Fermion-Chern-Simons Theory of the Half-Filled Landau Level*

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Recent experiments have shown that a. quantized Hall plateau can occur in double layer systems when the total Landau Icvel filling factor is $\nu = 1/2$, though there is no plateau at $\nu = 1/2$ or v = 1/4 in a normal single layer system. For the single layer system, considerable insight has been provided by a theory based on tlie fermion Chern-Simons picture, where tlie electrons are transformed into fermions that carry two flux quanta of a Cliern-Simons gauge field. A similar picture can he used to cliaracterize ground states which have been proposed for tlie two layer system.

I. Introduction

During tlie course of tlie past few years, as experiments have continued to reveal the structure of electronic states in a. partially filled Landau level, a variety of theoretical approaches have been developed to understand these systems. One of tlie most useful of these approaches employs a singular gauge transformation to convert the electrons to a system of particles interacting with a Chern-Simons gauge field[1-11]. In this description, a flux tube containing an integer number $\tilde{\phi}$ of quanta of tlie Chern-Simons magnetic field is attached to each particle. If $\tilde{\phi}$ is an even integer [5-11], then the transformed particles obey Fermi statistics. The motivation for employing this singular gauge transformation is that for various rational values of the Landau-level filling factor v, with an appropriate choice of ϕ, if one treats the transformed system in a simple Hartree approximation, the resulting ground state is nondegenerate, and therefore lias a. reasonable chance of being a. good first approximation to tlie true ground state of the system. Moreover, one may hope to calculate corrections to the ground state and study the dynamic response of tlie system by using standard techniques of diagrammatic perturbation theory, beginning with tbe Hartree ground state [9,10,12]. As an important example, if $\nu = p/(2p+1)$, where p is a positive or negative integer, and if we choose $\tilde{\phi} = 2$, then the mean-field ground state of the transformed system is a collection of fermions in an *effective* magnetic field whose strength is such that exactly $|p|$ Landau levels are filled hy ferinions. The ground state is, therefore, stabilized by an energy gap separating it from the excited states. This provides a natural explanation for the most prominent fractional quantized Hall states, wliich are observed at these filling fractions. At the mean-field level, the fermion Chern-Simons description is essentially equivalent to Jain's composite fermion description of these quantized Hall states [8].

In a recent paper (HLR), P.A. Lee, N. Read, and tlie present author employed the fermion Chern-Simons method to analyze the properties of a single layer system, at v = 1/2 and at various other even fractioiis, where the quantized Hall effect has not been observed [10]. At v = 1/2, if one cliooses $\tilde{\phi} = 2$, the average Cliern-Simons field just cancels the external magnetic field, so that the Hartree ground state is just a filled Fermi sea of particles, in zero magnetic field, with Fermi wavevector $k_F = (4\pi n_e)^{1/2}$. (Here n_e is the real density of the electroiis. We assume tliat tlie electron spins are fully aligned by the Zeeman field.) Although there is no energy gap in this case, the density of states for low energy particle-hole excitations is small, so that

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there is reason to hope that the mean-field ground state may be stable with respect to the particle-particle interactions, similar to tlie case of an ordinary Fermi liquid. The detailed analysis of HLR gives rise to predictions for various properties of the $\nu = 1/2$ system, which seem to be in excellent qualitative agreement with experiments and with exact calculations of finite systems. The most striking of these predictions is an explanation for the surface acoustic wave anomalies observed by Willett aiid coworkers [13,14]. A summary of the most important results of the fermion Chern-Simons theory in the single layer system will be given in Section II below.

As is now well known, a quantized Hall plateau a.t total filling $\nu = 1/2$ has recently been observed in certain double layer systems 113 groups at Princeton and Bell Laboratories [15,16]. A plateau at. filling fraction $\nu = 5/2$ mas observed earlier in single layer systems by Willett *et al.* [17]. Although various explanations foi these states have been advanced, there remains a considerable amount of debate about the precise form of the ground state in various cases. I am iiot able to settle these questions, but I will try to outline in Section III below how the various postulated quantized Hall states may be at least formulated in terms of the fermion Chern-Simons picture as states with various forms of BCS pairing among the particles near tlie Fermi surface. This suggests a simple phase diagram foi how tlie various states may be connected.

