Electrons in Non-Homogeneous Magnetic Fields*

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The successful growth of a thin film ferromagnetic material on top of a semiconductor has opened a new aiid exciting field in solid state physics. Lithographic patterning of the ferromagnetic film will allow one to construct nonhomogeneous rnagnetic fields on a length scale of nanometers which may interact with a two-dimensional electron gas underneath the film. The electrons moving from a zero magnetic field region towards a nonzero magnetic field region will feel this region as a barrier. New systems are proposed consisting of magnetic tunneling barriers: 1) magnetic quantum wires, 2) double tunneling harriers, 3) magnetic dots, 4) magnetic superlattices, etc. The forrn of the equivalent potential that corresponds to a magnetic barrier depends on the wavevector of the incident electron. This renders the transmission through such structures an inherently *two-dimensional* process since the tunneling probability depends not only on the electron's energy but also on the direction of its wavevector. Pronounced resonances are obtained for the tunneling probability and tlie conductance of a resonant tunneling device coiisisting of such magnetic barriers. Such systems can be used as an electron wavevector filter. The energy spectrum (bound and scattered states) for these systems is obtained and the nature of the states is discussed.

I. Introduction

The behavior of electrons in *macroscopically* homogeneous inagrietic fields has been used extensively to obtain experimental information on properties of charge carriers[1] like e.g. their density and the Fermi surface (through the Shuhnikov de Haas (SdH) effect), and their mass (e.g. using cyclotron resonance). Scattering of electrons on magnetic impurities form the other limit in which electrons feel locally (on an angstrom scale) strong nagnetic fields (i.e. *microscopically* inhomogeneous) which may act as scattering centers in e.g. diluted seininiagnetic materials[2].

Between these limits lie inhomogeneous inagnetic fields on the *nanometer* scale. They have recently been realized with the creation of magnetic dots[3], integration of ferromagnetic materials with semiconductors[4-6] where patterning of such films was recently demonstrated experimentally[7]. This new technology will add a new functional dimension to the present semiconductor technology and will open new avenues for new physics and possible applications like, switches based on the Lorentz force and nonvolatile memories based on the Hall voltage generated by a local magnetic field. A different route to create inhomogeneous magnetic fields is through the integration of superconducting materials with semiconductors. This was realized experimentally using type II superconductors which was deposited on a Si-MOS[8] or a GaAs/AlGaAs-heterojunction[9,10]. Magnetic flux lines penetrate the two-dimensional electron gas (2DEG) acting as nanoscale scattering centers for the electrons[11-13], offering the possibility to study weak localization[10] and the dynamics of vortices[14]. Using lithographic techniques these superconducting films can be patterned into any desired form. The ge-

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ometry of tlie patterning determines the geometry of the inhomogeneous magnetic field.

In general tlie shape anisotropy of tlie magnetic film (or the stripes) will force the magnetization in the plane of tlie film. Other inechanisms can be active whicli can lead to a magnetization vector perpendicular to the film, whicli is the situation we are mostly iiiterested in. Out-of-plane magnetization has been realized in ultrathin layers of Fe on Ag[15] or Cu[16], compounds such as MnAlGa[17], Co/Ni[18] multilayers, and ultrathin MnGa films[5] and the metastable τ-MnAl phase[4] whicli can be grown epitaxially on GaAs/AlAs heterostructures using MBE.

The creation of superlattices by an inhomogeneous magnetic field was proposed theoretically in Refs. [19] and [20]. Vil'ms and Éntin[21] presented a theoretical analysis of tlie energy spectrum of 2D electrons near domain walls aad in a. system of parallel magnetic strips. Transport of a 2DEG in tlie presence of a perpendicular magnetic field modulated weakly and periodically along one direction was stuclied in Ref. [22] and receiitly tried unsuccesfully experimentally by Yagi and Iye[23]. The generalization to 2D magnetic field modulation is given in Ref. [24]. Recently Van Roy *et al*[25] studied the geometric factors controlling the magnitude of the demagnetizing field of ferromagnetic thin films with perpendicular magnetization. Different geoinetries were studied and they found that a grating-type structure rvith periodicity of a. few $100nm$ to $1\mu m$ would give tlie maximum magnetic field strength in the underlying semiconductor heterostructure. Müller[26] considered a different system in which a. 2DEG strip is placed in a perpendicular magnetic field which increases linearly along one direction. He showed that this system has a remarkable time-reversal symmetry. The present authors[27] studied different systems consisting of magnetic barriers and considered also tunneling through double magnetic barriers[28]. A quasi-analytic solution to a model barrier system was found in Ref. [29].

In tlie present paper we will consider different configurations of nonuniform magnetic fields in which the noiiuniformity is only along one direction and has a typical length scale of tlie order of nanometers. The electron spectrum of a 2DEG in simple magnetic structures like, magnetic step (Sect. 111), magnetic barrier (Sect. IV) and magnetic well (Sect. V) is considered and discussed. The similarities and differences between similar potential problems is pointed out. We consider electron tunneling through structures of magnetic barriers and in particular resonant tunneling. In contrast with tunneling through electric barriers, the tunneling probability depends not only on the electron's energy but also on the direction of its wavevector. This render the tunneling an *inherently two-dimensional* process. Furthermore we found that the magnetic barriers possess wavevector filtering properties.

II. Nonhomogeneous magnetic field profiles

As shown in Fig. 1 a magnetic barrier can be created by the deposition, on top of a heterostructure, of a ferromagnetic stripe with magnetization (a) perpendicular and (b) parallel to the 2DEG, (c) of a conducting stripe with a current driven through it, and (d) of a type I superconducting plate interrupted by a stripe. In all cases the 2DEG is situated at a distance z_0 below the stripe whose thickness and height are d and h, respectively.

For tlie case of magnetic stripes we consider the following Maxwell equations

$$div\vec{B} = -4\pi div\vec{M} = 4\pi\rho_M(\vec{r}),$$
$$\vec{B} = -grad\Phi_M \ , \tag{1}$$

which can be integrated and results into

$$\Phi_M(\vec{r}) = \int d^3r' \frac{\rho_M(\vec{r}')}{|\vec{r}-\vec{r}'|} \ . \tag{2}$$

For illustrative purposes we consider perpendicular magnetization (Fig. 1(a)) in which the width of the magnetic strip d is very small such that we can replace it by a dipole line with magnetic charge density:

$\rho_M(\vec{r}) = -M_0\delta(x)\frac{d}{dz}\delta(z)$. Integrating (2) results in the magnetic potential

$$\Phi_M(\vec{r}) = \frac{2M_0 z}{x^2 + z^2}. \qquad (3)$$

which leads to the magnetic field distribution

$$B(x) = 2M_0 \frac{z_0^2 - x^2}{(z_0^2 + x^2)^2}, \qquad (4)$$

and the vectcr potential

$$A(x) = 2M_0 \frac{x}{z_0^2 + x^2}. \qquad (5)$$

If we have stripes of width d instead of wires we have to integratc Eq.(2) numerically in the region $-d/2 \leq x' \leq d/2$. It turns out that in the lirnit $z_0, h \ll d$ the magnetic field distribution takes tlie simple form (h is the height of the inagnetic strip)

$$B(x, z_0) = B_0\left(K(x + d/2, z_0) - K(x - d/2, z_0)\right) \quad (6)$$

with $B_0 = Mh/d$ and $K(x, z) = 2xd/(x^2 + z^2)$ which is depicted in Fig. 2(a) for three values of z_0. Similarly for parallel magnetization we found $K(x,z) = -zd/(x^2 + z^2)$ and which is depicted in Fig. 2(b) for three values of z_0.

