The Effect of Chaotic Dynamics in Resonant Tunneling'

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Resonant tunneling spectroscopy is used to investigate the energy level spectrum of a wide potential well in the presence of a large magnetic field oriented at angles θ between 0° and 90° to the normal to the plane of the well. The systeni used for the investigation is a double barrier resonant tunneling device incorporating a wide GaAs quantuin well and two (AlGa)As tunnel barriers. In the tilted field geometry, the current-voltage characteristics exhibit a large number of quasi-periodic resonant peaks, even though the classical motion of electrons in the potential well is chaotic. The voltage range and spacing of the resonances both change drainatically with θ. We give a quantitative explanation for this behaviour by considering the classical period of unstable periodic orbits within the chaotic sea of the potential well.

I. Introduction

The quantum mechanical description of classically chaotic systems is one of the most interesting and challenging problems in contemporary physics[1,2]. An impressive body of theoretical work already exists. In particular, the periodic classical orbits which can exist in the chaotic regime have been related to both the energy levels and wavefunctions of the corresponding quantum mechanical picture[1,2]. However, to date the range of experiments studying quantum chaos has been limited. Most experimental work has focused on the spectroscopy of the highly excited states of atoms in high magnetic fields[3] and on the ionisation of atoms in strong microwave fields[4]. In condensed matter physics, recent experiments have explored the effect of chaos on quantum transport[5-7].

In this paper we describe a new and flexible experiment for studying electrons in a quantum mechanically confined system under conditions when their motion is classically chaotic. Fig 1(a) shows two potential barriers, with a uniform electric field I? along tlie normal to the barrier interfaces ($-\vec{E} \parallel \vec{x}$). Electrons moving classically hetween the barriers undergo specular reflection froni the left-hand (einitter) and right-hand (collector) barrier. If a magnetic field $\vec{B}$ is applied at an angle $0 = 0$ or $0 = 90°$ to $\vec{x}$ the electron orbits are periodic. At intermediate tilt angles the electron motion comprises a constant acceleration along B and a cycloidal drift in the orthogonal plane. Specular reflection at the barrier interfaces repeatedly resets the boundary conditions for the cycloidal motion and, when the cyclotron radius is sufficiently small, produces orbital segments which rapidly becorne uncorrelated and are collectively chaotic. For general initial velocity at the emitter barrier, the motion is aperiodic. However, as shown in Fig. 2(a), unstable periodic orbits do occur for certain starting velocities. If the initial velocity is changed by only 0.1%, as in Fig. 2(b), the niotion becomes rapidly irregular. This extreme sensitivity to the initial conditions is the defining characteristic of classical chaos.

At high quantum numbers (n $\geq$ 220) the energy-level spectrum of the potentiai well is very complicated[8] corresponding to the classically chaotic domain. By incorporating the potential well and sur-

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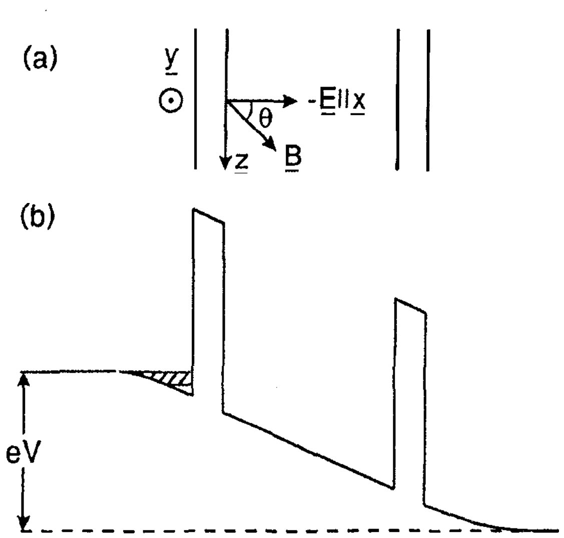

Figure 1: (a) Plan view of tlie classical double barrier system in the tilted field geometry. (b) Schematic conduction band profile of a resonant tunneling diode. Tlie hatched region represents the 2D accumulation layer.

