Plasmon-Polaritons Spectra in Quasi-Periodic Semiconductor Superlattices

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A comparative theoretical analysis is carried out for the spectra of plasmon-polaritons of multiple semiconductor layers in quasi-periodic arrangements obeying the Fibonacci and Thue-Morse sequences, respectively. We consider building blocks composed of two-dimensional electron gas (2DEG) separated by semiconductor media of alternating thicknesses and dielectric functions. The spectra of both bulk and surface plasmon-polaritons are conveniently derived by using a transfer matrix treatment with a model frequency-dependent dielectric function including the effect of retardation.

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As a result of recent advances in experimental techniques there is a continuing interest to investigate the properties of collective excitations, such as plasmon-polaritons modes, in multilayered systems (for a review see Ref. 1). In particular the physical properties of a new class of artificial layered media, the so-called quasi-periodic superlattice, have also attracted a lot of attention recently. The fabrication of such structures was pioneered by Merlin et al.[2], who grew a quasi-periodic GaAs-AlAs superlattice from two distinct building blocks, each having one or more layers of different materials with different thickness, arranged according to a Fibonacci sequence. Theoretical plasmon-polaritons spectra in these structures were recently successfully reported by the authors[3,4].

In this communication we compare our results for the spectra of both bulk and surface plasmon-polaritons in Fibonacci superlattices to another type of quasi-periodic structure which obeys the Thue-Morse sequence. This structure has been reported by Humlicek et al.[5], who presented measurements of the reflectance spectra for the Thue-Morse GaAs-AlAs quasi-periodic layered films deposited on the GaAs substrate. They were concerned with the electron-hole excitations in this structure. Later, Tao and Singh[6] presented a comparative theoretical analysis of the reflectance and transmission spectra of multiple layers arranged in the periodic and the Thue-Morse quasi-periodic pattern, with a good agreement between their theory and the experimental spectra obtained by Humlicek et al.

We consider the two building blocks α and β as shown in Fig. 1, to set up the quasi-periodic superlattice. Each block consists of a two-dimensional electron gas (2DEG) charge sheet and a layer of medium **A** or **B**, which may have different thicknesses a and b, and dielectric function $\epsilon_A(\omega)$ and $\epsilon_B(\omega)$. They may also have a volume density of charge. The Fibonacci sequence is here described in terms of a series of generations that obey the following recursion relations:

$$S_n = \,,\ _{-1},\ _{-2}. \quad \text{for} \quad n \geq 2 \qquad (1)$$

with $S_0 = \beta$ and $S_1 = \alpha$.

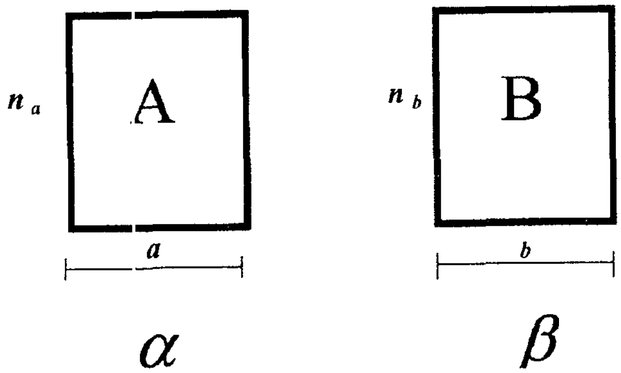

Figure 1: The two building blocks α and β of the quasi-periodic superlattice. Here A and B are dielectric layers, with thickness a and b, and the 2DEG layers have carrier concentrations n_a and n_b per unit area.

On the other hand, the Thue-Morse sequence is arranged according to the recursion relations:

$$\begin{aligned} S_n &= S_{n-1} S_{n-1}^T \quad for \quad n \geq 1 \\ S_n^T &= S_{n-1}^T S_{n-1} \end{aligned} \tag{2}$$

with $S_0 = \alpha$ and $S_0^T = \beta$.

In order to find the bulk and surface plasmon-polaritons spectra, we follow the general formalism of Ref. 7, where a convenient description of the spectra is carried out by using a transfer matrix treatment to simplify the algebra, which can be otherwise quite involved. We consider a semi-infinite superlattice structure where the cartesian axes are chosen in such a way that the z-axis is normal to the plane of the layers. The structure is terminated at the plane $z = 0$, with the half-space $z < 0$ filled with a material that has a frequency-independent dielectric function ϵ_S.

