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Magneto-Optical Properties of Coupled Double Quantum Wells *

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In tliis talk we review tlie magneto-optical properties associated witli excitons in coupled double quantum well structures. Iii particular, we study tlie bindiiig energies of both the light-hole and the heavy-hole excitons in a.symmetric double-quantum-well in the presence of a magnetic field applied parallel to tlie growtli clirection. Exciton wave functions are expressed as combinations of Gaussian basis orbitals, witli variationally determined expansion parameters. By varying tlic inter-well potential barrier width and height (hence the inter-well coupling), we obtain exciton binding energies ranging in character from those for a strongly coupled double-well to tliose for a system of two isolatecl siiigle wells. The behavior of tlie exciton binding energies as functions of the inter-well coupling, well sizes and the magnetic field is consistently described witli our formalism. The application of the magnetic field leads to stronger confinement of tlie excitonic wave functions and hence enhances tlie exciton binding energies. And finally, the calculated rcsults are compared with tlie available experimental data.

I. Introduction

Double quantum well structures have attracted a good deal of atteiition, both experimentally and theoretically.[1–7] A douhle quantum well (DQW) is a semiconductor structure in which two siiigle quantum wells are separated by only a thin potential barrier across which electrons and holes from one well can tunnel into tlie other. As in siiigle quantum wells, tlie electrons ancl hcles confined in a DQW can form excitons due to their mutual Coulomb attraction. The electro-optical propt rties of such excitons promise applications in high speed spatial-light modulators and switches.[8] One advantage tliat a DQW structure offers over the siiigle quantum wells is the enhanced excitonic electro-optic response.[9]

A magnetic field applied parallel to the growtli direction has an additioiial confining effect on clectrons and holes in tlie quantum wells, aiid is expected to modify exciton binding energies in tlie DQW.[10,12] Together with the effects of the confinement and iiiter-well coupling (through tunneling across tlie potential basrier) provided by a DQW, we have an interesting physical system in wliich these competing factors influence those exciton cliaracteristics determining the excitonic electro-optical properties of the DQW. Although tlie properties of excitons in DQWs have been studied by several groups,[13–15] a systematic investigation of the effects of a magnetic field on them has begun only recently.[16] 111 addition, there have been some apparently conflicting results as to how inter-well coupling would qualitatively affect exciton binding energies in a DQW in tlie weak ancl strong inter-well coupling limits. A yualitative and a quantitative study is desired to gain knowledge of these aspects and to clear up ambiguities about the role played by inter-well coupling in affecting exciton binding energies in a double quantum well.

In tliis paper we review a formalism to calculate exciton binding energies and oscillator strengths in DQW structures in the presence of a magnetic field directed along tlie growtli axis.[16–21] The formalism is applied to a GaAs-$Al_xGa_{1-x}As$ DQW for various physical parameters. In Sec. II, we describe this formalism, in

*Invited talk.

which we solve for electron and hole wave functions for the double well potential profile; take mixing of electron and hole wave functions of neighboring subbands into account; express the exciton internal-state wave function in terms of Gaussian orbitals and determine expansion parameters and excitoii binding energies variationally. In Sec. III, we show that our formalism correctly describes exciton binding energies for all inter-well coupling strengths, and discuss exciton binding energies as a function of the quantum confinement, tlie magnetic field and the inter-well coupling. In Sec. IV, we provide a summary of our results, and discuss possible further extensions.

II. Formalism

We consider a DQW consisting of two identical GaAs layers sandwiched between two semi-infinite $Al_xGa_{1-x}As$ slabs, with a tliin layer of $Al_xGa_{1-x}As$ between them. A uniform magnetic field B is applied perpendicular to the layers (in tlie growth direction).

The Hamiltonian of the electron-hole system is[20]

$$\begin{aligned} H &= H_e\left[-i\hbar\nabla+\frac{e}{c}\mathbf{A}\right]-H_h\left[i\hbar\nabla+\frac{e}{c}\mathbf{A}\right] \\ &+ V_e(z_e)+V_h(z_h)-\frac{e^2}{\kappa_0|\mathbf{r}_e-\mathbf{r}_h|} \end{aligned} \quad (1)$$

where $V_e(z)$ and $V_h(z)$ are respectively the potential profiles for the electrons and holes, A = (B x r)/2 is the vector potential of the magnetic field B, κ_0 is the dielectric constant of the layers (assumed to be uniform here), $\mathbf{r}$, and $\mathbf{r}_h$ are the electron and hole positions. The electron Hamiltonian H_e is adequately described by an effective mass approximation, using parabolic bands. The hole Hamiltonian H_h is the 4 × 4 Kohn-Luttinger Hamiltonian.[21] To gain physical insight with a tractable inodel, we assume parabolic hole bands in the x − y plane aiid in the z-direction and retain only diagonal terms in H_h, thereby ignoring coupling between the heavy and tlie light hole bands. Following a standard procedure to separate the constant center-of-mass motion of an electron-hole pair in the x − y plane, we define the reduced mass of an electron-hole pair $\mu_\pm$ with $\mu_\pm^{-1} = m_e^{-1} + (\gamma_1 \pm \gamma_2)m_0^{-1}$, where m_0 is the free electron mass, m_e is the effective electron mass and γ_1 and γ_2 are the Kohn-Luttinger band parameters, the (+) sign corresponds to the heavy-hole exciton, (-) sign to the light-hole exciton. We then scale all lengths in the exciton Bohr radius $a_\pm = \kappa_0\hbar^2/\mu_\pm e^2$, and energies in the exciton Rydberg $R_\pm = e^2/2\kappa_0 a_\pm$, to obtain the dimensionless form of the Hamiltonian