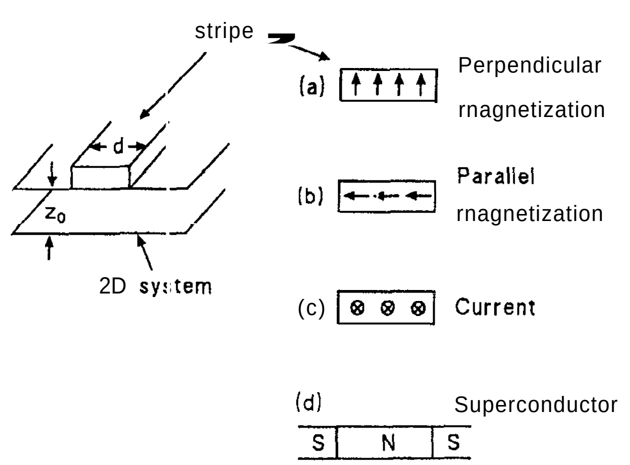

Figure i: Sectional view of several systems for producing non-lioniogeneoiis rnagnetic field profiles in the plane of tlie 2D electroii gas.

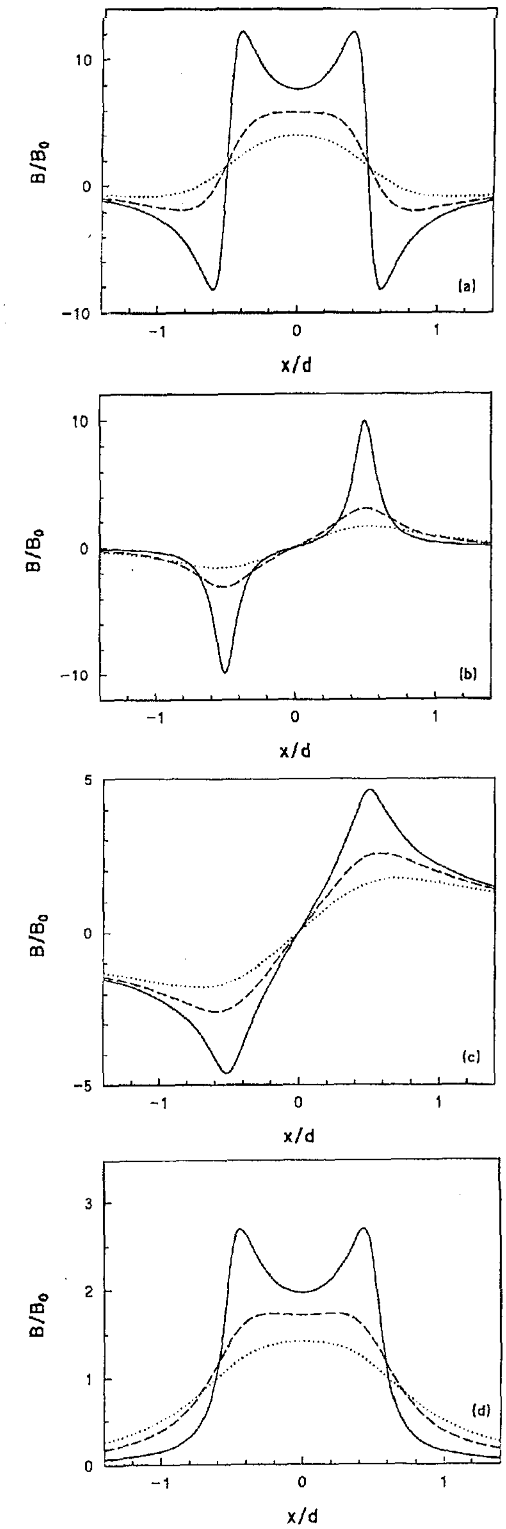

Figure 2: Magnetic field under tlie stripe corresponding to the four different configurations as given in Fig.1. The magnetic field is given at different distances from the magnetic stripe: $z_0 = 0.1$ (solid curve), $z_0 = 0.3$ (dashed curve), and $z_0 = 0.5$ (dotted curve).

For a wire with a current $\mathbf{I}$ going through it (Fig. 1(c)) the magnetic field is determined by the Maxwell equation: $rot\vec{B} = 4\pi\vec{j}/c$ which results in the angular magnetic field component $B_\phi = 2I/cr$ at a radial distance r from the wire. In the plane of the 2DEG a distancc z_0 from the wire this leads to the magnetic field profile

$$B(x) = B_z(x, z_0) = B_0 \frac{x}{z_0^2 + x^2}, \tag{7}$$

with $B_0 = 2I/c$ which is depicted in Fig. 2(c). The vector potential is obtained by integrating tlie equation $\vec{B} = rot\vec{A}$ which gives

$$A(x) = A_y(x, z_0) = \frac{1}{2} B_0 \ln\left(\frac{z_0^2 + x^2}{z_0^2 + R^2}\right), \tag{8}$$

where R is some distance away from tlie wire where the opposite current is flowing. For a strip of finite width d such that z_0, h $\ll d$ we can cast the result into the form of Eq.(6) with $K(x,z) = \ln[(x^2 + z^2)/d^2]$. This magnetic field profile is shown in Fig. 2(c).

For a superconducting stripe in a magnetic field we have to solve the Maxwell equations in the superconductor: $\vec{j} = -(ne^2/mc)\vec{A}$, and $\nabla^2\vec{H} = (1/L_0^2)\vec{H}$, where $L_0^2 = mc^2/4\pi ne^2$ and we made use of $rot\vec{H} = -(4\pi ne^2/mc^2)\vec{A}$. Outside tlie superconductor we have the equation: $\nabla^2\Phi_M = 0$, with tlie boundary condition on the superconductor $B_n = -\partial\Phi_M/\partial n = 0$. This 2D potential problem can be solved by the conformal mapping $x +$ **ir** $= sin(\pi w)$ where me introduce the complex potential W $= \Phi_m + i\Psi$. The solution of the problem is W = iw and the magnetic potential is given by $\Phi_M = ReW = -Im(\arcsin(x +$ iz)) which results into the magnetic field B $= B_0 Re(1/\sqrt{1-(x+iz)^2})$ as shown in Fig. 1(d).