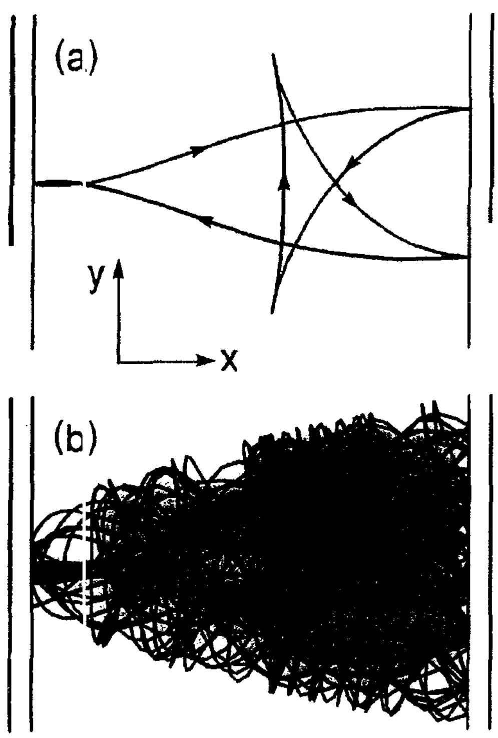

Figure 2: (a) Projection on tlie x-y plane of a star-shaped periodic orbit in a 120 nm wide potential well with $E = 2.3 \times 10^6\ Vm^{-1}$, $B = 11.4T$, and $0 = 20°$. (b) If the starting speed is changed by 0.1%, tlie orbit is irregular over the time interval corresponding to 50 traversals of tlie periodic orbit in (a).

rounding barriers in a double-barrier tunnel structure, the energy level pattern is qualitatively unchanged but we may now probe this spectrum using resonant tunneling spectroscopy.

Our structure consists of a GaAs quantum well of width $w = 120$ nm enclosed by two $Al_{0.4}Ga_{0.6}As$ tunnel barriers of thickness $b = 5.6$ nm (Fig 1(b)). Full details of the layer composition are given elsewhere[9]. Electrons tunnel into the quantum well from the occupied states of a two-dimensional accumulation layer. The resonant peaks in the current-voltage characteristic $I(V)$ are superimposed on a monotonically increasing background which we suppress by taking second derivative plots, d^2I/dV^2. Fig 3 shows measurements of $dG/dV = S^21/dV^2$ for a range of tilt angles with $B = 11.4T$.

At $\theta = 0$, a series of resonant tunneling peaks is observed with a voltage spacing $AV \simeq 30$ mV[10]. The beating, at around 550 mV and 850 mV, is due to an "over the barrier" quantum interference effect at the collector barrier[10]. Oscillatory structure in $I(V)$ persists up to tilt angles of just over 50°, but it decreases in amplitude and changes its periodicity with increasing 0. For $0 \leq 0 < 10°$ the voltage separation AV decreases gradually as $\cos^2\theta$. This behaviour can be understood semiclassically[11]: in a large magnetic field tilted at small angles, the electrons orbit about the inagnetic field direction so that the effective traversal length (effective well width) becomes $w/\cos\theta$. At larger tilt angles the oscillatory structure changes dramatically. In particular, at $\theta = 15°$, the spacing, AV, of tlie oscillations decreases by a factor of ~ 2 to a value of ~ 12 mV at voltages above ~ 460 mV, with the more widely spaced oscillations persisting at lower bias. The boundary between these two regions is very distinct and inoves steadily to lower bias with increasing angle. At $0 \geq 25°$, the shorter period oscillations dominate the $I(V)$ curves in the range of voltage up to 800 mV. For $15° < \theta < 50°$, their period is almost constant. For angles between 50° and 70°, no regular