$$\begin{aligned} H &= H'_e(z_e)+H'h(z_h)-\left[\frac{\partial}{\rho\partial\rho}\rho\frac{\partial}{\partial\rho}+\frac{\partial^2}{\rho^2\partial\varphi}\right] \\ &+ \gamma L_z+\frac{\gamma^2}{4}\rho^2-\frac{2}{\sqrt{\rho^2+z^2}}, \end{aligned} \quad (2)$$

where $\mathrm{p} = \sqrt{(\chi_e-\chi_h)^2+(y_e-y_h)^2}$ is the in-plane distance between a pair of electron and hole. $z = z_e - z_h$,Lz is the z-component of the angular momentum, and γ is the first Landau level expressed in R*, $\gamma = e\hbar B/2\mu_\pm cR_\pm$. The Hamiltonian H above is grouped into three terms, nainely the electron part H'_e, the hole part H'_h aiid the exciton part H^B_{ex}, $\mathrm{H} = H'_e + H'_h + H^B_{ex}$, where

$$H'_e(z_e) = -\frac{\mu_\pm}{m_e}\frac{\partial^2}{\partial z_e^2}+V_e(z_e), \quad (3a)$$

$$H'_h(z_h) = -\frac{\mu_\pm}{m_\pm}\frac{\partial^2}{\partial z_h^2}+V_h(z_h), \quad (3b)$$

$$H^B_{ex} = -\left[\frac{\partial}{\rho\partial\rho}\rho\frac{\partial}{\partial\rho}+\frac{\partial^2}{\rho^2\partial\varphi^2}\right]+\gamma L_z+\frac{\gamma^2}{4}\rho^2-\frac{2}{\sqrt{\rho^2+z^2}}, \quad (3c)$$

and $m_{+(-)}$ is the heavy (light) hole mass defined as $m_\pm^{-1} = (\gamma_1 \mp 2\gamma_2)m_0^{-1}$.

Wave function $\Psi(\mathbf{r}_e,\mathbf{r}_h)$ of the electron-hole system is solved from the Schrödinger equation $H\Psi(\mathbf{r}_e,\mathbf{r}_h) = E\Psi(\mathbf{r}_e,\mathbf{r}_h)$ where E is the total energy. We write $\Psi(\mathbf{r}_e,\mathbf{r}_h)$ in the following form to express the explicit dependence on z_e, z_h and on the relative distance $\mathrm{r} = \mathbf{r}_e - \mathbf{r}_h$ [22,23]

$$\Psi(\mathbf{r}_e,\mathbf{r}_h) = \sum_{k,l=1} A_{kl}F_e^k(z_e)F_h^l(z_h)\phi(\mathbf{r}), \quad (4)$$

where $\phi(\mathbf{r})$ is the wave function describing the internal state of an exciton: $F_e^k(z_e)$ is the k$^{\mathrm{th}}$ electron subband wave function, and $F_h^l(z_h)$ the l^{th} hole subband wave function, A_{kl} are the expansion coefficients to be determined; both $F_e^k(z_e)$ and $F_h^l(z_h)$ are normalized. Eq. (4) is a good approximation to the exciton wave function as long as the difference between the subband levels that are included in the summation and those that are not

is larger than tlie exciton binding energies. Tlie two wave functions in the z-direction are determined by tlie following two equations,

$$H'_e F^k_e(z_e) = E^k_e F^k_e(z_e), \tag{5a}$$

$$H'_h F^l_h(z_h) = E^l_h F^l_h(z_h), \tag{5b}$$

in which E^k_e and E^l_h are the electron and holc subband energies.

We first solve for subband envelope functions $F_e(z_e)$ and $F_h(z_h)$.[16] Next we express tlie exciton internal-state wave function $\phi(\mathbf{r})$ in terms of Gaussian orbitals ancl use a variational calculation to determine the expansion parameters and the exciton binding energies.

$$\phi(\rho,\varphi;z) = \sum_{i=1} c_i R_i(\rho,\varphi)\xi_i(z), \tag{6a}$$

where $c_i(i = 1,\mathrm{n})$ are the expansion coefficients, $R_i(\rho,\varphi)$ and $\xi_i(z)$ are respectivcly the basis functions in the $x-\mathfrak{z}$ plane and in the z-direction,

$$R_i(\rho,\varphi) = \frac{e^{im\varphi}}{\sqrt{2\pi}}\rho^{|m|}\exp[-(\alpha_i+\beta)\rho^2], (m = 0,\pm1,\pm2,...), \tag{6b}$$

and

$$\xi_i(z) = \exp[-(\alpha'_i+\delta)z^2], \tag{6c}$$

where $\boldsymbol{3}$ and δ are tlie variational parameters, α_i and $\alpha'_i(i = 1.n)$ are sets of constants.[24,25] For excitons in the double quantum well, 3 and δ are varied to adjust these Gaussian basis functions to minimize the total energy E.