Tlie magnetic field produced by the stripes, in units of B_0, is shown in Fig. 2 for tliree differeiit depths: $z_0 = 0.1$ (solid curve), $z_0 = 0.3$ (dashed curve) and $z_0 = 0.5$ (dotted curve). The smaller z_0, i.e., the closer the 2DEG is to the stripes, tlie sharper tlie magnetic barrier structure. With increasing z_0 the magnetic field profile becomes gradually smoother. Concurrently with the inagnetic field profile one expects a scalar (electric) potential, which can be short circuited by putting a non-magnetic metallic film[23] between the 2DEG and the patterned layer and which therefore will be neglected.

III. Motioii in nonhomogeneous inagnetic fields

We consider a 2DEG moving in the (x, y) plane with a magnetic field B along the z-direction. In the single-particle approximation such a system is described by the hamiltonian

$$H = \frac{1}{2m}(\mathbf{p} + \tfrac{e}{c}\mathbf{A})^2 \tag{9}$$

We take the vector potential in the Landau gauge $\mathbf{A} = (0, A, 0)$ and the magnetic field is uniform along the y-direction but modulated along the x-direction, and thus

$$B_z = B(x) = \frac{d}{dx}A(x), \tag{10}$$

where we have the magnetic field profiles of previous section in mind.

Let us introduce the following characteristic parameters: i) the frequency u, $= eB_o/mc$ with B, some typical magnetic field, and ii) the length $\ell_B = \sqrt{\hbar c/eB_o}$. For GaAs and an estimated $B_0 = .1$T we have $\ell_B = 813$Å, $\hbar\omega_c = .17$meV, and $\ell_B\omega_c = 1.4 m/sec$. From now on we will express all quantities in dimensionless units: 1) the magnetic field $B(x) \to B_o B(x)$, 2) the vector potential $A(x)$ **-** $B_o l_B A(x)$, **3)** the time $t \to t/\omega_c$, 4) the coordinate $\mathbf{r} \to l_B \mathbf{r}$, 5) the velocity $\mathbf{v} \to l_B\omega_c \mathbf{v}$, and 6) the energy $E \to \hbar\omega_c E$.

In these dimensionless units the two-dimensional (2D) Schrödinger equation becoines

$$\left\{\frac{\partial^2}{\partial x^2} + \left(\frac{\partial}{\partial y} + iA(x)\right)^2 + 2E\right\}\Psi(x, y) = 0. \tag{11}$$

Because of the special forin of the gauge the system is translational invariant along the y-direction and as a consequence we can choose the following form for the wavefunction

$$\Psi(x, y) = e^{-iqy}\psi(x) \tag{12}$$

where $-q = k_y$ is the wavevector of the electron in the y-direction which is a conserved quantity. This does not imply that v_y is conserved. Tlie wavefunction $\psi(x)$ actually satisfies tlie following 1D Schrödinger equation

$$\left\{\frac{d^2}{dx^2}-(A(x)-q)^2+2E\right\}\psi(x)=0 \qquad (13)$$

where the function

$$V(x)=\frac{1}{2}(A(x)-q)^2 \qquad (14)$$

can be interprcted as a q-dependent electrical potential. Note that in the case of one-dimensional (1D) magnetic field modulation studied in the present paper there is an analogy between tlie magnetic fielcl and tlie potential given by tlie following relation

$$B(x)=\frac{1}{\sqrt{2V(x)}}\frac{dV(x)}{dx}. \qquad (15)$$

A jump in tlie magnetic field will result in a discontinuity in the derivative of the poteiitial $V(x)$.

IV. Magnetic step

First, let us consider tlie most simple shape for a. nonhomogeneous magnetic field: tlie magnetic step. In this situation the magnetic field fills the half space $x>0$ which is described by $B(x)=\theta(x)$ with the corresponding vector potential $A(x)=x\theta(x)$, wliere $\theta(x)=1(x\geq 0),0(x<0)$ is the step-function. There are two different cases which we have to consider.

Case 1 ($q>0$). The potential $V(x)=\frac{1}{2}(x\theta(x)-q)^2$ has the forn- of an asymmetric quantum well which deepens with increasing q. It is well-known[30] that such a. well can have a bound state if tlie well is sufficiently deep. Thus, we have to consider separatelly: a) E $<q^2/2$ where bound eigenstates are expected to appear in the region $x\sim q$, and b) E $>q^2/2$ which corresponds to scattered states, describing tlie electron reflection by the magnetic step. Slie appearence of bound states makes this system essentially different from the usual potential stcp problem where only scattered states exist.

For Case 2 ($q<0$) tlie equivalent poteiitial is a constant $V(x)=q^2/2$ for $x<0$, and a barrier $V(x)=\frac{1}{2}(x\theta(x)+|q|)^2$ in the region $x>0$ which is unbounded for x $\to\infty$. In this case there are only scattered states which corresponds to electron reflection by the magnetic barrier.

For $x>0$ the Schrodinger equation takes the form

$$\left\{\frac{d^2}{dz^2}-\frac{z^2}{4}+p+\frac{1}{2}\right\}\psi(z)=0. \qquad (16)$$

with $z=\sqrt{2}(x-q)$ and $p=$ E $-$ 0.5. The solutions of it are tlie Weber functions[31] $D_p(z)$ which have the following asymptotic behaviour: $D_p(z)|_{z\to+\infty}\to 0$ In tlie region $x>0$ the wavefunction becomes $\psi(x)\sim D_{E-0.5}(\sqrt{2}(x-q))$, up to a normalization constant, while for $x<0$ tliere is no magnetic field and the wavefunction is proportional to $\psi(x)\sim\exp(x\sqrt{q^2-2E})$ when $E<q^2/2$. Matching the wavefunctions and its first derivative at $x=0$ we obtain tlie following equation

$$\sqrt{q^2-2E}=\frac{d}{dx}\ln D_{E-0.5}(\sqrt{2}(x-q))\Big|_{x=0}. \qquad (17)$$

whose solutions lead to the electron eigenvalues E $=E_n(q)$ with tlie corresponding wavefunction $\psi_{nq}(x)$.

