Tetracritical Behaviour in a Spin-3/2 Quantum Chain

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Finite-Size-Scaling and Conformal Invariance are used in order to find the phase diagram and critical exponents of a quantum spin chain with spin S = 3/2. The model has a tetracritical point besides critical lines. The conformal anomaly and anomalous dimensions of some primary operators are calculated at the tetracritical point.

Conformal invariance enable us to classify a large number of field theories in two dimensions[1,2], however it is not obvious that there exist Statistical Mechanics models realizing such field theories. Meanwhile severals examples of Statistical Mechanics models described by such field theories are known[3]. In particular Andrews et al.[4] and Huse[5] introduced the RSOS model which gives a realization of the minimal series[2] with central charge

$$c = 1 - \frac{6}{(K+2)(K+3)} \quad ; \quad K = 1,2,3,\dots \quad . \tag{1}$$

However the quantum Hamiltonian associated to the RSOS model are rather complicated.

Recently Sen[6] introduced a serie of simple spin-S Hamiltonians. Based in a Zamolodchikov's Landau-Ginzburg theory he conjectured that, at special multicritical points, these models also give realizations of the minimal series in Eq. (1). These models are defined by the Hamiltonian

$$H[S] = \sum_{i=1}^{L}\left[\frac{1}{2}(S_i^z - S_{i+1}^z) + \gamma S_i^x + \sum_{n=1}^{[S]} a_{2n}(S_i^z)^{2n}\right] \tag{2}$$

where S_i^x, S_i^y and S_i^z are spin-S representations of $SU(2)$ algebra acting on site i of an L sites chain and periodic boundary conditions (PBC) are imposed. The integer K in Eq. (1) are related to the spin S in Eq. (2) by $K = 2S$. The degree of the polynomial is $[S]$, the largest integer less than or equal to S. According to Ref.[6], such Hamiltonian present a multicritical point where $K+1$ phases are indistinguishable. If S = 1/2 then $K = 1$ and c = 1/2. Then $H_{[S]}$ reduces to the Ising Hamiltonian. In the case S = 1 we have $K = 2$ and $H_{[S]}$ is given by

$$H_{[1]} = H_{[3/2]} = -\sum_{i=1}^{L}[S_i^z S_{i+1}^z - (1+a_2)(S_i^z)^2 - \gamma S_i^x] \ , \tag{3}$$

which is a particular case of the Blume-Emery-Griffths model[7], having a tricritical point governed by a Conformal Field Theory (CFT) with c = 7/10.

In case S = 3/2 the Hamiltonian is the same as in Eq. (3) and a tetracritical point is expected with c = 4/5[6]. This tetracritical point is the crossing point where two second order lines end up in a first order transition line.

In this work, by using Finite-Size-Scaling[8] (FSS) and the Conformal Invariance (CI) predictions for finite systems[9,10,11], we studied the Hamiltonian $H_{[3/2]}$ for S = 3/2 to test the conjectures of Ref. [6].

First, in order to calculate the phase diagram we locate the second order lines by using FSS. These lines can be found by extrapolating the sequence of curves obtained from the relation

$$LG_L(\gamma, a_2) = L'G_{L'}(\gamma, a_2) \ , \tag{4}$$

where L and L' are the sizes of two finite chains with L' = L+1 and $G_L(\gamma, a_2)$ is the mass gap of the Hamiltonian $H_{[3/2]}$ evaluated at (γ, a_2) for the finite chain of size L.

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The spectral calculations were done numerically using the Lanczos algorithm[12] for lattices sizes up to L = 11. Beyond the translation invariance the Hamiltonian $H_{[3/2]}$ commutes with the parity operator

$$P = \prod_{i=1}^{L} \frac{1}{3}[4(S_i^x)^3 - 7S_i^x] \qquad (5)$$

and consequently we can separate the Hilbert space into disjoint sectors labelled according to the momentum $q = \frac{2\pi}{L}l$, with $l = 0, 1, 2, ..., L - 1$, and parity $p = \pm 1$ eigenvalues.