II. The Single Layer System

We summarize here some of the key results of the analysis of HLR [10] for a fully polarized single-layer system at $\nu = 1/2$.

The Chern-Simons theory begins with an exact unitary transformation. We define a transformed wavefunction

$$\Psi_{tr}\{\vec{r}_i\} \equiv \Psi_e\{\vec{r}_i\} \prod_{i<j} \left[\frac{z_i - z_j}{|z_i - z_j|}\right]^{\tilde{\phi}} , \qquad (2.1)$$

where Ψ_e is the electron wavefunction, $z_j = x_j + iy_j$ is the electron position in complex notation, and $\tilde{\phi}$ is an integer. If $\tilde{\phi}$ is cveii, tlie transformation preserves the fermi statistics: the transformed wavefunction must change sign when two particles are interchanged. (If $\tilde{\phi}$ is chosen to be odd, then fermions are converted to bosons, and *vice versa.*) We shall choose $\tilde{\phi} = 2$.

Under tlie unitary transforination (2.1), the electron Hamiltonian is transformed to the form

$$\begin{aligned} H_{tr} &= \sum_i \frac{|\vec{p}_i + e\vec{A}(r_i) - \vec{a}(r_i)|^2}{2m} \\ &+ \frac{1}{2}\sum_{i<j} v(\vec{r}_i - \vec{r}_j) , \end{aligned} \qquad (2.2)$$

where v is the Coulomb interaction, $\vec{A}$ is the external vector potential, and $\vec{a}(\vec{r})$ is a Chern-Simons vector potential, given by

$$\vec{a}(\vec{r}) \equiv \tilde{\phi} \sum_{\vec{r}_j \neq \vec{r}} \frac{\hat{z} \times (\vec{r} - \vec{r}_j)}{|\vec{r} - \vec{r}_j|^2} . \qquad (2.3)$$

The mean field approximation for the ground state of the Hamiltonian (2.2) is then ohtained hy ignoring the Coulomb interaction, and by replacing the true Chern-Simons magnatic field $b(\vec{r}) \equiv \vec{\nabla} \times \vec{a}(\vec{r})$ by its mean value, $\langle b \rangle = 2\pi\tilde{\phi} n_e$.

The density and current response functions have been ohtained using the Random Phase Approximation (RPA) or time-dependent Hartree approximation. Here the transformed fermions are treated as free particles which respond to the self-consistent Chern-Simons electric and magnetic fields $\langle \vec{e}(\vec{r},t) \rangle$ and $\langle b(\vec{r},t) \rangle$, as well as to tlie external electromagnetic field and the self-consistent Coulomb potential of the particles [9,10,12]. The equations for $\langle b \rangle$ and $\langle \vec{e} \rangle$ are

$$\langle b \rangle = 2\pi\tilde{\phi}\langle \rho \rangle , \qquad (2.4)$$

$$\langle \vec{e} \rangle = -2\pi\tilde{\phi}\hat{z} \times \langle \vec{j} \rangle , \qquad (2.5)$$

where $\langle \rho \rangle$ and $\langle \vec{j} \rangle$ are the particle density and current, respectively, and $\tilde{\phi} = 2$. The system is found to be "compressible" at long wavelengths, which means more precisely that the static density response function $\chi_{\rho\rho}(q)$ is determined hy the diverging Coulomb interaction for $q \to 0$:

$$\chi_{\rho\rho}(q)^{-1} \sim v(q) = \frac{2\pi e^2}{\epsilon q} \qquad (2.6)$$

Tlie frequency-dependent density response function $\chi_{\rho\rho}(q,\omega)$ has, in addition to tlie polc at the cyclotron frequency that exhausts the f-sum rule for $q \to 0$, a diffusive pole at a low frequency $\omega = -i\gamma_q$ which we write ill the form