Once we know the eigenvalues and the corresponding wavefunction we can obtain the other characteristics of tlie bound states. The average electron velocity of tlie hound state along the inagnetic step (y-direction)

$$-v_n(q)\equiv\int_{-\infty}^{\infty}dx\psi_{nq}^2(x)(q-A(x))=\frac{d}{dq}E_n(q), \qquad (18)$$

wliere tlie minus sign results from the definition q $=-k_y$. Tlie mean electron position along the z-axis $X_n(q)$ has the simple form

$$\begin{aligned}X_n(q) &= \int_{-\infty}^{\infty}dx\,x\psi_{nq}^2(x)\\ &= q+\left(1+\frac{1}{2q\sqrt{q^2-2E_n(q)}}\right)v_n(q). \qquad (19)\end{aligned}$$

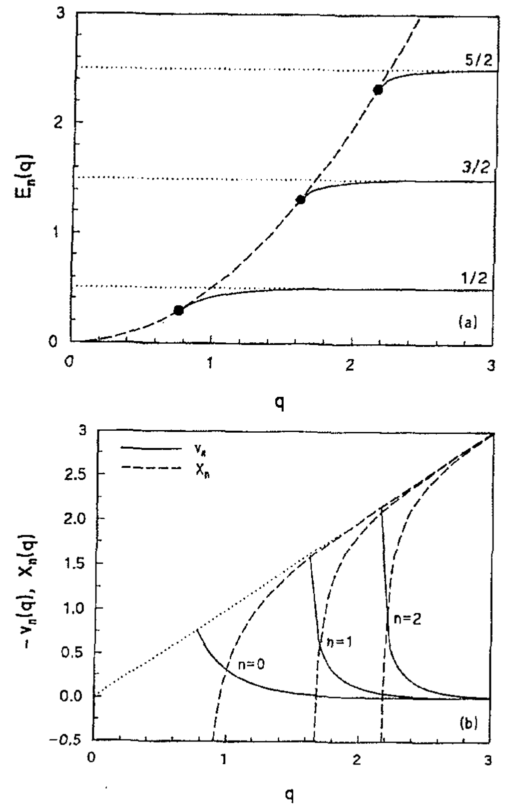

Figure 3: a) tlie energy spectrum for tlie bound states (solid curves), and b) the corresponding average velocity along the rnagnetic step $v_n(q)$ (solid curves) and tlie electron avcrage position $X_n(q)$ along the x-axis (dashed curve) for tlie magnetic step case.

The numerical results of tlie solution of Eq. (17) are depictecl in Fig. 3(a) by the solid curves, for tlie lowest three eigenvalues. Tliese curves start at a certain q-value (denoted by tlie solid dot in Fig. 3), which is a function of n. The corresponding results for the average electron velocity $v_n(q)$ (solid curves) and mean electron position $X_n(q)$ (dashed curves) are shown in Fig. 3(b). Notice tliat the eigenvalues asymptotically, i.e. $q \rightarrow \infty$, reach the values $(n + 1/2)$ for Landau levels in a.homogeneous magnetic fielcl as it should be. In this asymptotic limit tlie mean electron position approaclies $X_n(q) \sim q$ and the average electron velocity tends to zero. In this limit tlie electron is situated far from the magnetic step and is not influenced by the $x < 0$ region. Witli decreasing q tlie electron wavefunction starts to feel tlie magnetic step: 1) its energy decreases, because part of the wavefunction will be situated in a region with zero magnetic field where the electron will have a smaller kinetic energy, 2) its average position is less than q, because the wavefunction is sucked into the x < 0 region, and 3) its velocity increases and the electron runs along the step. From Figs. 3(a) and 3(b) we notice tliat the width of the transition region, i.e. tlie q-region where E„ < $(n + 1/2)$, is narrower witli increasing n. The above properties of these bound states forces us to make the analogy with edge states[32]. Nevertheless there are a number of differences: 1) tlie available q-space for edge states increases with increasing Landau level number n which is opposite to the behavior of tlie present bound states, 2) tlie direction of the velocity is opposite as compared to those of the usual edge states: and **3)** the magnitude of tlie velocity satisfies $|v_n(q)| \leq q$ which is different from edge states which do not have an upper bound on their velocity.

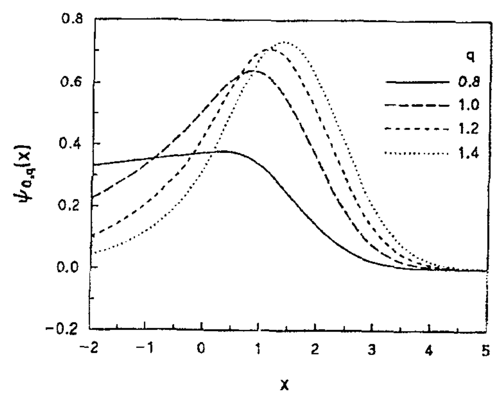

Figure 4: The electron wavefunction for the lowest bound state for different values of the electron momentum in the y-direction (q) in tlie case of a magnetic step.

From Fig. **3** we notice that there exist critical values q_n^* such that for $q < q_n^*$ no bound states are found. Tliese points are indicated by the dots on Fig. 3(a) and are situated on the free electron spectrum curve $E = q^2/2$ (dashed curve in Fig. 3(a)). For the plotted curves we found the critical values: $q_0^* = 0.768$, $q_1^* = 1.623$. $q_2^* = 2.155$ at which the eigenenergy curve $E_n(q)$ is tangent to the $E = q^2/2$ curve. At these points the electron velocity equals the free electron value: $-v_n = q$, and $X_n(q) \rightarrow -\infty$. The electron wavefunction $\psi_{n,q}(x)$ is shown in Fig. 4 for the n = 0 case ancl clifferent values of the wavevector q. This figure nicely illustrates the increasing leakage of the wavefunction into tlie $x < 0$ region with decreasing q-value and the concomittant increasing asymmetry of tlie wavefunction.

Notice that in the present magnetic step case tlie transmission coefficient is always zero. Independent of the strengtli of the magnetic field and tlie magnitude of tlie electron energy an electron, impeding on tlie magnetic barrier will always be reflected, which is a consequence of the Loreiitz force acting on tlie electroii. In this respect this system is different from the textbook potential step problem in which the reflection coefficient becomes different from zero when tlie electron energy is larger than the potential barrier height.

V. Magnetic barrier

The magnetic step can be used as a building block, from which more complicated structures can be build. As a first example we consider tlie magnetic barrier in which tlie magnetic field is different from zero in a strip of width d. In this case the magnetic field has the following form in dimensionless units: $B(x) = \theta(d^2/4 - x^2)$, and we choose the vector potential as follows: $A(x) = -d/2$ ($x < -d/2$), x ($|x| \leq d/2$), $d/2$ ($x > d/2$). Tlie analogous potential $V(x)$ of (14) depends on tlie value of the wavevector q: wlien $|q| < d/2$ the potential consists of an asymmetric well of finite height, and when $|q| > d/2$ it is a gradual step. The problem is symmetric under tlie substitution $q \rightarrow -q$ (and $x \rightarrow -x$) and consequently we may liinit ourselves to tlie case $q \geq 0$.

There are tlirec clifferent energy regions important to us: 1) $0 \leq E \leq (d/2-q)^2/2$ where bound eigenstates can exist, 2) $(d/2-q)^2/2 < E \leq (d/2+q)^2/2$ whicli is the reflection region, and **3)** $(d/2+q)^2/2 < E$ where the electron is transmitted through tlie magnetic barrier.