The ground state (GS) is a zero-momentum state with parity +(-) for latices sizes L even (odd), and the first excited state is also a zero-momentum state but with opposite parity.

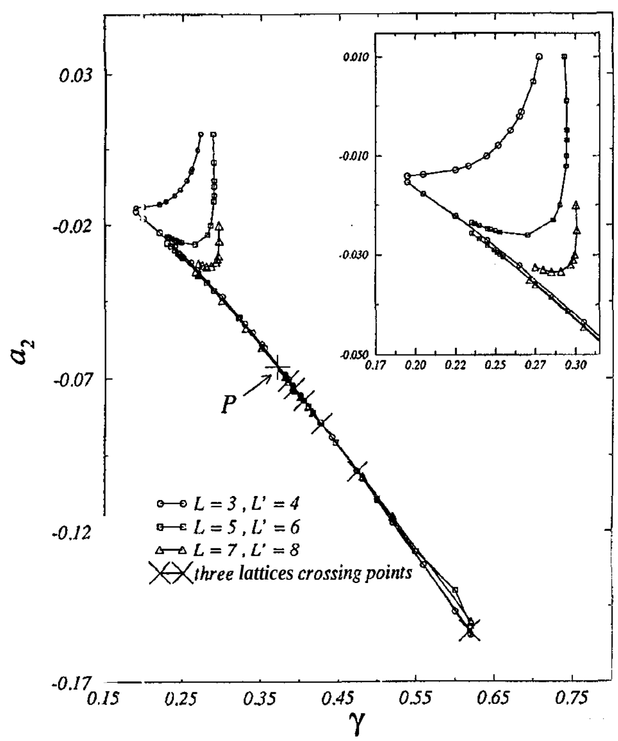

Figure 1: Curves satisfying Eq. (4) for some values of L and, in a large scale, the region where the intersections of curves occur. Three lattices crossing points satisfying Eq. (6) and the tetracritical point P = (0.3702,-0.0661)is also showed.

In Fig. 1 we show in the parameter space of y and a_2 the curves satisfying Eq. (4) for some values of L. We also show in a large scale the region where the intersections of these curves occur. Notice verify the qualitative agreement of Fig. 1 with the phase diagram proposed in Ref. [6].

The tetracritical point **P** in Fig. 1, where a conformal anomaly $c = 4/5$ is expected[6], is obtained by the intersection of the two second order phase transition lines. Unfortunately, as we can see in Fig. 1, in the region where these intersections occur, the second order lines are almost parallel, which make difficult a high precision determination of the finite-size estimative of the point **P**. This difficulty obviously will be reflected in the convergence of the numerical sequence used for the bulk limit ($L \rightarrow \infty$) evaluation of **P**.

An alternative way[13], which give us better estimatives of the tetracritical point **P**, is obtained by using the sequence of points calculated by a generalization of Eq. (4) according to

$$LG_L(\gamma, a_2) = L'G_{L'}(\gamma, a_2) = L"G_{L"}(\gamma, a_2) \ , \qquad (6)$$

where L" = L' + 1 = L + 2. This condition is satisfied at the crossing point obtained by using Eq. (4). These crossing points are denoted in Fig. 1 and we show their coordinates in Table 1. The tetracritical point is obtained by extrapolating the coordinates of these estimators. Unfortunately, due to the small number of points these extrapolations are poor, specially for the sequence related to the coordinate y. The best estimatives we get for these extrapolations are $y = 0.36 \pm 0.01$ and $a_2 = 0.065 \pm 0.002$.

Table 1 - Three lattices crossing points satisfying Eq. (6) and extrapolated values.