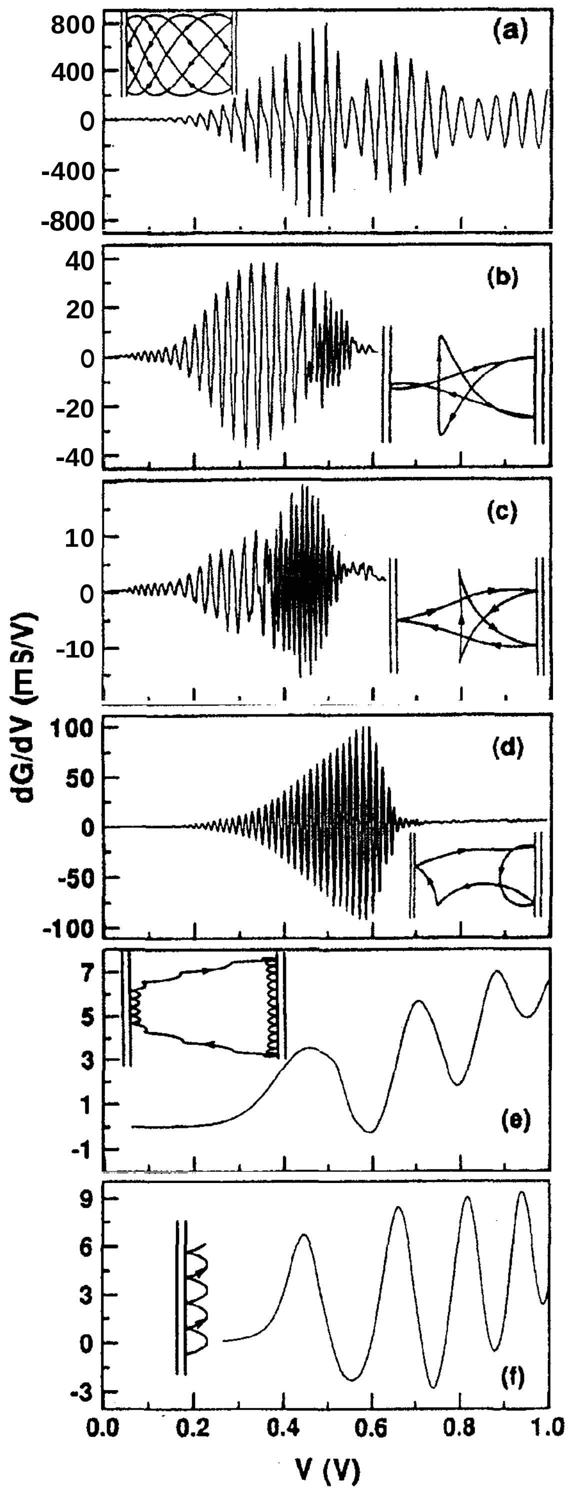

Figure 3: dG/dV plots for a resonant tunneling diode containing a 120 nm wide quantum well with B = $11.4T$ and (a) $\theta = 0°$, (b) $6 = 15°$, *(c)* $6 = 20°$, (d) $\theta = 40°$, *(e)* $6 = 80°$, (f) $\theta = 90°$. The resonances are related to tlie classical periodic orbits (inset) projected onto the x-y plane.

oscillatory structure is observed. For $0 > 70°$ a new series of much more widely-spaced oscillations is observed in dG/dV. The voltage separation of these resonances decreases a.s 0 approaches 90°, c.f. Figs 3(e) & (f).

In a. classical picture, electrons in the potential well execute regular lielical orbits for $0 = 0$(Fig 3(a)) and skipping orbits for $\theta = 90°$ (Fig 3(f)). Quantisation of these orbits in the quasiclassical regime gives an energy-level splitting $\Delta\epsilon = h/T$, where T is the periodic time for return to the emitter barrier. This is related to the voltage separation between adjacent peaks in the dG/dV plots by $\mathrm{A V} = f\Delta\epsilon/e$, where the factor f ~ 2 depends on the potential distribution in the structure. The larger AV values at $0 = 90°$ compared with $\theta = 0$ are attributable to the shorter return times of the skipping orbits[12].