First let us concentrate on the situation in which we have bounded electron states. In this case the electron wavefunction in the barrier regioii, i.e. $|x| < d/2$, is a linear combination of Weber functions

$$\psi(x) = aD_{E-0.5}(\sqrt{2}(x-q))+bD_{E-0.5}(\sqrt{2}(q-x)) \quad (20)$$

which we must match (and its first derivative) to the free electron wavefunctions at the points $x = \pm d/2$. This matching results into the equation

$$F^+(-q+d/2)F^-(q+d/2)-G^+(q-d/2)G^-(-q-d/2) = 0 \quad (21)$$

where

$$F^{\pm}(z) = \sqrt{(q \pm d/2)^2/2 - E}\,D_{E-0.5}(\sqrt{2}z)+D'_{E-0.5}(\sqrt{2}z) \quad (22)$$

and

$$G^{\pm}(z) = \sqrt{(q \pm d/2)^2/2 - E}\,D_{E-0.5}(\sqrt{2}z)-D'_{E-0.5}(\sqrt{2}z). \quad (23)$$

Eq. (21) was solved numerically. The results for a wide magnetic barrier ($d = 5$) are shown in Fig. 5 by the solid curves which end at the solid dots. The latter are situated on the $E = (q - d/2)^2/2$ curve (dashed curve). Notice that the spectrum resembles the one of the magnetic step case (see Fig. 3(a)) with the distinction that the latter has an infinite number of branches while the one for a magnetic barrier has a finite number of bound states for each q. For $d = 5$ there are only three branches in tlie energy spectrum. The number of energy branches decreases with decreasing barrier width d. Irrespective of the value of d there is always *at least one* cliscrete energy value for $q = 0$. This is a consequence of the fact that for $q = 0$ the potential $V(x)$ is one-dimensional and symmetric. Such a potential is known to have at least one discrete eigenvalue[30] irrespective of tlie size of the potential well. The value of the lowest branch in the spectrum is plotted in Fig. 6 for $q = 0$ as function of the barrier width d. Notice that wlien $d < 1$ (i.e. when the magnetic barrier width is less than the magnetic length l_B) the eigenvalue approaches $E_0(q = 0) \approx (d/2)^2/2$ which is shown by the long-dashed curve in Fig. 6. Although the electron is bound to tlie barrier, in the case of small d-values the electron wavefunction is situated mainly outside the barrier and consequently its energy approaches the height of the potential well $V(d/2)$. The width in q-space (Δq) of the lowest energy branch is also given in Fig. 6. It is seen that this width decreases rapidly to zero when $d < 1$ and in thc opposite case (when $d \to \infty$) it asymptotically reaches the line $\Delta q/2 = d/2 - q_0^*$ (short-dashed line). Where $q_0^* = 0.768$ is the value as obtained froin tlie magnetic step spectrum. Another distinction as compared to tlie magnetic barrier spectrum (see Fig. 3(a)), is that the energy eigenvalues are smaller in magnitude than those in the magnetic step case.

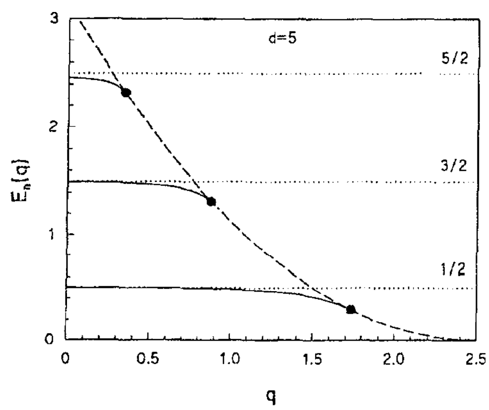

Figure 5: The energy spectrum for tlie bound states (solid curves) in a magnetic barrier of width $d = 5$. Dashed curve $E = (d/2 - q)^2/2$ indicates tlie free electron spectruiii.

For tlie unbounded states we have calculated the transmission coefficient which now depends not only on the electron energy but also on tlie electroii wavevector q in the y-directioii. In tlie present case tunneling is a two-dimensional process in which the total electron wavevector and tlie electron energy is conserved but the direction of tlie wavevector is altered. A contour plot of the transmission coefficient $T(q, \mathrm{E})$ versus initial electron velocity components (v_x, v_y) is shown in Fig. 7 for a magnetic barrier of width $d = 5$. The quantum transition coefficient is zero above tlie line $v_y = (v_x^2 - \mathrm{d}^2)/2\mathrm{d}$ which is the result one would obtain from classical mechanics and which defines a semi-infinite transmission window. Below this line we have classically $\mathrm{T} = 1$, but quantum mechanically $T(q, \mathrm{E})$ gradually increases with increasing electron cnergy. For rather thick barriers (as in the case of $\mathrm{d} = 5$) there is some additional structure at low energy which is enlarged in the inset of Fig. 7. There is an additional peak around $(v_x, v_y) = (0.3, -2.5)$ which is a consequence of tlic presence of a virtual energy level above the quantum well $V_q(x)$.

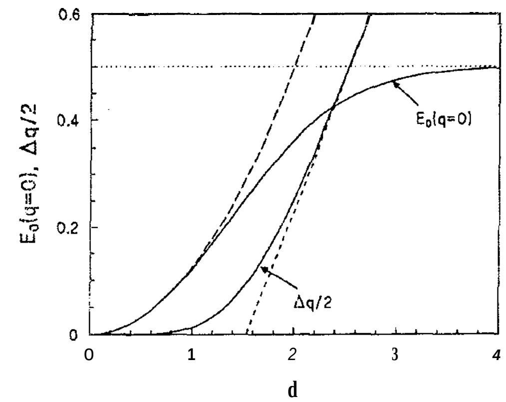

Figure 6: Tlie lowest eigenvalue of the bound state in a magnetic barrier as function of the barrier width for $q = 0$ and the width (Δq) of tlie lowest energy branch in q-space (solid curves). The long-dashed curve indicates the height of tlie potential $V_{q=0}(x = d/2) = \mathrm{d}^2/8$ and the short-dashed line $\Delta q/2 = d/2 - q_0^*$ indicates the asymptotic value of that width defined from tlie magnetic step spectrum.

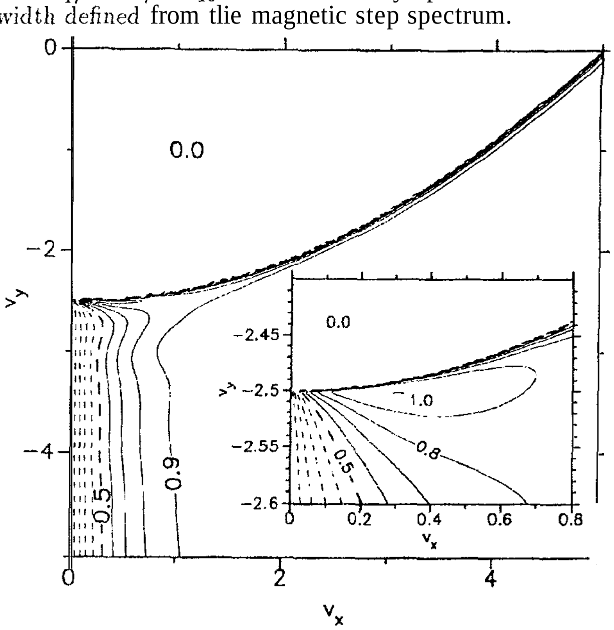

Figure 7: Contour plot of the transmission coefficient through a magnetic barrier of width $d = 5$ in the incident electron velocity (v_x, v_y)-space.

