Interpolated Nambu-Jona-Lasinio Model

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We investigate the breaking of chiral symmetry in the Nambu-Jona-Lasinio model using the linear 6-expansion. It is shown that within this approach quantities like the quark condensate and the pion decay constant, calculated to lowest order, depend only on the scale of chiral symmetry breaking. We compare different ways of evaluating a physical quantity with the δ-expansion.

I. Introduction

The possibility that the symmetry brealtdown of the standard model be a dynamical mechanism involving the top quark[1], has led to a revival of the Nambu-Jona-Lasinio model (NJL)[2]. This model was proposed in tlie early sixties and introduced the idea of dynamical symmetry breaking. It reproduces chiral phenomenclogy with a minimal number of pararneters and is therefore of great iriterest in the study of hadron properties at low energies, where it can be thought as an effective model for QCD. At low energies (around 1 GeV), QCD has a non-perturbative character, and the study of the NJL, model requires the use of non-perturbative techniques. Conventionally it has been treated in a large-N_c approximation[1,3] or in the Hartree-Fock (H-F) approximation[4]. In this paper we investigate the NJL model using an alternative artificial (non-perturbative) rnethod known as the 5 expansion. Since the model is non-renormalizable it will be regarded here as a "toy" model, illustrative of chiral symmetry breaking but which does not have to be necessarily trusted in quantitative detail. The δ expansion was first introduced by Bender et al.[5] who chose to rewrite the interaction term of the ϕ^4 theory as

$$\mu^{1-\delta}\frac{\lambda}{4!}\phi^{2(1+\delta)} , \tag{1}$$

where μ is an arbitrary parameter introduced in order to balance the dimensions and λ is the coupling constant. This way of interpolating the Lagrangian, with 6 as an exponent that modifies the interaction, became known as the logarithmic 6-expansion. There is also a variant of the 5-expansion in which δ enters linearly in the action

$$S_\delta = \delta S + (1-\delta)S_0 , \tag{2}$$

where S is the original action and S_0 is a free action (quadratic in the fields) that contains the arbitrary mass parameter μ. This new way of interpolating the action, known as the linear δ expansion, was proposed by Duncan and Moshe[6] in the context of the Gross-Neveu model. For example, a ϕ^4 interaction is then replaced by

$$\delta\frac{\lambda}{4!}\phi^4 - (1-\delta)\frac{\mu}{2}\phi^2 . \tag{3}$$

It is clear from (1) and (3) that at $\delta = 0$ the interaction term is a mass shift (the theory is free) and that at 6 = 1 we recover the original interaction term, In both cases the theory is expanded as a power series in 6 up to some finite order, and a physical quantity (P) calculated in this way will depend in 6 and μ because the Feynman rules generated by S_δ are 6 and μ dependent. We then set 6 = 1, the value at which we retrieve our original theory, but P remains a function of μ (this would not be tlie case if we were performing a calculation to all orders in 6, because the exact result is μ independent). In the original calculations of Ref. [5] μ was set to unity, and the improvement resulting from the expansion in 0 was purely on tlie basis of new terms introdueed by writing $\phi^{2(1+\delta)}$. However, μ is arbitrary and there is no justification for setting $\mu = 1$ a priori. The arbitrariness of μ (which has to be confronted in both the logarithmic and tlie linear version of tlie 6 expansion) could be regarded as a drawback of the theory. However, it can be turned to advantage by choosing μ optimally in some way. The most general method of fixing μ relies on the Principle of Minimal Sensitivity (PMS) introduced by Stevenson[7], the philosophy being that if an approximant depends on unphysical parameters tlien their values should be chosen so as to minimize the sensitivity of the approximant to small variations of these parameters. So, the essence of the PMS is to require a quantity P, calculated with tlie δ expansion, to be evaluated at a stationary point $\bar{\mu}$ satisfying

$$\frac{\partial P(\mu)}{\partial \mu}|_{\bar{\mu}} = 0 \ , \qquad (4)$$

when $\delta = 1$. This procedure forces the solution to the at least locally (at the stationary point) independent of μ. The logarithmic S expansion improved by the PMS was first employed by Jones and Monoyios[8]. Applying the niethod to field theories in zero and one dimension space time they have obtained results which are more accurate than tlie ones obtained with fixed μ.