$$\gamma_q = q^2 v(q) \sigma_{xx}(q) \ , \qquad (2.7)$$

where $\sigma_{xx}(q)$ is the wavevector-dependent longitudinal conclictivity (we assume $\vec{q} \,||\, \hat{x}$). According to the RPA, for a system without impurities, $\sigma_{xx}(q)$ is given by [10]

$$\sigma_{xx}(q) = \frac{e^2}{8\pi\hbar} \frac{q}{k_F} \qquad (2.8)$$

More generally, if impurity scattering is taken into account, we expect that (2.8) applies for $q \gg \ell^{-1}$, where ℓ is tlie transport mean-free path at $\nu = 1/2$. Foi $q \to 0$, the conductivity goes to a finite value which may be obtained by replacing q on the right hand side of (2.8) by $(2\ell^{-1})$. The value of ℓ is expected to be much smaller than the transport mean free path in zero magnetic field. This is because the dominant mechanism for scattering of carriers at $\nu = 1/2$ comes from static fluctuations of the Chern-Simons magnetic field due to inhomogeneities in the electron density induced by random variations in tlie clensity of charged impurities in the doping layer, a mechanism wliicli does not occur for electrons in zero magnetic field [18]. A crude estimate of ℓ, at $\nu = 1/2$, was obtained by assuming that the charged impurities are uncorrelated within the doping layer, and are eyual in number to the electrons in the conducting layer. If scattering is treated in tlie Born approximation, one finds a value of ℓ which is just equal to tlie setback distance d_s of the doping layer in this moclel [10]. Experiments suggest tliat our crude estimate for ℓ is about a factor of three smaller than the actual values in the highest mobility samples [14,19].

An important effect arising from dynamic fluctuations of tlie Chern-Simons vector potential is a large renormalization of tlie effective mass of the transformed fermions. If the bare mass is small, so that the cyclotron energy is large compared to the scale of the electron-electron interactions, then the effective mass becomes independent of the bare mass, and is determined by the electron-electron interaction. Using a self-consistent analysis based on the leading diagrams in perturbation theory, HLR propose that there is a logarithmic divergence of the effective mass at the Fermi energy for Coulomb interactions, and a stronger power-law divergence for short range interactions, but that the most essential features of Fermi liquid theory are preserved in either case. Note that expressions (2.6)-(2.8) for the density response function and tlie conductivity are independent of the electron mass, and we believe that they are not affected by tlie divergent mass renormalization. (Tlie results for the mean-free path in the presence of impurities are also independent of the electron mass.)

One place where the effective mass enters directly is in the expression of HLR for the energy gaps $E_g^{(\nu)}$ for the principal quantized Hall states at $\nu = p/(2p+1)$. For an interaction that beliaves like $e^2/\epsilon r$ at large distarices, HLR predict the following asymptotic form for the energy gap at large p:

$$E_g^{(\nu)} \sim \frac{4}{\pi} \frac{e^2}{ti''} \frac{1}{D(\ln D + C)} \ , \qquad (2.9)$$

where $D = |2p+1|$ is the denominator of the fraction and C is a constant which depends on the short distance behavior of tlie poteiitial. (This formula is based on a self-consistent analysis of the leading correction to the quasiparticle self energy arising from interactions with fluctuations in the transverse gauge field; it is possible that it inay be modified by other singular contributions.) A good fit to numerical estimates [20] of the energy gaps at $\nu = 1/3$, $2/5$, and $3/7$, for a pure Coulomb interaction, may be ohtained by choosing $C \approx 2.5$ in that case. The effects of finite layer thickness and inter-Landau-level mixing, which occur in any real sample, would tend to increase the value of C still further. An energy gap of the form (2.9), with a relatively large value of C, also gives a good fit to the data of Du *et al.* [21], provided that one accepts the proposal of those authors that the effects of impurity scattering may be taken into account by subtracting a constant Γ, independent of ν, from the theoretical energy gap.