VI. Magnetic wire

The inverse situation of the previous problem is the magnetic well case which we will discuss now. Because of the essential 2D character of the electron motion in a magnetic field we should rather speak of a magnetic wire. In dimensionless units the magnetic field is given by: $B(x) = 0$ ($|x| \leq d/2$), 1 ($|x| > d/2$), aiid the corresponding vector potential is: $A(x) = x - d/2$ ($x > d/2$), 0 ($|x| \leq d/2$), $x + d/2$ (z $< -d/2$). Thc value of the vector potential is now unbounded, i.e. $A(x)|_{x\to\pm\infty} \to \pm$m: and as a consequence tlie potential satisfies: $V(x)|_{x\to\pm\infty} \to \infty$ which implies that the electroii motion is confined in tlie x-direction and *all the states are bound* at least in this direction.

The corresponding wavefunctions are constructed by matching the quasi-free electron wavefunction in tlie region $|x| < d/2$ with tlie Weber functions: $\psi(x) = D_{E-0.5}(\pm\sqrt{2}(x \mp d/2 - q))$, which are valid in the regions $|x| > d/2$. This matching of the wavefunction and its first derivative leads to the following algebraic equation for tlie eigenvalues

$$\left[\cos(kd)D_{E-0.5}(\sqrt{2}q) - \frac{\sqrt{2}}{k}\sin(kd)D'_{E-0.5}(\sqrt{2}q)\right]D'_{E-0.5}(-\sqrt{2}q)$$
$$+\left[\cos(kd)D'_{E-0.5}(\sqrt{2}q) + \frac{k}{\sqrt{2}}\sin(kd)D_{E-0.5}(\sqrt{2}q)\right]D_{E-0.5}(-\sqrt{2}q) = 0\ , \qquad (24)$$

where $k = \sqrt{2E - q^2}$ for $2E > q^2$ and $k = i\sqrt{q^2 - 2E}$ for $2E < q^2$ in which case the trigonometric functions should be replaced by their corresponding hyperbolic functions.

The results of tlie numerical solution of this equation are presented in Figs. 8(a) for a wide well (i.e. d = 5), and (h) for a narrow well (i.e. d = 1). In the wide well case (Fig. 8(a)) there are clearly two distinct regions which are separated by the free electron eiiergy $E = q^2/2$ curve (dashed curve in Fig. 8(a)). For $E \ll q^2/2$ the energy spectrum consists of Laiidau levels. Thc electron is mainly located in tlie barrier where there exists a. uniform magnetic fielcl. For small q-values, i.e. $E \gg q^2/2$, the spectrum consists of baiids with free electron-like motion in the y-direction. This is similar to the case of the well-known quantum wire with electrical potential barriers. When we decrease the width of tlie well the two regions are less clistiiict as is apparent in Fig. 8(b) for the case of d = 1. For $d = 1$ tlie well is narrower than the width of tlie electron wavefunction and consequently there is always an appreciable overlap of the wavefunction with tlie magnetic barrier regioii. Notice that tlie energy levels have almost no dispersion. The different behavior between tlie two cases is also illustrated in Fig. 9 where the electron velocity is shown for the different states. Notice that tlie velocity exliibits a niaximum near $E = q^2/2$ and it diminishes fast for $q \gg \sqrt{2E}$ which is the region where the electron is mainly located inside the magnetic barrier. Notice that for wide wells, i.e. see the d = 5 case, the velocity curve $v_n(q)$ can have several local maxima's which is a consequence of the repulsion of tlie different energy levels as seen in Fig. 8(a). In the case of tlie usual quantum wiie constructed from walls consisting of potential barriers the electroii velocity is $v_n = \hbar k_y = -q$ and is independent of the energy level index n and is a uniform increasing function of the electron wavevector. The behavior of $v_n(q)$ as depicted in Figs. 9 is also different from the one of edge states in which $v_n(q)$ is a uniform increasing function of q.

The density of states (DOS) for the two cases is depicted in Fig. 10. Notice that like for the quantum wire case tlie DOS exhibits singularities at the onset of each energy level. But there is a difference, the width in energy space of each level is finite and bounded by a singularity in thc DOS. Suppose we have a systeni in

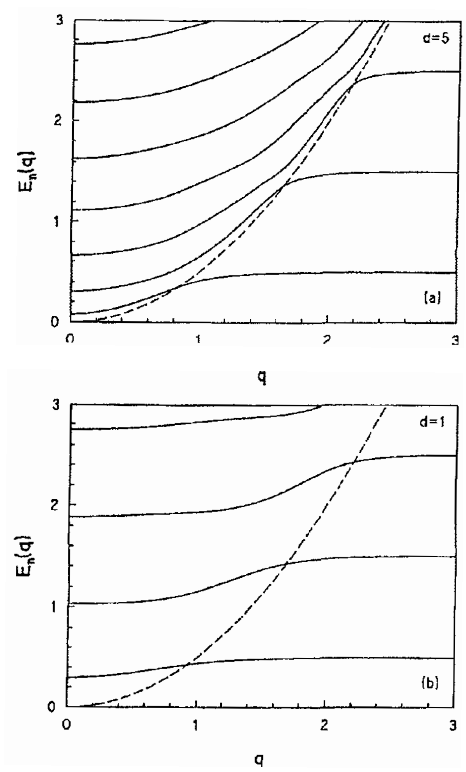

Figure 8: The energy specruin of a magnetic well for two clifferent values of the width: a) $d = 5$, and b) $d = 1$.

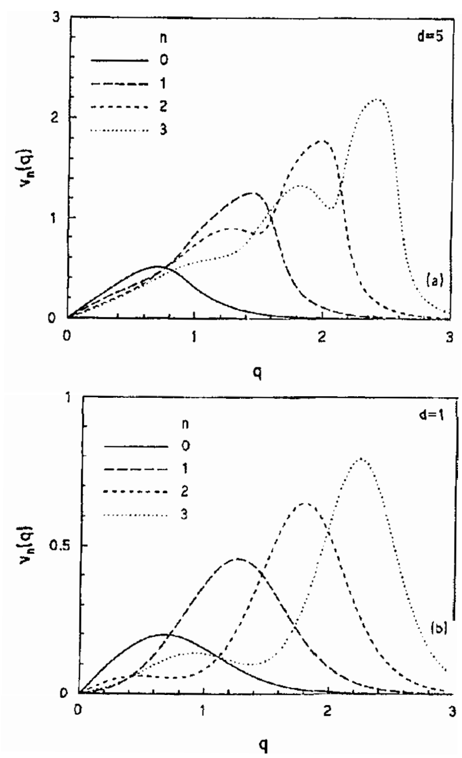

Figure 9: The electron average velocity corresponding to the energy spectrum of Fig. 6.