When first introduced in Ref. [6], the linear version was not used in conjunction with tlie PMS. Instead, μ was fixed to be the vacuum expectation value of an auxiliary field. However, this choice was based on a previous knowledge of the δ expansion for the effective potential a.t large-N. Motivated by the results obtained by combining the 6 expansion with the PMS, the authors of Ref. [8] also investigated the breaking of discrete chiral symmetry in the Gross-Neveu model[9]. The calculation of the effective potential was carried out with tlie linear δ expansion exactly as in Ref. [6], but μ was fixed via PMS producing encouraging results. Being capable of generating all sorts of non-perturbative functions from what is only a slightly modified perturbation calculation, the PMS then became a very useful ingredient in the linear 6 expansion.

It is important to note that the unphysical parameter can, and should, be adjusted to suit the energy scale of tlie process in question. There is no reason why it should be assigned a universal value to be used when calculating different processes. Choosing the optimisation point separately for each individual physical quantity means that might become a different function of the original parameters each time the condition (4) is imposed. Two recent works[10,11] prove the convergente of the optimized linear expansion (at least in low dimensions) giving a calculational basis to a method which has philosophical appeal apart froin being simple, precise and applicable to any order of perturbation theory.

In the study of the chiral symmetry breaking (CSB), the linear δ-expansion and the PMS condition have been used to calculate the effective potential of tlie Gross-Neveu model (discrete CSB)[9,12,13], of the Abelian version of the Nambu-Jona-Lasinio model in 1+1 dimensions and of the non-Abelian Gell-Mann and Lévy σ-model (continuous CSB)[14].

Here we use the 6-expansion and the PMS to fix the parameters of the model, by using empirical inputs related to the pion; and to calculate physical quantities that are directly related to the breaking of chiral symmetry, namely the quark condensate and the constituent quark mass. An advantage of the method is that it provides a clear way of fixing the parameters and does not put any restriction on the magnitude of N_c, i.e., diagrams belonging to different orders in a $1/N_c$ calculation appear at the same order in S. The fact that the PMS allows us to express μ as a function of the original parameters can also be regarded as an advantage because it can lead to an economy of the number of inputs needed for the computation of certain physical quantities at the leading order. In section II we review the general features of the model. The modified (interpolated) NJL Lagrangian is presented in section III. In section IV we employ the δ-expansion to calculate the relevant physical quantities, and to discuss alternative ways of performing a calculation with the method. The conclusions are presented in section V.

II. The model

The NJL Lagrangian is given by

$$\mathcal{L} = \bar{q}(i\not{\partial} - m_c)q + G[(\bar{q}q)^2 - (\bar{q}\gamma_5\vec{\tau}q)^2] \,, \tag{5}$$

where the fields $q(x)$ represent up and down quarks, m_c is the current quark mass¹ and (in 3+1 dimensions) G is a positive coupling constant with $(\text{mass})^{-2}$ dimensions indicating that this is a non-renormalizable theory which requires the introduction of a cut-off Λ. This cut-off can be regarded as an effective, if crude, implementation of the known short distance behaviour of QCD within the model[15]. In the chiral limit ($m_c = 0$) Eq.(5) is invariant under the chiral transformation

$$q \rightarrow \exp(i\gamma_5\vec{\tau}\cdot\vec{\alpha})q \,. \tag{6}$$

In the standard H-F and large-N_c approaches the calculated physical quantities involve the full quark propagator

$$iS^{-1}(p) = \not{p} - m_c - M_{dyn}(M_q) \,, \tag{7}$$

where $M_{dyn}(M_q)$ is the dynamical mass, induced by quantum fluctuations, calculated using Eq. (7). The constituent (physical) mass is

$$M_q = m_c + M_{dyn}(M_q) \,. \tag{8}$$

For $m_c = 0$, the leading large-N_c result (which is equivalent to the Hartree approximation) reads

$$M_q = G\frac{3}{\pi^2}M_q\left[\Lambda^2 - M_q^2\ln\left(1+\frac{\Lambda^2}{M_q^2}\right)\right] \,, \tag{9}$$