Let us assume that the electromagnetic mode is p-polarized in the absence of any external magnetic field. The 2DEG charge sheet at each interface is considered to be due to the presence of a current density

$$J_{jx} = i\omega\epsilon_0\sigma_j E_{jx}, \tag{3}$$

where

$$\sigma_j = \frac{n_j e^2}{m_j^* \omega^2 \epsilon_0}, j = a \text{ or } b \tag{4}$$

Here, n_j is the carrier concentration per unit area, e is the electronic charge, and m_j^* is the effective mass of the charge carrier; ϵ_0 is the vacuum permittivity.

For the periodic system $\alpha\beta\alpha\beta\ldots$, the bulk dispersion relation is simply given by[7]:

$$\cos(QL) = Tr(\overleftrightarrow{\mathbf{T}}_{\alpha\beta}) \tag{5}$$

where Q is the Bloch wavenumber, $L = a + b$ is the size of the periodic superlattice unit cell, and $\overleftrightarrow{\mathbf{T}}_{\alpha\beta}$ is a unimodular transfer matrix defined by:

$$\overleftrightarrow{\mathbf{T}}_{\alpha\beta} = \overleftrightarrow{\mathbf{N}}_a^{-1} \overleftrightarrow{\mathbf{M}}_b \overleftrightarrow{\mathbf{N}}_b^{-1} \overleftrightarrow{\mathbf{M}}_a \tag{6}$$

Here, the matrices $\overleftrightarrow{\mathbf{N}}_j$ and $\overleftrightarrow{\mathbf{M}}_j$ ($j = a, b$) are defined by:

$$\overleftrightarrow{\mathbf{N}}_j = \begin{vmatrix} 1 & 1 \\ \epsilon_j' - \sigma_j & -\epsilon_j' - \sigma_j \end{vmatrix} \tag{7}$$

$$\overleftrightarrow{\mathbf{M}}_j = \begin{vmatrix} f_j & f_j^{-1} \\ \epsilon_j' f_j & -\epsilon_j' f_j^{-1} \end{vmatrix} \tag{8}$$

where:

$$\epsilon_j' = \epsilon_j / \alpha_j \tag{9}$$

$$\alpha_j = \begin{cases} [k_x^2 - \epsilon_j(\omega/c)^2]^{1/2} & \text{if} \quad k_x > \sqrt{\epsilon_j}\omega/c \\ i[\epsilon_j(\omega/c)^2 - k_x^2]^{1/2} & \text{if} \quad k_x < \sqrt{\epsilon_j}\omega/c \end{cases} \tag{10}$$

In general, we assume for media **A** and **B** that ϵ_j may be frequency dependent, having the form:

$$\epsilon_j = \epsilon_{\infty j}[1 - \{\omega_{pj}^2 / \omega(\omega + i\Gamma_j)\}] \tag{12}$$

where ω_{pj} is the plasma frequency, Γ_j is a damping factor, and $\epsilon_{\infty j}$ is the background dielectric function of the $j-th$ cell.

The implicit dispersion relation for the surface modes can now be obtained through the application of the usual electromagnetic boundary conditions at $z = 0$ and at each of the interfaces between layers to yield[7]:

$$T_{11} - T_{22} + T_{12}\lambda - T_{21}\lambda^{-1} = 0 \tag{13}$$

with

$$\lambda = \frac{\epsilon_a' + \epsilon_s'}{\epsilon_a' - \epsilon_s'} \tag{14}$$

Here T_{ij} are the elements of the $\overleftrightarrow{\mathbf{T}}_{\alpha\beta}$ matrix

For quasi-periodic systems, the general results (5) and (13) for the bulk and surface plasmon-polaritons modes still hold, provided a suitable transfer matrix is considered. For the Fibonacci sequence, these transfer matrices are[3]:

$$\begin{aligned}\overleftrightarrow{\mathbf{T}}_{S_1}\ (\overleftrightarrow{\mathbf{T}}_{S_0}) &= \overleftrightarrow{\mathbf{N}}_j^{-1}\overleftrightarrow{\mathbf{M}}_j,\ j = a(b)\\ \overleftrightarrow{\mathbf{T}}_{S_2} &= \overleftrightarrow{\mathbf{N}}_a^{-1}\overleftrightarrow{\mathbf{M}}_b\overleftrightarrow{\mathbf{N}}_b^{-1}\overleftrightarrow{\mathbf{M}}_a\\ \overleftrightarrow{\mathbf{T}}_{S_{k+2}} &= \overleftrightarrow{\mathbf{T}}_{S_k}\overleftrightarrow{\mathbf{T}}_{S_{k+1}},\ \text{for}\ k \geq 1\end{aligned} \tag{15}$$

