet shows the differential cross sections with (solid curves) and without (dotted curves) reorientation coupling. (b) The predicted magnetic substate probabiities, P(M), for the case where the transition form factor has been adjusted to provide a reasonable fit to the Coulomb-nuclear interference region but with no reorientation coupling.

In Figs. 10(a) and 10(b) the predictions of the coupled channels calculations with Coulomb and nuclear excitation are shown. Fig. 10 (a) shows the case where reorientation coupling was included. In Fig. 10(b), a modified transition form factor was used which provided a "reasonable" description of the Coulomb-nuclear interference region. The two M state populations in 10 (a) and 10(b) are seen to be remarkably similar.

These analyses suggest that the sensitivity of the magnetic substate population to the nature of nuclear coupling can be probed only in the Coulomb-nuclear interference region. This is a consequence of the tidal symmetry of the nuclear amplitudes. If different transition form factors are chosen to fit the Coulomb-nuclear interference region, all of them will predict the same M-state population, ***even* if *they disagree at large angles*** where they are dominated by nuclear excitations.

VI. Summary and conclusions

There is considerable experimental evidence that heavy-ion transition amplitudes, under certain kinematic conditions, possess an additional symmetry termed "tidal symmetry". This is a consequence of the isocentrifugal approximation and due to the dominance of the reaction amplitudes by large impact parameters. The validity of this approximation and the tidal symmetry is governed by the following conditions.

(i) Nuclear excitations to strongly collective states at energies close to the Coulomb barrier satisfy tidal symmetry. The reactions need to be nearside dominated.

(ii) Even at energies close to the Coulomb barrier, tidal symmetry is strongly violated by Coulomb multipole forces.

(iii) Tidal symmetry will be violated by spin and momentum dependent forces.

There is further need to explore the validity of the isocentrifugal approximation at energies near the Coulomb barrier where the coupling may become "effectively" spin and/or momentum dependent due to strong coupling to transfer channels. In particular, particle-gamma correlation may be more sensitive to the transition potentials than the measurement of the M-state populations.

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