and a non-trivial solution ($M_q \neq 0$) corresponds to CSB. To see more clearly the dependence of M_q on the parameters we rewrite Eq. (9) as

$$\frac{\alpha}{G\Lambda^2} = 1 - \frac{M_q^2}{\Lambda^2}\ln\left(1+\frac{\Lambda^2}{M_q^2}\right) \,, \tag{10}$$

from which we see that for $G\Lambda^2 > \alpha_{\text{crit}}(\sim 3.29)$ the quark acquires a mass. When the breaking of the symmetry occurs, the pion appears as a collective mode of zero mass (Goldstone boson) in the chiral limit. In fact, it can be shown that in this approach the existence of a Goldstone boson leads to the gap Eq. (10)[2]. Conventionally the parameters of the model (G and Λ) are fixed in such a way as to reproduce the empirical values of the decay constant of the pion (f_π = 93MeV) and the pion mass away from the chiral limit (m_π = 138 MeV), as in Ref. [3], or (as in Ref. [4]) to reproduce the empirical values of f_π and $<\bar{q}q>_0 = -(225 \pm 25\ \text{MeV})^{3}$. As already mentioned, both the H-F and the large-N_c methods utilise the full fermion propagator, Eq.(8), in the evaluation of these physical quantities, which therefore become a function of M_q. Then, either G is traded for M_q via the gap equation[4] or M_q has its

¹We consider an unbroken flavour group $SU(2)_F$ with $m_c^u = m_c^d$.

value restricted by a previous knowledge of the empirical value of tlie constituent quark mass[15]. An excellent pedagogical review of the NJL model can be found in Ref. [16].

III. The interpolated Lagrangian

To apply the δ-expansion we first rewrite Eq. (5) as

$$\mathcal{L}_\delta = \bar{q}(i\not{\partial} - m_c)q + \delta G[(\bar{q}q)^2 - (\bar{q}\gamma_5\vec{\tau}q)^2] - (1-\delta)\mu\bar{q}q \;. \quad (11)$$

It is clear from Eq. (11) that at $\delta = 1$ we regain the original Lagrangian, Eq. (5). which (for $m_c = 0$) is invariant under transformation (6) and that at 6 = 0 we have a free massive tlieory which is not chirally invariant (even when m, = 0).

In the 6-expansion approximation we deal with a bare quark propagator of the form

$$iS^{-1}(p) = \not{p} - \mu' \;, \quad (12)$$

where $\mu' = m_c + \mu$. Tlie Lagrangian of Eq. (11) generates a new quadratic vertex of weight $i\delta\mu$ and tlie original Feynman rules for tlie four-fermion vertices of Fig. 1 are now multiplied by 6.

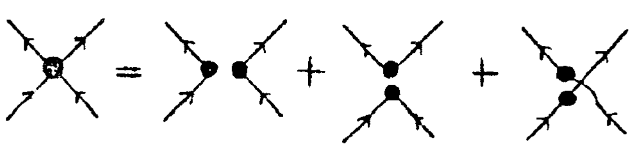


Figure 1: Double dot representation of the four-fermion vertice.

IV. Evaluation of physical quantities

In this section we will use the 6-expansioii to calculate tlie quark condensate, f_π (to fix A), the constituent quark mass and tlie quark-antiquark scattering amplitude in the pseudoscalar channel (to fix G). All tlie loop integrals are regularized by a covariant cut-off. In the case of loops involving two propagators [$S(p)$ and $S(p \pm q)$] we follow tlie procedure of Bernard et al.[15], namely, we introduce Feynman parameters, perform a change of variable on tlie momentum integration so as to get an expression depending only on p^2 and then cut off. When calculating quantities away from the chiral limit, we have m_c as an extra input; its value will be fixed with tlie aid of the Gell-Mann-Oakes-Renner relation. At lowest order ($O(\delta^0)$) the quark condensate is given by the first diagram of Fig. 2