The linear wavevector dependence of $\sigma_{xx}(q)$, predicted by (2.8) for $\nu = 1/2$, is just what is needed to explain the anomalous surface acoustic wave propagation, seen at short wavelengths by Willett *et al.* [14]. The absolute values of $\sigma_{xx}(q)$ extracted by Willett *et al.* froin their data are larger than the theoretical values obtaincd from (2.8). however, by a. factor of ≈ 2. The theory of HLR also predicts that the width of the anomaly should depend linearly on q as the magnetic field is varied away from the field corresponding to $\nu = 1/2$. This is in good agreement with tlic experimental observations.

Quasiparticle states for tlie transformed fermions which lie close to the Fermi energy should not have a. significant overlap with the wavefunction of a. single electron added to the ground state of a $\nu = 1/2$ system. A recent analysis by He, Platzman and Halperin [22], building on the results of HLR, suggests that the spectral density $A(\omega)$ for the electron Green's function vanishes as $e^{-\omega_0/|\omega|}$, for $|\omega| \to 0$, wliere ω_0 is a. constant. Following this analysis, they predict a. pseudogap in $A(\omega)$, which is in reasonable agreement with recent tunneling experiments [23].

The general methods of HLR can be applied to various other even-denominator fractions, including $\nu = 1/4, 3/4, 3/2, 3/8$, etc. Chklovskii and Lee have shown, however, that a more sophisticated analysis is necessary to understand the value of tlie electrical conductivity a_r at the higher order even fractions, because the Born approximation for scattering becomes quite poor in this situation [24].

III. Double Layer Systems

As a model to describe a double layer system, ive shall introduce an "isospin" index $\mathbf{r} = \pm 1$, which distinguishes between the two layers, in addition to the position $\vec{r}$ in the x-y plane. The Coulomb interaction between two electrons then has different forms $V_{++}(\vec{r}-\vec{r}')$ and $V_{+-}(7-\vec{r}')$, depending on whether the two electrons are in the same or in different layers [25]. In the simplest case where ench separate layer is considered to be of zero thickness, me may write

$$V_{++}(r) = e^2/\epsilon r \,, \tag{3.1}$$

$$V_{+-}(r) = e^2/\epsilon(r^2 + d^2)^{1/2} \,, \tag{3.2}$$

where d is the separation between the layers. In addition. we introcluce a term to represent tunneling between the layers, which we write as

$$H_t = -tI_x \,, \tag{3.3}$$

where t is the tunneling matrix element, and $\mathbf{I}_x$ is the x component of the total isospin operator $\vec{\mathbf{I}}$. We assume that the actual spins of tlie electrons are completely polarized in the direction of tlie magnetic field, and we consider only the case where there is a mirror symmetry between the two layers. In our discussions we coiisider that the system employed in [15], consisting of a single wide quantum well in which the self-consistent Coulomb potential creates a barrier in the middle of the well, with maxima in the electron density at the two edges, is equivalent to a. double layer system with a relatively large value of the tunneling matrix element t.

We shall limit our discussions here to the case where the *total* filling factor ν is equal to $1/2$; i.e. there is a total of one electron per flux quantum in the two layers combined. Then, if the system is confined to the lowest Landau level, there are essentially two dimensionless parameters in our model $\tilde{d} \equiv d/\ell_0$, and $\tilde{t} \equiv t/(e^2/\epsilon\ell_0)$.

Let us first consider the case where $\tilde{d} = 0$, so that $V_{++} = V_{+-}$. If $\tilde{t}$ is also equal to zero, then the Hamiltonian H_0 possesses full $SU(2)$ symmetry in the isospin $\vec{I}$. In fact, H_0 is equivalent to the Harniltonian for a single layer system with two spin states and no Zeeman term to split the degeneracy. The simplest assurnption (though not universally believed [26]) is that the ground state of the single layer system would be completely polarized at v = 1/2, even in the absence of Zeeman interactions. If this is the case, then for the two layer system with $\tilde{d} = 0$. tlie effect of H_t, for any positive value of $\tilde{t}$, is siinply to align the isospin polarization in the x-direction. Specifically, this means tliat. evcry electron is restricted to the isospin state $I_x = 1/2$, i.e., the lowest subbanc , which is the even combination of states in tlie two layers. Since the Hamiltonian is equivalent to that of a fully polarized single layer system, we expect, as discussed in Section II above, that the ground state can be described by gauge transformed fermions with a single Fermi surface, having $k_F = (4\pi n_e)^{1/2} = \ell_0^{-1}$, and no quantized Hall eífect.