Although the energy spectrum of a classically chaotic systein is complex, regular clusterings of levels occur which are related to the unstable periodic orbits of tlie system. This produces oscillatory structure in the smoothed density of states and the energy period $\Delta\epsilon_p$ is related to the unstable orbit period T_p by $\Delta\epsilon_p = h/T_p$[1,2]. Our experiments probe the density of states of tlie quantum well so the voltage spacings AV are directly related to the unstable periodic orhits by $\mathrm{AV} = fh/eT_p$. We have therefore used numerical simulations to search for periodic orbits in a potential well which has impenetrable barriers but is otherwise identical to that in the device. The starting velocities at the emitter barrier are chosen to be consistent with the momenta of the occupied states in the accumulation layer. The orbital paths are then obtained by numerical solution of Newton's equations, including specular reflection at the barrier interfaces. For $0 < 15°$, the allowed periodic orbits are all modified corkscrew trajectories which collide only once with each barrier per period, as shown in Fig. 4(a). We stress that Fig. 4 shows projections of the orbits in the x-y plane and that tlie orbits are in fact three-dimensional corkscrew segments. When $0 = 15°$, modified corkscrew orbits exist for V < 5(0 mV. In addition, a.secorid type of periodic orbit occurs, a.s shown in Fig. 4(b) for V = 200 mV. Electrons n these orbits make two successive collisions with the collector barrier before returning to their starting point on tlie emitter barrier. The path between the two successive collisions on the collector barrier (dotted curve in Fig. 4(b)) almost intersects with the einitter barrier. Tlie period T_D for these orbits is almost twice that for the modified corkscrew orbits. However, for V = 200 mV, $T_D \sim 1.2$ ps is much greater than tlie LO phonoii emission rate $\tau_{LO} \sim 0.2$ ps[13] of liot electrons in GaAs. At this low voltage the energy-level clusters corresponding to the classical period-doublecl orbit are tlierefore not resolved in the dG/dV plot. As V is increaçed, the orbital segment between successive collisions with the collector barrier is pulled away from the emitter barrier so that tlie total path lengtli and orbital period both fall. The closely-spaced energy level clusters associated witli the period-doubled orbit dominate tlie resonant. structure in dG/dV for V > 500 mV, when T_D becomes comparable with τ_{LO}. The orbital path then passes close to the centre of the quantum well between successive collisions with the collector barrier as shown in Figs. 3(b) and 4(c). Electrons in these star-shaped orbits therefore travel almost twice tlie distance in the x-direction as electrons in tlie modified corkscrew orbits. Tlie period for tlie star-shaped orbits (0.6 ps for V = 400 mV, 0 = 20°) is alniost twice that for the corltscrew orbits (0.37 ps for V = 400 mV, 0 = 2C°). The separation of energy level clusters corresponding to the star-shaped orbits is therefore approximately half that for the corltscrew orbits. This is the origin of the recluction of AV by a factor ~ 2, which is observed with increasing voltage for tilt angles in the range $15° \leq \theta \leq 24°$. From the above orbital periods we calculate values of AV = 28 mV and 17 mV for resonantes associated mith the corltscrew and star-shaped orbits respectively. These values are in good agreement with the measured peak spacings of 30 mV and 12 mV. Within the allowed range of initial velocities the corltscrew and star-shaped orbits coexist over a limited range of 0 and bias and the two periods observed in dG/dV at around 350 mV for $\theta = 20°$ provide evidente for this. Similar star-shaped orbits occur for tilt angles in the range 15° < 6 < 50° and account for all the high- frequency resonant structure (AV ~ 15 mV) in dG/dV. Although the shape of the orbits changes with θ and V, the topology does not (compare Figs. 3(b)-(d)), and the period remains ~ 0.6 ps, which accounts for the slow variation of AV.