For DQWs consisting of narrow wells with strong interwell couplings (for center barriers of small widths or low heights), effects of the coupling between neighboring subbands on exciton binding energies are shown to be small, tlierefore it is sufficient to assume exciton to be associated with a single electron subband and a single hole subband.[17] In general, however, single-subband description of excitons in a. double quantum well is inadequate ancl can lead to qualitatively misleading results. When tmo single quantum mells are separated by a potential barrier, the wave functions in these wells are scrambled to form a "bonding" (even-parity) ancl an "anti-bonding" (odcl-parity) combination (total) wave function. If the barrier is thin *and* tlie wells are narrow, the single-well wave functions are strongly modified hy the presence of the neighboring well because of the tunneling of electron (hole) across tlie potential barrier. As a result, the bonding and anti- bonding total wave functions have significantly diffcrcnt subband levels. In other words, subband levels in such a thin-barrier, narrow-well DQW are non-degenerate. When tlie harrier is thicker *or* wells are wider, the single-well wave functions are essentially confined to one single-well and are therefore diminishingly affected by tlie presence of its neighboring well. Bonding and anti-bonding combinations would yield similar subband levels, with one slightly lower and one slightly higher than the isolated single-well subband levels. All subband levels are *almost* doubly-degenerate. *A* consistent description of excitons in DQW structures should therefore include *pairs* of subband levels to properly account for contributions to exciton bincling energies from both the even-parity and the odd-parity suhhand wave functions.

In what follows, we calculate the properties of the ls exciton associatecl with tlie first two electron and hole subbands ($A_{kl} = 0, k, l > 2$). In a system with a thin barrier and narrow wells, the separations between tlie adjacent subband levels are large compared with the expected exciton binding energies and there is little iriter-subband coupling, i.e., $A_{11} \approx 1$ ***and*** $A_{kl} \to 0((k+1) > 2)$. As $L_b \to \infty$, these subband levels become degenerate and coupling between them becomes important, i.e., all A_{kl}'s would play comparable roles. In the absence of the Coulomb interaction ancl without mixing of the even-parity and odd-parity subbands, the total energy E is just the sum of first electron and hole subband energies $E^{(1)}_e$, $E^{(1)}_h$ and the Landau level energy γ. The Coulomb interaction between the electron and hole lowers the total energy and leads to the formation of excitons. The binding energy of the lowest lying exciton E_B is defined in $E_B = E^{(1)}_e + E^{(1)}_h + \gamma - \mathrm{E}$.

The total variational wave function corresponding to the ls state is $\Psi(\mathbf{r}_e,\mathbf{r}_h) = \sum_{k,l;i} A_{kl}c_i\eta^{kl}_i(z_e,z_h,\rho)$ in which $\eta^{kl}_i(z_e,z_h;\rho) = F^k_e(z_e)F^l_h(z_h)R_i(\rho)\xi_i(\rho)(z_e - z_h)$ is tlie non-orthogonal basis wave function set. We determine the expansion coefficients from the Schrodinger equation,

$$\sum_{j;k'l'} H_{ij}(kl,k'l')A_{k'l'}c_j = E\sum_{j;k'l'} U_{ij}(kl,k'l')A_{k'l'}c_j, \tag{7}$$

where $U_{ij}(kl,k'l')$ and $H_{ij}(kl,k'l')$ are the overlap matrix and the Hamiltonian matrix defined below,

$$U_{ij}(kl,k'l') = 2\pi\int_{-\infty}^{\infty}dz_e\int_{-\infty}^{\infty}dz_h\int_{-\infty}^{\infty}\rho d\rho\eta_i^{kl}(z_e,z_h,\rho)\eta_j^{k'l'}(z_e,z_h,\rho), \tag{8a}$$

$$H_{ij}(kl,k'l') = 2\pi\int_{-\infty}^{\infty}dz_e\int_{-\infty}^{\infty}dz_h\int_{-\infty}^{\infty}\rho d\rho\eta_i^{kl}(z_e,z_h,\rho)H\eta_j^{k'l'}(z_e,z_h,\rho), \tag{8b}$$

By parities of tlie wave functions, we have $U^{ij}(kl,k'l') = 0$ and $H^{ij}(kl,k'l') = 0$ if $k+k'+1+1$ is an odd integer.

The total energy E is an eigenvalue determined by the following eigensystem equation,

$$\begin{bmatrix} \Pi_{ij}(11,11) & \Pi_{ij}(11,22) & 0 & 0 \\ \Pi_{ij}(11,22) & \Pi_{ij}(22,22) & 0 & 0 \\ 0 & 0 & \Pi_{ij}(12,12) & \Pi_{ij}(12,21) \\ 0 & 0 & \Pi_{ij}(12,21) & \Pi_{ij}(21,21) \end{bmatrix} \begin{pmatrix} A_{11}c_j \\ A_{22}c_j \\ A_{12}c_j \\ A_{21}c_j \end{pmatrix} = 0, \tag{9}$$

where $\Pi_{ij}(kl,k'l') = H_{ij}(kl,k'l') - \lambda U_{ij}(kl,k'l')$ are real n x n symmetric matrices. Notice that tlie expansion coefficieiits for the subband wave functions and tliose for the Gaussian orbitals are determined at the same time by Eq. (9), which is actually a superposition of two independent eigensystems. The eigensystem is solved by a generalized Rayleigh quotient itcration method.[26] By choosing tlie appropriate eigenvalue λ and minimizing it as a function of tlie variational parameters, we obtain tlie total energy E of tlie exciton ground state and the exciton binding energy E_B. The wave function Ψ is simultaneously determined by the corresponding eigen-vector $A_{kl}c_i$ for the given E, subject to the normalization conditioii $<\Psi|\Psi>= 1$.