L, L', L"	γ	a_2
2,3,4	0.61867	-0.153
3,4,5	0.473	-0.10033
4,5,6	0.4266	-0.084526
5,6,7	0.404	-0.077047
6,7,8	0.39166	0.073026
7,8,9	0.384	-0.070553
extrap.	0.36±0.01	-0.065±0.002

A further analysis of Table 1 show us that these points obey, with a very good fitting, the following relation

$$a_2(\gamma) = 0.036883 - 0.23522\gamma - 0.1159\gamma^2 \ , \qquad (7)$$

with squares mean deviations 3% and 8% for a_2 and y, respectively. On the other hand in Ref. [6], based in a pesturbation exparision of $H_{[3/2]}$ up to order γ^4, the point **P** should also lay in the curve

$$a_2(\gamma) + -\frac{1}{2}\gamma^2 + \frac{25}{192}\gamma^4 \ . \tag{8}$$

An estimative of **P** is then obtained by equating Eqs. (7) and (8). Apart from spurious solutions we obtain **P** = (0.3702, −0.0661), which we believe, considering the small number of data available, to be our best estimative for this tetracritical point.

Once the point **P** is calculated the next step towards the verification of the conjecture of Ref. [6] is the calculation of the conformal anomaly and dimensions the operators of the underling CFT governing the large physics at this point. This is done by exploiting at point **P** the consequences of CI in the finite-size-corretions of the eigenspectra. The conformal anomaly c can be obtained, for periodic chains, from the finite-size-corrections of the GS energy $E_0(L)$, i. e.

$$\frac{E_0(L)}{L} = e_\infty - \frac{\pi c \zeta}{6L^2} + o(L^{-2}) \ , \tag{9}$$

where e_∞ is the bulk limit of the GS energy per particle. In Eq. (9) ζ is the sound velocity (non-universal) and can be calculated from the mass gap associated to the first excitated state with momentum $2\pi/L$ in the sector with the same parity of the GS, i.e.,

$$\zeta_L = \frac{L}{2\pi}(E_0(L) - E_{1,0}) \overset{L\to\infty}{\longrightarrow} \zeta \ . \tag{10}$$

Using Eqs. (9) and (10) we obtain for several points in the second order phase transitions lines of Fig. 1 a value close to c = 1/2 (Ising). As we move in Fig. 1 along the second-order transition line, we verify around the point **P** a clear cross-over (finite-size-effect) of our estimatives of the conformal anomaly, and at the point **P** we obtain: e_∞ = −0.2512, ζ = 0.67 and c = 0.76. Clearly this is very different of the value c = 0.5 of the Ising line and due to numerical instability in our extrapolations, it is also compatible with the conjectured value[6] c = 0.8. A better numerical test of the universality class of the point **P** is obtained calculating the anomalous dimensions of the field theory associated to this point. Associated to the dimension x_ϕ of a given primary operator of the theory ϕ, with spin s_ϕ, there is a tower of eigenstates with energy and momentum given by

$$E_{m,m'}(L) - E_0(L) = \frac{2\pi\zeta}{L}(x_\phi + m + m') + o(L^{-1}) \ , \tag{11}$$

and

$$P_{m,m'}(L) = \frac{2\pi}{L}(s_\phi + m + m') \ ; \ m, m' = 0, 1, 2, \ldots \tag{12}$$

Using these expressions we obtained finite-size estimators for the several dimensions associated to the eigenenergies of $H_{[3/2]}$. In particular the lowest gap (with opposite parity of the GS), gives the dimension x_{ϕ_1} = 0.052, and in the sector with the parity of the GS the lowest gap give us x_{ϕ_2} = 0.130. These values are much smaller than the values along the second-order transitions lines (c = 0.5), which are x_{ϕ_1} = 0.125 and x_{ϕ_2} = 0.5. According to the conjecture of Ref. [6] these gaps should be compared with the two magnetic operators of the modular invariant series[10,14] $A(5)$, with dimensions x_{ϕ_1} = 1/20 = 0.05 and x_{ϕ_1} = 2/15 = 0.1333.... As we can see the agreement with our results, taking into account the small number of lattices, is good.

In conclusion, our results for spin 3/2 and early results for S = 1 are clear indications in favour of the conjecture of Ref. [6], In other words, the Hamiltonian $H_{[S]}$ gives us statistical mechanics models with multicritical points governed by the modular invariance series $A(K+2)$ of CFT, having central charge given by Eq. (1) with $K = 2S$. After the completion of our calculations we became aware of a preprint[15] with results in agreement with our conclusions.

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