On the other hand, for quasiperiodic Thue-Morse superlattice we have:

$$\begin{aligned}\overleftrightarrow{\mathbf{T}}_{S_0} &= \overleftrightarrow{\mathbf{N}}_a^{-1}\overleftrightarrow{\mathbf{M}}_a\\ \overleftrightarrow{\mathbf{T}}_{S_1} &= \overleftrightarrow{\mathbf{N}}_a^{-1}\overleftrightarrow{\mathbf{T}}_\beta\overleftrightarrow{\mathbf{M}}_a\\ \overleftrightarrow{\mathbf{T}}_{S_2} &= \overleftrightarrow{\mathbf{N}}_a^{-1}\overleftrightarrow{\mathbf{T}}_\alpha\overleftrightarrow{\mathbf{T}}_\beta\overleftrightarrow{\mathbf{T}}_\beta\overleftrightarrow{\mathbf{M}}_a\end{aligned} \tag{16}$$

and, in general

$$\overleftrightarrow{\mathbf{T}}_{S_n} = \overleftrightarrow{\mathbf{N}}_b^{-1}\overleftrightarrow{\mathbf{T}}_\alpha\overleftrightarrow{\mathbf{T}}_\beta\overleftrightarrow{\mathbf{T}}_\beta\ldots\overleftrightarrow{\mathbf{T}}_\alpha\overleftrightarrow{\mathbf{T}}_\beta\overleftrightarrow{\mathbf{T}}_\beta\overleftrightarrow{\mathbf{M}}_b \quad \text{for}\ n\ \text{odd}\ (n \geq 3) \tag{17}$$

$$\overleftrightarrow{\mathbf{T}}_{S_n} = \overleftrightarrow{\mathbf{N}}_a^{-1}\overleftrightarrow{\mathbf{T}}_\alpha\overleftrightarrow{\mathbf{T}}_\beta\overleftrightarrow{\mathbf{T}}_\beta\ldots\overleftrightarrow{\mathbf{T}}_\beta\overleftrightarrow{\mathbf{T}}_\alpha\overleftrightarrow{\mathbf{T}}_\alpha\overleftrightarrow{\mathbf{M}}_a \quad \text{for}\ n\ \text{even}\ (n \geq 4) \tag{18}$$

Here,

$$\overleftrightarrow{\mathbf{T}}_\gamma = \overleftrightarrow{\mathbf{M}}_j\overleftrightarrow{\mathbf{N}}_j^{-1}. \tag{19}$$

where $\gamma = \alpha$ for $j = a$, and $\gamma = \beta$ for $j = b$.

Now, we present some numerical examples to illustrate the bulk and surface plasmon-polariton spectra. In what follows we assume physical parameters typical of electron concentrations in GaAs-AlGaAs superlattices, and we assume the medium outside the superlattice to be vacuum. We take thicknesses corresponding to $a = 40$ nm and $b/a = 2$. Also we considered the following physical parameters in the numerical calculations: $\epsilon_{\infty a} = 12.9$, $\epsilon_{\infty b} = 12.3$, $n_a = n_b = 6 \times 10^{15}\text{m}^{-2}$, and $m_j = 6.4 \times 10^{-32}$kg $(j = a, b)$. We allow ω_{pa} to be different from zero only in medium A, corresponding to a volume charge density in the medium. The damping factor is taken to be zero in both media.

Figure 2 shows the plasmon-polariton spectra for the Fibonacci sequence S_4. The bulk bands are the

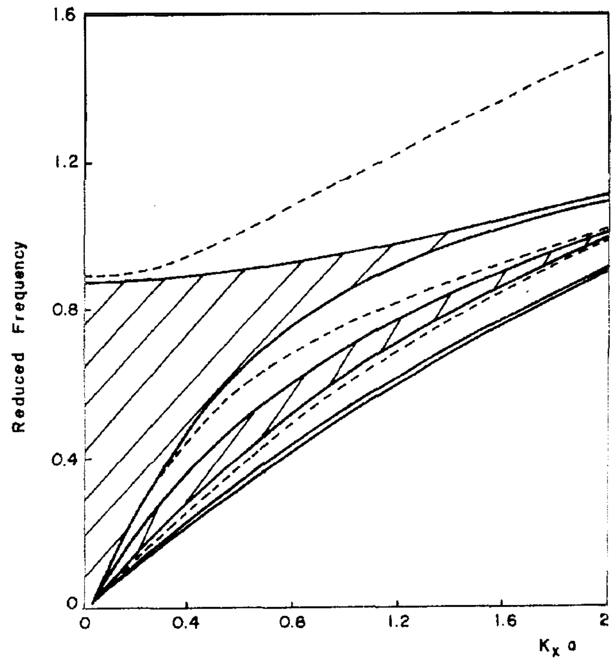

Figure 2: Plasmon-polaritons spectra, p-polarization, for the Fibonacci sequence S_3 'The physical parameters used here are described in tlie main text.