III. Results and discussion

We have calculated the binding energies of tlie heavy-hole exciton and the light-hole exciton as functions of tlie magnetic field, the well width L_w and tlie center barrier thickness L_b of a. symmetric GaAs-$Al_xGa_{1-x}As$ double quantum well. The values of physical paraineters pertaining to GaAs used in our calculations are: $m_e = 0.067$ mo, $\kappa_0 = 12.5$, $\gamma_1 = 7.36$, $\gamma_2 = 2.57$.[27] The values for the heavy-hole (J, $= \pm\frac{3}{2}$) exciton are $m_+ = 0.45\ m_0, \mu_+ = 0.04 m_0, a_+ = 165$ Å, $R_+ = 3.49$ meV; tliose for the light-hole (J, $= \pm\frac{1}{2}$) exciton are $m_- = 0.08\ m_0, \mu_- = 0.05\ m_0, a_- = 131$ Å, $R_- = 4.39$ meV. We use an empirical formula $\Delta E_g = 1.33x + 0.22z^2$ (eV) to determine the band gap discontinuity,[28] with 60% of ΔE_g contributing to the conduction hand discontinuity ΔE_c and 40% to the valente band discontinuity ΔE_v. Mole fraction $x = 0.3$ is used for all Al coiicentrations. Differences between material parameters of GaAs and those of $Al_xGa_{1-x}As$ are not included in tlie calculations. Potential for electroiis, $V_b^e = V_w^e = 257$meV; for holes $V_b^h = V_w^h = 171$ meV. For computational simplicity, we have used $\alpha_i' = \alpha_i$ and have chosen α_i's from tlie results of Huzinaga[26] on energy levels of a hydrogen atom using the Gaussian basis orbitals. It was also sufficient to set $6 = 0$ and use only oiie variational parameter β.

In Fig. 2, we compare binding energies of the heavy-hole exciton in a DQW with $L_b = 100$Å ($\approx 0.6a_+$) with tliose iii a single well of width L_w and those in a DQW with $L_b = 0$ (which makes the DQW equivalent to a. single well of width $2L_w$). The binding energies in the single wells are calculated both with and without mixing of subband wave functions. The increase in excitoii binding energies in single quantum wells is negligible when subband mixing is included, for all well sizes shown in the figure. In the case of the DQW with

$L_b = 100$ Å, however, binding cnergies obtained with mixing of two electron and two hole subbands are significantly higher than tliose obtained using only one electron and one hole subband (therefore ignoriiig subband inixiiig) at all well widths, except for very narrow wells (L, $\lesssim 25$Å). Such a thick barrier effectively prevents the electron and hole tunneling from one well to the other, so the correct wave function should be a single-well wave function. A single-subband wave function, with c. definite parity in tliis symmetric DQW, on the other hand, unrealistically forces the electron and hole to be present in both wells, therefore reducing the probability of finding an electron in tlie vicinity of a hole and vice versa, which leads to the underestimation of exciton binding energies in a. double quantum well. Tlie result of Kamizato and Matsuura (KM hereafter)[25] using the single-subband treatment therefore gives a misleading impression that wells separated by such a thick barrier are still strongly coupled. Our results correctly show that with such a thick barrier, the two quantum wells are effectively decoupled for L„ $\gtrsim$ 40 Å, which is physically consistent with what one finds in experiments.

In Fig. 3, we compare binding energies of the heavy-hole exciton in a DQW calculated by us, with those obtained by KM with and without subband mixing, and those by Dignam and Sipe (DS hereafter)[15] with subband mixing, a.s a function of tlie barrier thickness L_b. It is evident that the two-subband treatment by DS seriously underestimates the binding energy in the strong inter-well coupling limit ($L_b \lesssim 0.2a_+$) and can not recover the fact that at $L_b = 0$, tlie DQW is simpiy a single well of width $2L_w$. On the otlier hand, the two-subband DS result in tlie weak inter-well coupling limit approaches that obtained by KM without subband mixing, which can not recover the single-well result at large barrier thicknesses eitlier. It appears that although the DS two-subband treatment works in tlie intermediate inter-well coupling strengths, it overestimates the strength of the inter-well coupling in both tlie strong and weak inter-well coupling limits. Our result agrees with that of KM including subband mixing at both strong and weak inter-well coupling limits. It is also evident that our formalism gives higher binding energies for all inter-well coupling strengths. Furthermore, our formalism correctly describes the behavior of exciton binding energies when the additional confining effect of the magnetic field is also included.

In Fig. 4(a), we show variation of the binding energy E_B of the heavy-hole exciton as a function of well widths L, for several different combinations of the barrier thickness L_b and the magnetic field B. The results of E_B for $L_b = 0$ have been cornpared with those of Greene and Bajaj for exciton binding energies in single quantum wells in a magnetic field,[10] based on the expansion of the exciton wave function into Gaussian basis orbitals. Tliose for zero magnetic field (B = O) and $L_b \ll a_+$ have been compared with the results obtained by KM for exciton binding energies in a symmetric double quantum well, with material parameters roughly corresponding to those of heavy-hole excitons in GaAs-$Al_xGa_{1-x}As$ quantum wells.[25] The agreement in both cases is excellent, as expected.