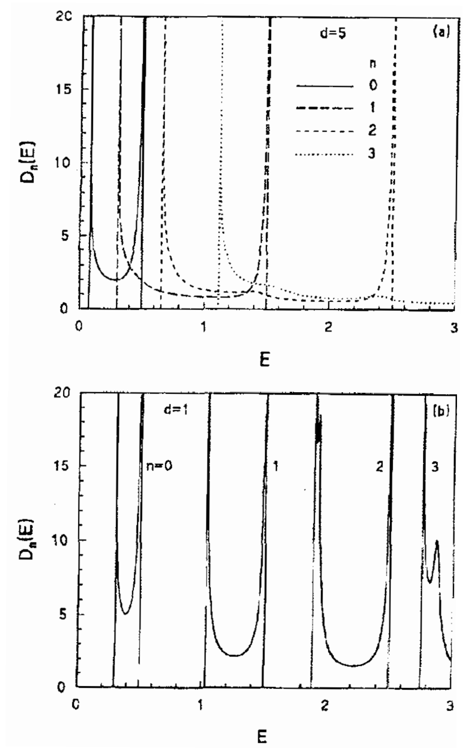

Figure 10: The density of states of the electron states in the magnetic wells correspoiiding to Fig. 6.

which we are able to increase the Fermi energy gradually. Starting from zero we first populate the quantum wire states, the electrons are mainly situated in the well region. Further increasing the Fermi energy we see that for d = 5 we first start to populate the next energy level which consists initially of states located inside the well. For $d = 1$ on the other hand we start to populate states which are situated in the magnetic barrier region and which are nothing else then 2D Landau states. Thus by changing the Fermi level we are able to have 1D states or 2D states at the Fermi level which will have considerable influence on the electrical properties of the system. The 1D states are quasi-free while the 2D states are localized on Landau orbits and can only move if scattering is involved.

VII. Resonant tunneling structures

In previous sections we have made a detailed study of the nature of the electron states in different magnetic barrier structures. In this section we will consider different tunneling structures where we will focus on the tunneling current going through it.

For simplicity we now consider electron tunneling through a. magnetic barrier of constant height B_0 and width $d = x_+ - x_-$ surrounded by regions of zero magnetic field. The free electron wavefunction on the left side of the barrier (z < x_-) is $\psi_-(x) = Ae^{ik_-(x-x_-)} + Be^{-k_-(x-x_-)}$ and on the right side of it (z > x_+) $\psi_+(x) = e^{ik_+(x-x_+)}$, where $k_\pm = \sqrt{2[E - V(\pm\infty)]}$ is the x component of the electron wavevector on the corresponding side of tlie barrier. Under tlie barrier there are two solutions for $\psi(x)$ which can be written as a linear combination of tlie Weber function $D_p(x)$ and its derivative D_p'(z). Next we construct tlie transition matrix

$$T(x, x_0) = \begin{pmatrix} u(x) & v(x) \\ u'(x) & v'(x) \end{pmatrix}, \qquad (25)$$

where we defined the functions $u(x) = c\{D_p'(\sqrt{2q})D_p(z) + D_p'(-\sqrt{2q})D_p(-z)\}$ and $v(x) = c\{D_p(\sqrt{2q})D_p(z) - D_p(-\sqrt{2q})D_p(-z)\}$, with p = E − 1/2 and $z = \sqrt{2}(x - q)$, which satisfies the boundary conditions $u(x_0) = 1$, $u'(x_0) = 0, v(x_0) = 0$, and $v'(x_0) = 1$. Matching the wave function at the edges of the barrier, $x_\pm$, by means of the above matrix we obtain

$$A = T_{11}^{-1} + \frac{k_+}{k_-}T_{22}^{-1} + i\left(\frac{1}{k_-}T_{21}^{-1} - k_+T_{12}^{-1}\right); \qquad (26)$$

the electron transmission through the barrier $t(E, q)$ is given by

$$t(E, q) = \frac{k_+}{k_|A|^2}, \qquad (27)$$

where T^{-1} stands for tlie inverse of the matrix T r $T(x_+, x_-)$. For complex structures involving several barriers of constant height, the total T matrix is a product of the T matrices that correspond to tlie separate barriers and the one describing tlie free electron propagation between tlie barriers. As for tlie electron current through such a structure! it can be calculated, in the ballistic regime, by introducing the conductance G as the electron flow averaged over half the Fermi surface[33]

$$G = G_0 \int_{-\pi/2}^{\pi/2} t(E_F, \sqrt{2E_F}\sin\phi)\cos\phi d\phi, \qquad (28)$$

where ϕ is the angle of incidence relative to tlie x direction. Further, $G_0 = e^2 m v_F \ell/\hbar^2$, where E is the length of tlie structure in the y direction and v_F the Fermi velocity.

To reveal the main qualitative features of tunneling through these barriers we restrict ourselves to: *i) a single* barrier which was already discussed in Sect. V, and *ii)* complex *structures* composed of rectangular magnetic barriers one example of which is shown in the inset of Fig. 11.

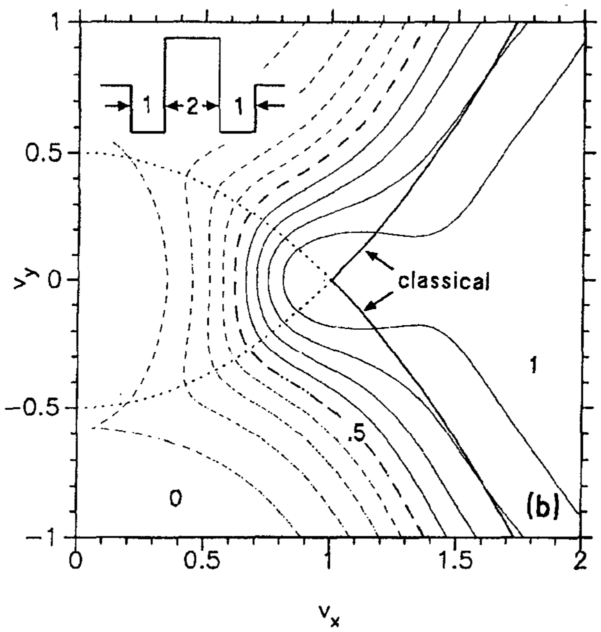

Figure 11: Contour plot of the electron transmission probability in the (v_x, v_y) plane for a more complex structure. The magnetic field profile of the corresponding magnetic barrier are shown in the inset of the figure.