Let us now consider the case where d is nonzero and $\tilde{t}$ is infinite. Every electron must have I, = 1/2, aiid hence all electrons have the same interaction, $\bar{V}(r) = \frac{1}{2}[V_{++}(r)$ + V+-(r)]. If (7 is very large, then $V_{+-} \approx 0$, and $\bar{V}(r) \approx \frac{1}{2} V_{++}(r)$. Therefore, for large $\tilde{d}$, tlie ground state is tlie same as for $d = 0$. and we expect to find a Fermi surface with no quantized Hall effect. (The only change from $\tilde{d} = 0$ is tliat the energy scale is reducecl by a factor of 2.)

According to the numerical calculations of Greiter, Wen and Wilczek [27] for a two layer system with $\tilde{t} = \infty$, there should exist an intermediate range $\tilde{d}_{\min} < \tilde{d} < \tilde{d}_{\max}$, where a. quantized Hall effect. does occur at $\nu = 1/2$. Their calculations suggest tliat tlie quantized Hall state has a very high overlap witli the so-called Pfaffian state, originally described by Moore and Read [5], and further analyzed by Greiter *et al.* [7]. From tlie point of view of tlie ground state symmetry, this state can also be understood in terms of tlie fermion Chern-Simons picture as a state where the fermions near to the Fermi surface are paired in a BCS-like state, with orbital angular momentum I, = −1, and isospin I, = 1. (In terms of untransformed electrons, the state may be crudely described as made up of pairs with angular momentum $\ell_z = 1$, which are then "condensed" into a Laughlin state of degrcc $m = 8$.) Moore and Read have suggested that the charged excitations of tlie Pfaffian state have a different kind of statistics from what might be expected in a simple pairing state, and perhaps there are other subtle differences as well. We shall not distinguish here, however, between the Pfaffian state and the Chern-Simons BCS state with pairing e, = −1 and I, = 1.

Let us next consider tlie case $\tilde{t} = 0$, $\tilde{d} \neq 0$. Now, I, is a. good quantum number of the system, and if there is equal population of the two layers, tlie ground state must have I, = 0. (For $\tilde{d} \neq 0$, the Hamiltonian does not commute with I_x. The ground state has (I,) = 0, for $t = 0$, but is not generally an eigenstate of I,.) In tlie limit $\tilde{d} \to \infty$, for $\tilde{t} = 0$, the system becomes two uncoupled layers, with v = 1/4 in eacli layer. Experiments on single layer systems show that there should be no quantized Hall effect in this case [14]. According to the theory of HLR, there should be a separate Fermi surface of transformed fermions in each layer (seeing separate Chern-Simons fields, with $\tilde{\phi} = 4$, in each layer), and a Fermi wavevector $k_F = (2\pi n_e)^{1/2} = (2\ell_0^2)^{-1/2}$.

Numerical calculations for systems witli $\tilde{t} = 0$ again indicate that for an intermediate range $\tilde{d}'_{\min}$ < d < $\tilde{d}'_{\max}$, there should exist a. quantized Hall plateau at v = 1/2 [28–30]. The ground state in this case has been found to have a high degree of overlap with the so-called 331 state, first proposed in 1983 as a possible generalization of Laughlin's wavefunctions to an even denominator fraction [31]. The 331 state has been characterized by various authors as a system of two types of fermion with a 2 x 2 matrix of Chern-Simons interactions [32]. However, the state may also be characterized iii tlie spirit of Greiter *et al.* [7] as a system of fermions coupled to a single Chern-Simons field (with coupling strength $\phi = 2$), whose ground state has BCS pairing with *i*, = −1 and $I_z = 0$.

What happens for intermediate values of $\tilde{t}$, when $\tilde{d} \neq$ O? A simple schematic phase diagram, compatible with our previous discussion, is presented in Fig. 1.