Again for $L_b = O$, as well width L, decreases, electron and hole wave functions first become compressed in tlie narrowing wells and exciton binding energy E_B climbs up due to the decreasing average distance between the electron and the hole, which is mainly determiiied by tlie well size L, in a given magnetic field, until E_B reaches a maximum. As L, further decreases, subband energies are pushed up and leakage of the wave functions into the barrier regions becomes significant, E_B begins to fall off rather rapidly as the exciton assumes more of a 3D-like nature.[24]

For $L_b \neq 0$, the binding energy is lower for small L, and higher for large L, in comparison to that in the DQW with $L_b = 0$. For narrow wells, the electron and hole wave functions spread throughout the DQW structure, and the presence of the barrier merely increases the average distance between the electron and hole, leading to a lower binding energy. As the wells become wider, however, the wave functions become more and more confined in one single well due to the presente of the barrier, the average distance between the electron and hole decreases, leading to a higher binding energy. At some well width L_c, the E_B curves in the DQW with $L_b \neq 0$ will cross over with that in the DQW with $L_b = 0$. Since a magnetic field provides an extra confinement of the wave function in the quantum well, such a crossover will occur at a smaller L_w^c at higher field strengths. Also notice that a shoulder develops in the binding energy curves. As L_b increases, this shoul-

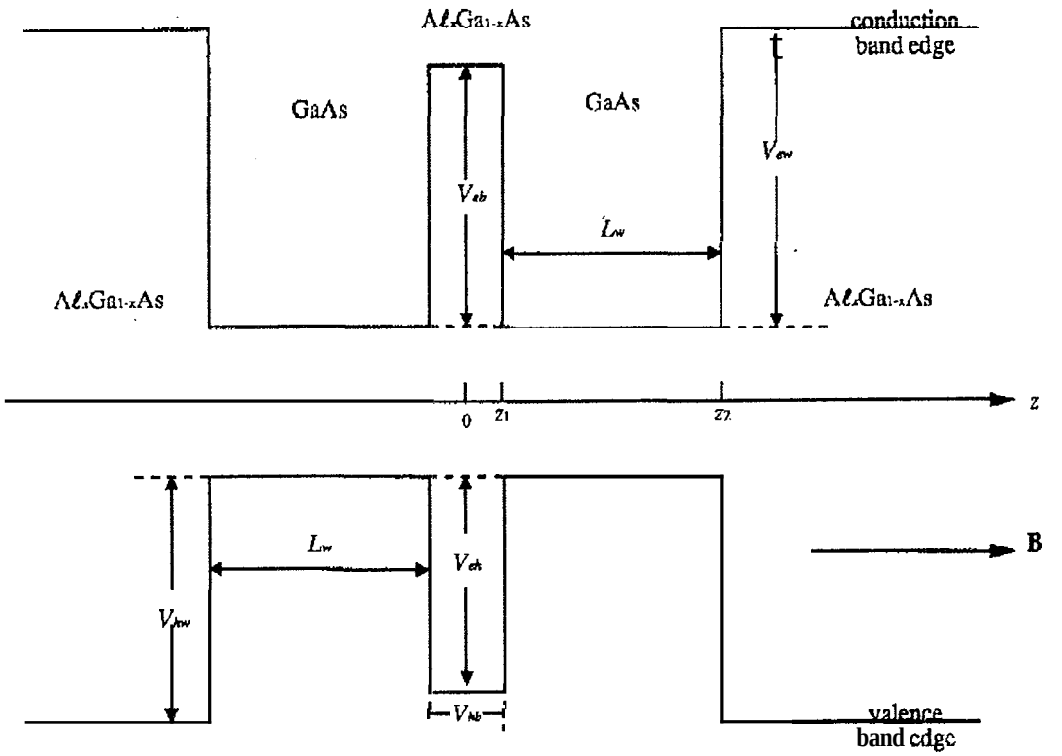

Figure 1: Schematic band cliagram of a symmetric GaAs-$Al_xGa_{1-x}As$ double quantum well and the applied magnetic field B in tlie growth direction.

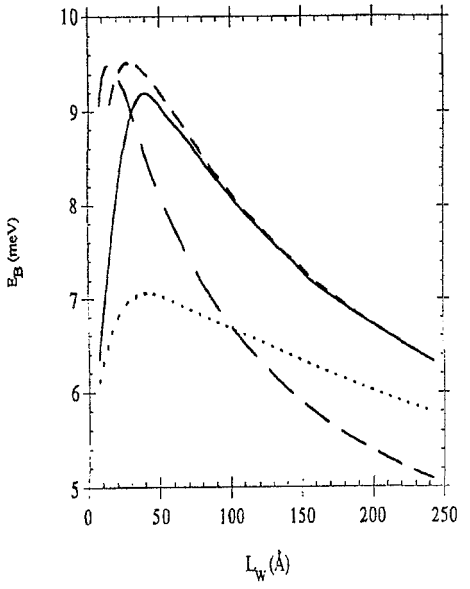

Figure 2: Comparison of binding energies of tlie heavy-hole exciton calculated, witli and without subband mixiiig, as function of well width $L_,$. Short-dashed line (- - -) is for a DQW witli $L_b = \infty$ (corresponding to an isolated single quantum well); long-dashed line (– –) for $L_b = 0$; solid line (—) for $L_b = 100\mathring{A}$ (with rnixing of two electron and two hole subbands); dotted line (- - - - -) for $L_b = 100\mathring{A}$ (only onc electron and one hole subband used in tlie calculation).