The contour plot of tlie transmission through a complex structure, shown in tlie inset of Fig. 11, is presented in the figure. Notice that the quantum and the classical calculation give drastically different results.

This complex structure can be used as a building block to make a double barrier-like structure which is composed of two units identical to that of Fig. 11 with a zero field region, of length L = 3, between them. We sliow only the velocity contour plot in Fig. 12 and the corresponding classical result. Again we see sharp resonances, the wavevector filtering properties, and the strong disimilarity between the quantum and classical results.

Having seen tlie transmission results, one may wonder to what extent their structure is reflected in measurable quantities which involve some kind of averaging. In Fig. 13 ive show the conductance, as given by Eq. (28), for the previous tunneling structure shown ill tlie inset of tlie figure, together with tlie corresponding classical result(dotted curves). Despite the averaging of $t(E, q)$ over lialf the Fermi surface, we have again strong resonant structure. This structure will become sharper if one can select the wavevectors that give the sharpest resonance in the transmission. In principle this can be achieved using quantum point contacts. As for the classical result, we see again that they are determined only by the first barrier in each structure.

Although our consideration of electron tunneling through the rectangular magnetic barrier structures gives only a qualitative picture, nevertheless these resonant tunneling spikes should be present in the more realistic cases with barriers of smooth shape, cf. Fig.2. Indeed these spikes do not depend on Lhe actnal shape of the magnetic barrier but only on the presence of barriers in tlie potential $V(x)$.

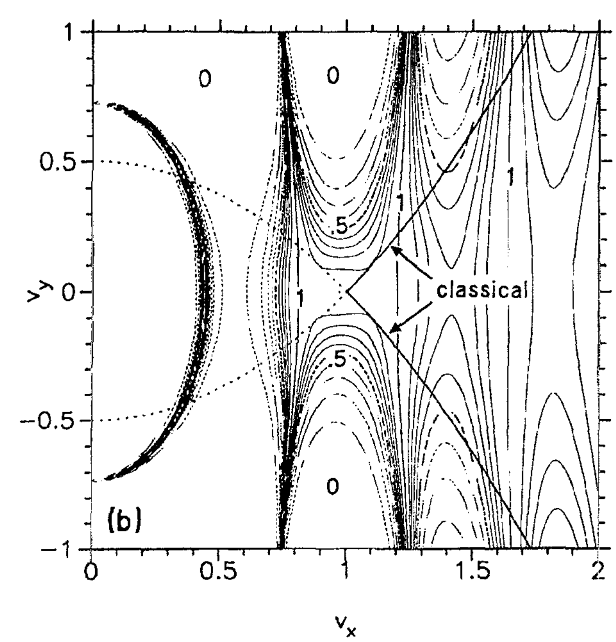

Figure 12: Contour plot of the electron transmission probability in the (v,, v,) plane for tlie resonant tunneling structure composed of the complex barrier structure of Fig. 11 in which tlic barricrs arc separated by the distancc L = 3.

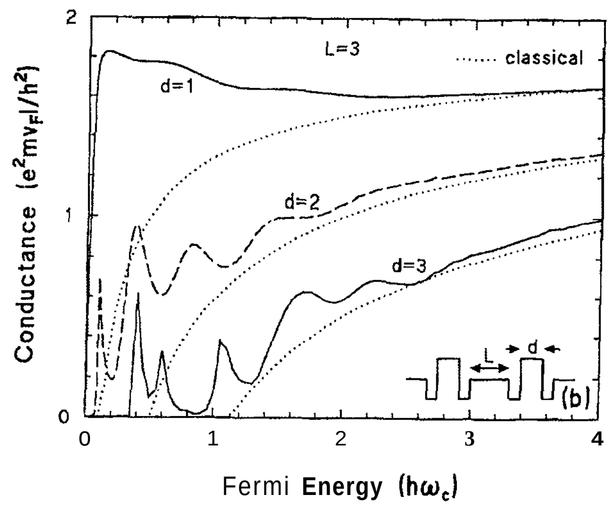

Figure 13: The conductance tlirough the barrier structure shown in the inset for different values of the barrier parameters. Tlie dotted curves show the conductance calciilated classically.

VIII. Conclusion

The spectra of electrons rnoving in 2D and interacting with nonhomogeneous magnetic fields was calculatcd. Different structures of nonhomogeneous magnetic fields in one clirection are considered. The similarities and differences between similar structures built from electrical potentials are pointed out. The motion in the present case is essential 2D while in the electrical potential problems often a separation of variables is possible which reduces the problem to 1D. In the present case the problem can be mathematically cast into a 1D problem but the physics and the motion stays essentially 2D. In the magnetic case the potential $V(x)$ appearing in the mathematical 1D problem depends on the electron wavevector (q) which maltes it inherently two dimensional even in the case of one-dimensional magnetic field modulations.

One of the interesting features of nonhomogeneous magnetic field structures is that a step in the magnetic field can bind electrons. The spectrum has bounded and unbounded (scattered) states. The wavefunction of tlie former are confined to the region with non-zero magnetic field. The discrete and continuum part of the spectrum overlap in an energy range. This is essentially different from potential steps which act always repulsively. As a consequence magnetic barriers can exhibit bound states and tunneling through them turns out to be much richer: for example tunneling can occur through such bound states which may lead to quasi resonances ir the traiismission coefficient. Tunneling is essentially a 2D process where only transmission is possible in a semi-infinite window in velocity space. Such a magnetic barrier structure can be usecl as a filter for electron wavevectors. A combination of such magnetic barriers will result in more coinplicated structures, like for example resonant tunneling structures and superlattices.

We found that the quantum transmission through magnetic-barrier structures: i) depends not only on the energy but also on the direction of the wavector, ii) possesses wavevector filtering properties, iii) shows well-pronounced resonances whereas the classical one does not, aiid iv) is drastically different frorn the classical transmission which is determined only by the sum of the barriers and is independent of the distance L between them.

We havc shown that the pliysics of electron transport in nonhomogeneouos magnetic fields is a rich subject. Furthermore one can think about creating magnetic dots, tlie theoretical analysis of which is in progress. Other possible systems are magnetic superlattices. In this case we may distiiict: 1) weak magnetic superlattices in which tliere is only a very weak modulation of the nagnetic field. Tliis problem was studied in Ref. [22] in which Weiss oscillations were predictecl in the magneto-resistance which are a. consequence of a commensurability between the period of the superlattice and the diameter of the cyclotron orbit, and 2) strong magnetic superlattices in which we may have: a) tlie situation of alternating magnetic wells and magnetic barriers such that tlie average magnetic field is zero. This system is now similar to the Kronig-Penney model, and b) the case with only magnetic barriers. Now the aveiage magnetic field is non zero and as a consequence the electric potential $V(x)$, Eq.(14), is unbounded and all the states will be localized in the direction of the superlattice. The study of these problems is in progress.

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