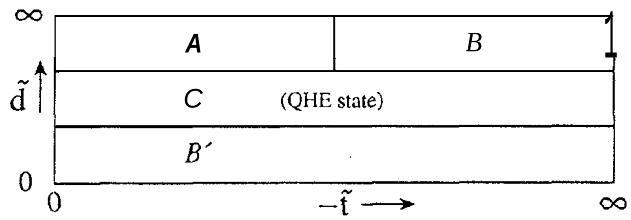

Figure 1: Possible schematic pliase diagram for the ground state of a two lager system at total filling $\nu = 1/2$. Variables $\tilde{d}$ aiid $\tilde{t}$ are respectively tlie separation between layers. in units of the magnetic length ℓ_0, and tlie tunneling strength between layers, in units of $e^2/\epsilon\ell_0$. Phase A has two essentially independent layers of filling factor $\nu = 1/4$, with a separate Fermi surface in each layer, and no quantized Hall effect. Phases B and B' behave like a single layer at v = 1/2, with electrons in tlie subband which is an even combination of states in tlie two layers. These phases have a single Fermi surface for tlie gauge transformed ferniions, and no quantized Hall effect. Phase C is a quantized Hall state which evolves continuously as a. function of $\tilde{t}$ from a state with tlie symmetry of the "331 state" at $\tilde{t} = 0$. to a state with the syrnmetrg of the "Pfaffian" state at $\tilde{t} = \infty$.

The phase labeled A consists of two essentially independent layers with $\nu = 1/4$ and a separate Fermi surface for transformed fermions in each layer. Phases R and B' have a large value of *(1,).* and contain a single Fermi surface for transformed fermions with isospin **I,** $= 1/2$, the even combination of states in the two layers.

The phase labeled C', which occurs for intermediate values of the parameter $\tilde{d}$, is a quantized Hall state. Within tlie fermion Chern-Simons picture, we characterize tlie entire phase as a.state with a. BCS gap at tlie Fermi surface due to pairing iii a state of isospin 1 aiid I, $= -1$. Specifically, we expect pairing of the form

$$c_{\vec{k}\tau} c_{\vec{k}'\tau'} \approx Q(\vec{r})(k_x - ik_y) f_{\tau\tau'} e^{i(\vec{k}+\vec{k}')\cdot\vec{r}} , \quad (3.4)$$

where $c_{\vec{k}\tau}$ is the annihilation operator for a transformed fermion with wavevector $\vec{k}$ aiid isospin τ, the wavevectors $\vec{k}$ and $\vec{k}'$ are close to the Fermi surface at diametrically opposite points, and $Q(\vec{r})$ is an order paramter whose pair correlation function $\langle Q^+(\vec{r})Q(\vec{r}')\rangle$ falls off at large separations as a power of $|\vec{r} - \vec{r}'|$ (i.e., the system has "quasi-long-range order" in the ground statc). Tlie matrix $f_{\tau\tau'}$ is symmetric in the isospin indices, and we hypothesize that it varies continuously a.s a function of tlie tunneling strength $\tilde{t}$ between the two limits:

$$f_{\tau\tau'} \rightarrow \delta_{\tau,-\tau'} , \quad \text{for} \quad \tilde{t} \rightarrow 0 ; , \quad (3.5)$$

$$f_{\tau\tau'} \rightarrow 1 , \quad \text{for} \quad \tilde{t} \rightarrow \infty , \quad (3.6)$$

corresponding to pairs with $I_z = 0$ and **I,** $= 1$, respectively. We also expect that the expectation value $\langle I_x \rangle$ for tlie total isospin of the electron system should increase continuously from **(I,)** $= 0$ at $\tilde{t} = 0$ to $\langle I_x \rangle = N/2$ (full polarization) at $\tilde{t} = \infty$.

Numerical calculations by He et *al.* [29,30] suggest tliat for tlie values of $\tilde{t}$ and $\tilde{d}$ which correspond to the Princeton and Bell Laboratories experiments, there is a high degree of overlap between the ground state at $\nu = 1/2$ and the 331 state, which has $I_z =$ Q Thus, it appears tliat there is only a. small amount of **(I,)** polarization, even for the Princeton experiment where $\tilde{t}$ is relatively large.