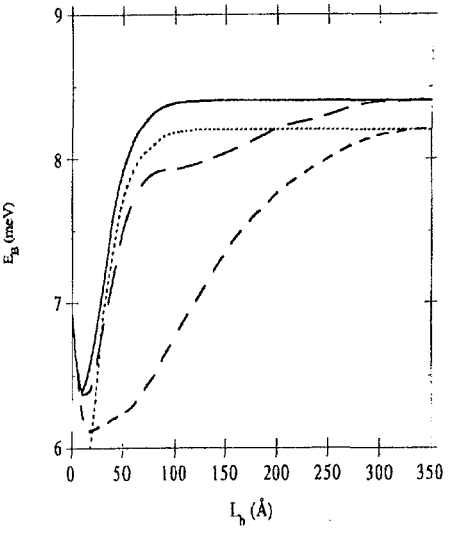

Figure 3: Coniparison of binding energies of tlie heavy-hole exciton calculated, with and without subband mixing, as function of barrier tliickness L_b. The well width is fixed at $L_, = 0.6a_+$. Solid line (–) is our result witli subband mixing included; dotted line (.....) by DS with subband mixing; short-dashed line (- - -) is by KM without subband mixing; long-dashed line (– –) by KM with subband mixing. Material parameters are as in KM.

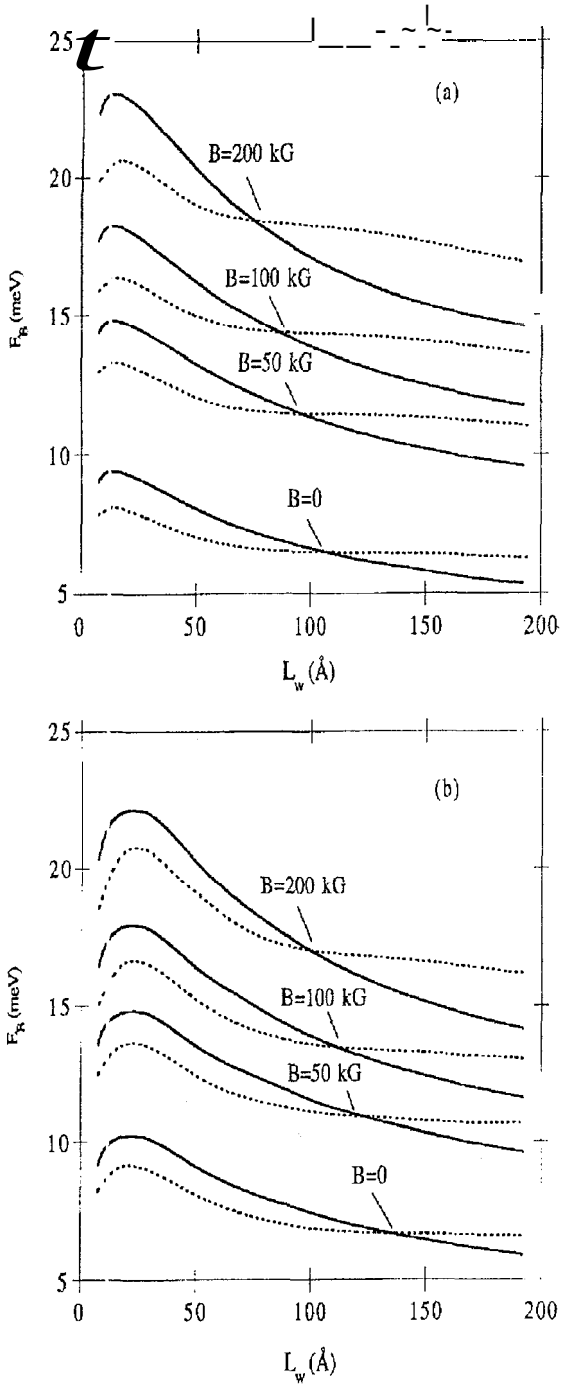

Figure 4: (a) Binding energy of tlie lieavy-liole exciton as a function of tlie well width L_w, (b) binding energy of tlie liglit-liole exc:ton as a function of tlie well width L,, with magnetic field B and barrier thickness L_b as the two other parameters. Material parameters are noted in text. Solid lines (—) are for $L_b = 0$ (corresponding to tliat in a single quantum well of width $2L_w$); dotted lines (.....) for $L_b =$ 25Å.

der becomes more evident and appears at smaller well widths L,. At the limit $L_b \to \infty$, it merges with the maximum that is caused by the leakage of the lowest subband wave functions into the barrier regions. This shoulder is attributed to mixing of wave functions of the odd-parity second subbands with those of the even-parity first subbands. For small L_b, $E^{(1)}$ and $E^{(2)}$ are significantly different, and wave functions of the second subbands are more spread out due to their higher energies. Binding energy associated with second subbands would rcacli the maximum at large well width L,. As L_b increases, tlie second subband comes down and eventually becomes degenerate with the first subband, the maximum in E_B caused by it coincides with that of the first subband.