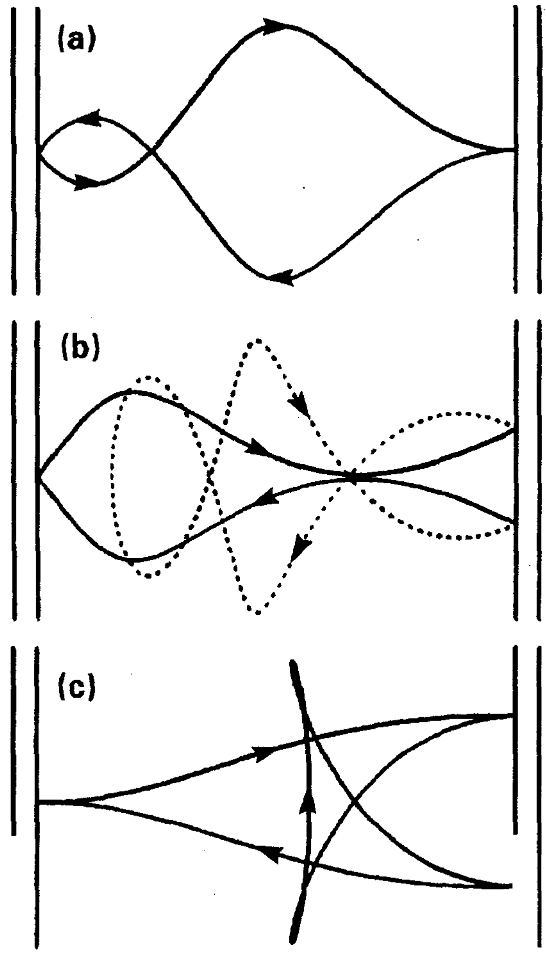

Figure 4: Periodic classical orbits calculated for a 120 nm wide well (B = 11.4T and $\theta = 20°$) projected onto the x-y plane. At low electric fields the corkscrew orbits (a) coexist with the period-doubled orbits (b). At high electric fields only tlie star-shaped orbits (c) exist.

For 50° < 0 < 70°, the classical star-shaped orbits are replaced by more complicated periodic orbits. However tlie period of these orbits greatly exceeds τ_{LO} so that no regular structure is resolved in dG/dV. For

0 > 70°, B_z is sufficiently liigh tliat tlie electrons in classical periodic orbits make several skips along the ernitter barrier before traversing the well, as shown in Fig. 3(e). The total orbital periocl T_p = 3 ps is much greater than τ_{LO} so that the associated energy level clusters are not resolved. However, the time interval τ_s of each skip along the einitter barrier (0.08 ps for θ = 80° and V = 590 mV) is almost 40 times smaller than T_p. We can give a quantitative explanation for tlie gradual reduction of AV with increasing 6 > 70° by using tlie skipping times τ_s rather than the total orbital period to determine the spacing of the clusters in tlie energy eigenvalue spectrum of tlie quantum well.

It slioulcl be noted tliat tlie range of allowed starting velocities does not give perfectly periodic orbits over tlie entire range of θ and applied voltage. However orbits which are almost periodic over several LO phonon emission times (> 10 ps) do occur over the measured range of tilt angles and bias voltages. Most of tlie resonant structure in $I(V)$ is presumably due to these almost-periodic orbits.

In summary, we have shown tliat tlie classical motion of electrons in a potential well is chaotic in the presence of a tiltecl magnetic field. Our studies liighlight the potential of resonant magnetotunneling spectroscopy for studying chaos in a system which shows clear quantuni behaviour. The resonances which appear in the current-voltage characteristics are associated with the periodic unstable orbits of tlie electrons in tlie quantuni well. These are analogous to the quasi-Landau resonances[14] in the absorption spectrum of highly excited, classically chaotic Iiydrogen atoms in strong magnetic fields. However, in our structures the device parameters can be tailored so that, in a classical picture, the electrons are injected into periodic orbits with specified topologies and return times.

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