The calculations of He *et al.* [30] support the conjecture that a quantized Hall state should exist for an intermediate range of separations d, for any value of the parameter $\tilde{t}$. The conjecture tliat the 331 state can be continuously connected with the quantized Hall state a.t $\tilde{t} = \infty$ is also compatible with the observation by Greiter et *al.* [7] that. tlie Pfaffian state is realized by taking tlie fully antisymmetric part of the spatial portion of the 331 wavefunction.

We do not address here the nature of the phase transitions between the various regions of Fig. 1. Of course, we cannot exclude at this stage the possibility tliat the actual phase diagram is more complicated, with various other intermediate phases occurring. Moreover, if our starting assumption, that the ground state for $\tilde{t} = \tilde{d} = 0$ has complete spontaneous alignment of the isospin vector $\vec{I}$, is *not* correct, then there must be a more complicated phase structure than we have indicated near the lower left corner of Fig. 1. Among the theoretical possibilities for the ground state at $\tilde{t} = \tilde{d} = 0$ are the following: (1) there inight be an isospin singlet ground state with some type of energy gap, which would thus exhibit a quantized Hall effect; (2) there might be an isospin singlet ground state with no energy gap, described within tlie fermion Chern-Simons picture as having a single Chern-Simons field, with $\tilde{\phi} = 2$, and a Fermi surface, with $k_F = (2\pi n_e)^{1/2}$, for each isospin state; or (3) the 331 state might exist as a stable ground state all the way down to tlie point $\tilde{t} = \tilde{d} = 0$. Since the 331 state is not an eigenstate of I^2, it cannot be the true ground state for a finite system at $\tilde{t} = \tilde{d} = 0$; however, it could be the ground state of an infinite system if there is a spontaneously broken isospin symmetry.

We note tliat BCS pairing, in tlie fermion Chern-Simons picture with $\tilde{\phi} = 2$, has also been used to discuss tlie spin-singlet "hollow-core" ground state of Haldane and Rezayi [33], originally proposed as an explanation for the quantized Hall state of a single layer at $\nu = 5/2$. (This is a state where tlic lowest Landau level is completely full and there is a one-half electron per flux quantum in tlie secoiid Landau levcl.) In this case the BCS pairing lias $\ell_z = -2$ for tlie transformed fermions, corresponding to pairs with $\ell_z = 0$ for the origiiial electrons [5,7].

Very recent numerical calculations by R. Morf [34] suggest tlict the correct ground state for $t = d = 0$ is an isospin singlet ground state with no energy gap, as in possibility (2) mentioned above. If this is correct, then thc lower left corner of Fig. 1 shoulcl contain a new phase D, having a single Chern-Simons field witli $\tilde{\phi} = 2$, and two fermi surfaces, witli radii k_{F+} arid k_{F-}, corresponding tc fermions witli $I_z = 1/2$ and $I_z = -1/2$, respectively. [illegible] is increased, the ratio k_{F-}/k_{F+} should decrcase continuously in phase D from tlie value unity, at $\tilde{t} = 0$, uiitil the boundary with phase B is encountered, where $k_{F-} = 0$. The sum $k_{F-}^2 + k_{F+}^2$ must be a constant, $4\pi n_e$.

Recent experimental results reported by Y.W. Suen *et al.* [35] suggest that in actual double layer systems, contrary to the model calculations of Ref. 27, tlie quantized Hall state may be absent regardless of the value of $\tilde{d}$, when $\tilde{t}$ is sufficiently large. If this is correct, then the QHE state C should not extend all the way to tlie right-hand boundary of Fig. 1. Phases B and B' may then be united into a single phase, connected in the right-hand portion of tlie figure.

IV. Conclusion

Although many details remain to be understood, it seems clear that the transformation to fermions with a Chern-Simons field is a powerful tool for understanding the behavior of electrons at $\nu = 1/2$, in hoth single and double layer systems.

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