In Fig. 4(b), we show values of the binding energy E_B of the light-hole exciton as a function of L„ . Qualitatively E_B behaves similar to tliat of the heavy-hole exciton. However, it is larger and reaches inaximum for larger L, as compared to that for the heavy-hole exciton.[29] Also, the values of light-hole exciton binding energy are higher than those obtained by Greene and Bajaj who used 85%-15% conduction-valence band offsets in their calculations,[10] as light-holes are now more severely confined in the quantum wells by higher potential barriers. Binding energies of the heavy-hole exciton associated with the lowest subband are not as sensitive to the change of band offsets used in calculations, since the heavier longitudinal mass results in stronger confinement of the heavy hole wave function in tlie quantum wells. However, for excitons associated with higher subbands, higher valence band offsets are expectcd to affect binding energies more significantly for both the light-hole and the heavy-hole excitons.

In all instances, the presence of a. magnetic field in tlie growth direction leads to higher exciton binding energies. In Fig. 5(a), we show the binding energies of tlie heavy-hole exciton, as functions of the magnetic field with several quantum well sizes and barrier widths $(L_{,,}, L_b)$. Similar results for the liglit-hole exciton are shown in Fig. 5(b). As the magnetic field increases, the in-plane radius ($\sim < \rho > = \sqrt{< \Psi|\rho^2|\Psi >}$) of the exciton is reduced, leading to stronger Coulomb attraction between the electron and the hole, and consequently higher binding energies. Our results on heavy-hole exciton fit ratlier well with those measured by Perry et

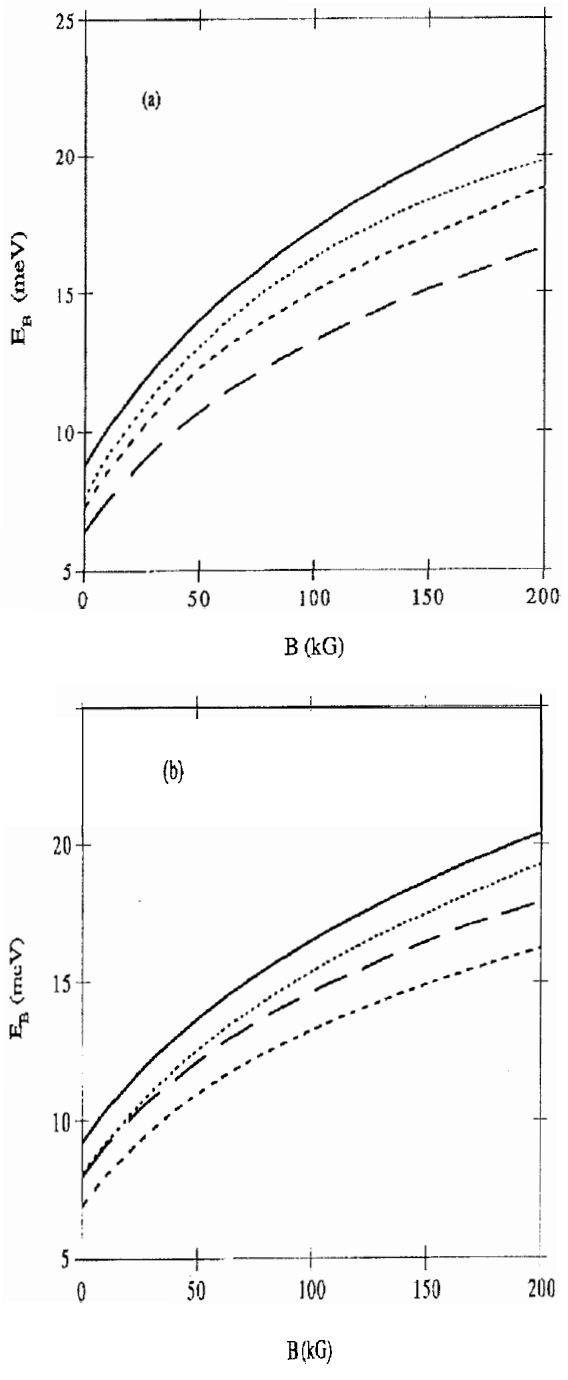

Figure 5: (a) Binding energy of the lieavy-liole exciton E_B as a function of tlie applied inagiietic field, (b) binding energy of tlie light-hole exciton E_B as a function of the applied niagnetic field B. The two parameters are LI,and $L_,$. Solid line (—) is for L_I, = 10 Å , $L_,$ = 10 Å ; long-dashed line (- -) for L_b = 50 Å , L_w = 10 Å; short-dashed line (- - -) for L_I, = 10 Å, L_w = 100 Å; dotted line (....) for L_b = 5 Å, $L_,$ = 100 Å.