shadowed areas bounded by $QL = 0$ and π, I, being the size of the superlattice unit cell, while the surface modes are represented by dashed lines. Here we plot the reduced frequency ω/Ω against a dimensionless wavevector $k_x a$, where:

$$\Omega = \left(\frac{n_a e^2}{m_a^* \epsilon_0 \epsilon_{\infty a} a}\right)^{1/2} \tag{20}$$

The plasmon-polariton spectra for the quasi-periodic Thue-Morse case are shown in Figures 3 and 4, representing n even (here equal to 2) and odd (here equal to 3), respectively. Again, we have plotted the reduced frequency ω/Ω against the dimensionless factor $k_x a$. We avoid to consider a larger number for n to illustrate oiir theoretical computation, since there is no important qualitative difference among the spectra as a direct implication of the increase of n (the main difference being the increase of the number of bulk bands and surface modes in the spectra, which of course means a more rich spectra, already discussed in previous papers in this subject[8]). However, there are important qualitative differences concerning the lowest plasmon-polariton surface mode in the quasi-periodic

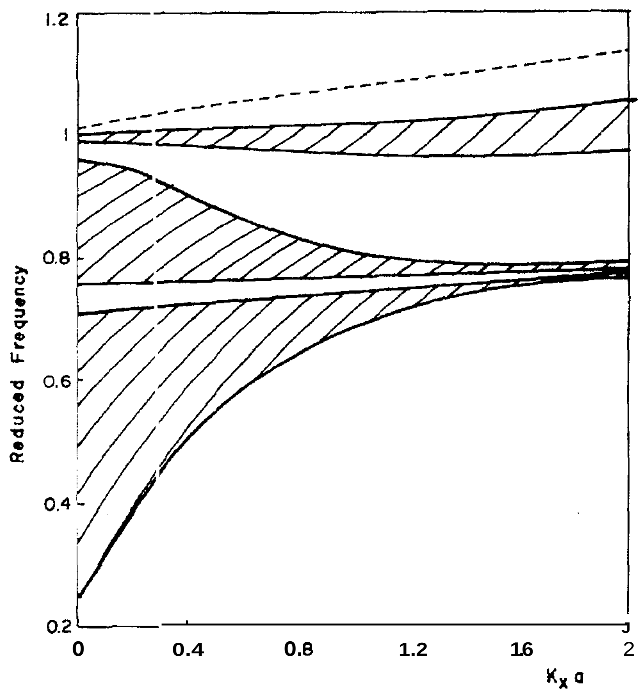

Figure 3: Plasmon-Polaritons spectra, p-polarization, for tlie Thue-Morse sequence S_2, and tlie parameters given ill tlie text.

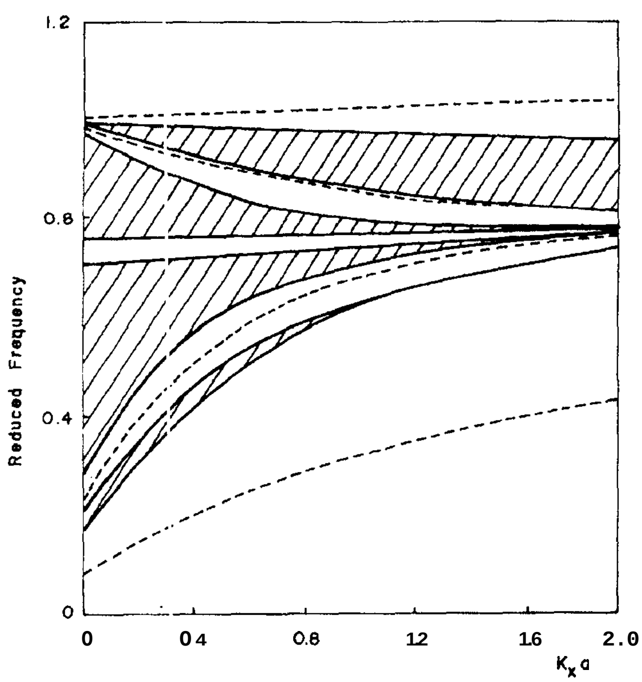

Figure 4: Same as in Fig. 3, but for the Thue-Morse sequence S_3.

Thue-Morse sequence for n odd. In the Fibonacci case and in the quasi-periodic Thue-Morse sequence for n even this mode is quite close to the lowest bulk band, while for n odd it is quite apart from the bulk band. This is due to the fact that when n is odd the first and last layers of S_n are the building blocks a and β, respectively, although for n even the first and last layers of S_n are of the type a.

In summary, we have presented a concise theoretical calculation for the plasmon-polariton spectra in quasi-periodic superlattices following the Fibonacci and Thue-Morse sequences. This work extends our previous papers in this subject[3 4].

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