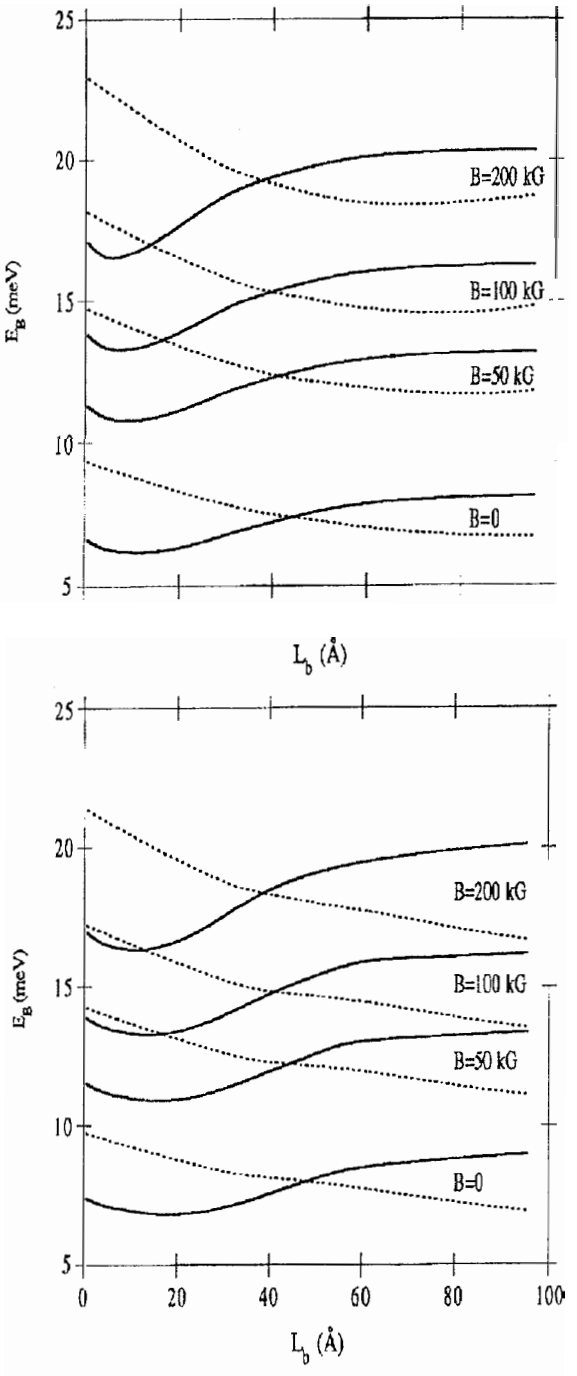

Figure G: (a) Binding energy of the lieavy-hole exciton as a function of tlie barrier thickness L_b, (b) binding energy of tlie light-hole exciton as a fiinction of tlie barrier thickness LI The other two parameters are the magnetic field B and the well widtli $L_,$. Solid lines (—) are for $L_,$ = 100 Å; dotted lines (....) for L, = 10 Å.

al.,[12] when appropriate material parameters are used in the calculations. As for the light-hole exciton, comparison with their data is more difficult, because of the ambiguity in identifying the correct exciton branches from their experimental results.

In Fig. 6(a), we show binding energies of the heavy-hole exciton as functions of barrier thickness L_b for several different values of (L_z, γ). Similar results for the light-hole exciton are displayed in Fig. 6(b). At $L_b = 0$, results for single quantum wells of width $2L_w$ are recovered, as we have noted earlier. The average distance between the electron and hole forming the exciton increases as L_b increases from 0, and as a result the binding energy E_B first decreases. For small barrier thicknesses, a significant portion of wave functions is present in the barrier regions. However, this leakage decreases sharply as well size increases.[24] Therefore for wider wells the rate at which the binding energy decreases as L_b increases is higher, as it is easier to separate the wave function in the two neighboring wells when the center barrier size L_b increases. By the same argument, the drop in E_B as L_b increases is also steeper for the heavy-hole exciton, since heavy-hole tunneling is more sensitive to changes in the barrier thickness.

As L_b further increases, coupling between the wells diminishes, and the binding energies will eventually increase and approach the isolated single well values $E_B(L_w)$. The E_B curves will bottom out at some barrier thickness L_b and then rise up. Again for wider wells, this minimum in E_B will occur at smaller barrier thickness L_b. For the same reason, binding energy E_B of the heavy-hole exciton will reach the minimum at smaller barrier thickness L_b than that of the light-hole exciton. Notice also that as the magnetic field increases, the exciton wave function spread is reduced, and as a result, the inter-well coupling decreases faster as the barrier thickness increases. The minimum of the binding energy occurs at smaller values of L_b.

It is worth pointing out that so far only our approach has produced consistent results for all barrier sizes. Although we have calculated only binding energies of excitons associated with the first electron and hole subbands, our formalism can be applied to excitons associated with other subbands.

IV. Summary and conclusions

In summary, we have reviewed a formalism to calculate exciton binding energies in symmetric double quantum wells in the presence of a magnetic field applied parallel to the growth axis. The extra quantum confinement provided by the magnetic field increases the exciton binding energies. Effects of inter-well (inter-subband) coupling on the light-hole and heavy-hole exciton binding energies in the double quantum wells are consistently included in our calculations for the first time. Effects of quantum confinement provided by the magnetic field and the potential wells, and those of tunneling across the center potential barrier on the exciton binding energies are discussed. In the limit of thick potential barriers, the even-parity and odd parity subband wave functions have degenerate energy levels; mixing of electron and hole subband wave functions strongly modifies the excitonic wave function and consequently lets one recover results for excitons in decoupled single quantum wells. We have shown that ignoring such subband mixing is a good approximation only for narrow-wells and thin-barrier double quantum well structures, and that a single-subband approach can lead to qualitatively misleading conclusions when applied to DQWs with wide-wells or thick barrier.

We have used the first two single electron and hole subbands in calculations of the exciton binding energies, and our results cover most cases one would encounter in experiments. While we have not included differences in the effective masses and dielectric constants across the GaAs-$Al_xGa_{1-x}As$ interfaces in our calculations, they have been shown by various authors to lead to only small increases in the exciton binding energies and therefore would not alter the conclusions presented here.

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