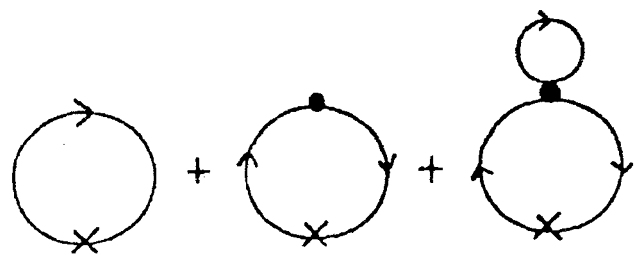


Figure 2: The quark condensate to $O(\delta)$

$$<\bar{q}q>_0 = tr\int_\Lambda d^4p \, \frac{i}{\not{p} - \mu + i\epsilon} \;, \quad (13)$$

which yields

$$<\bar{q}q>_0 = -\frac{N_c}{4\pi^2}\mu\left[\Lambda^2 - \mu^2 \ln\left(1 + \frac{\Lambda^2}{\mu^2}\right)\right] \;. \quad (14)$$

For $N_c = 3$ and upon applying the PMS condition as coiitained in Eqs. (4) to (14) we get

$$\bar{\mu} \approx \frac{\Lambda}{1.348} \;, \quad (15)$$

and

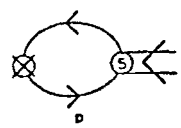
$$- <\bar{q}q>_0^{1/3} = 0.28942 \times \Lambda \;. \quad (16)$$

So, at this order $<\bar{q}q>_0$ is a function of Λ only, showing explicitly that the cut-off is related to the scale of CSB ($h \approx \Lambda_{CSB}$). The result of Eq. (16) also illustrates what was said in the introduction about the economy in the number of inputs needed to evaluate a physical quantity. If the large-N_c or H-F had been used in the same calculation, $<\bar{q}q>_0$ would be a function of Λ and M_q (or G).

To fix R we evaluate f_π using the Pagels and Stokar definition[17]

$$< 0|J^{b\nu}|\pi^a(q) >= -iq^\nu f_\pi \delta_{ab} \ . \tag{17}$$

The first contribution comes from



$$-iq^\nu f_\pi \delta_{ab} = tr \int_\Lambda d^4p S(p+q)(g_{\pi q}\gamma_5 \tau^a) S(p) \left(\frac{1}{2}\tau^b \gamma^\nu \gamma_5\right) \ . \tag{18}$$

In addition we use the Goldberger-Treiman relation obtained with the propagator of Eq. *(12)* to obtain

$$g_{\pi q} = \frac{\mu}{f_\pi(0)} \ . \tag{19}$$

Then, to lowest order Eq. *(18)* is written as

$$f_\pi(q^2) = -i12\mu' g_{\pi q} \int_\Lambda d^4p \int_0^1 d\alpha \frac{1}{[p^2 + q^2(1-\alpha)\alpha - \mu'^2]^2} \tag{20}$$

In tlie chiral limit it becomes

$$f_\pi^2(0) = \frac{3\mu^2}{4\pi^2}\left[\ln\left(1+\frac{\Lambda^2}{\mu^2}\right) - \left(1+\frac{\mu^2}{\Lambda^2}\right)^{-1}\right] \ . \tag{21}$$

The PMS condition

$$\frac{\partial f_\pi}{\partial \mu}|_{\bar{\mu}} = 0 \ . \tag{22}$$

shows that, at this order, $\bar{\mu}$ depends linearly Λ and lias the form

$$\bar{\mu} \approx \frac{\Lambda}{1.47} \ . \tag{23}$$

Then using the empirical value $f_\pi = 93$ MeV we obtain

$$\Lambda = 7.8 \times f_\pi = 725.7MeV \ , \tag{24}$$

to be compared to

$$\Lambda \approx \Lambda_{\rm CSB} \approx 12.56 \times f_\pi \approx 1GeV \ , \tag{25}$$

obtained by Manohar and Georgi[18]. It should be appreciated that a result similar to Eq. *(24)* cannot be obtained with tlie H-F or the large-N_c methods without a previous knowledge of the value of the constituent quark mass. The insertion of Eq. *(23)* into Eq. *(21)* allows us to plot f_π as a function of Λ only, as shown in Fig. *3*, where the δ-expansion result is contrasted with the H-F results of Ref. [4].

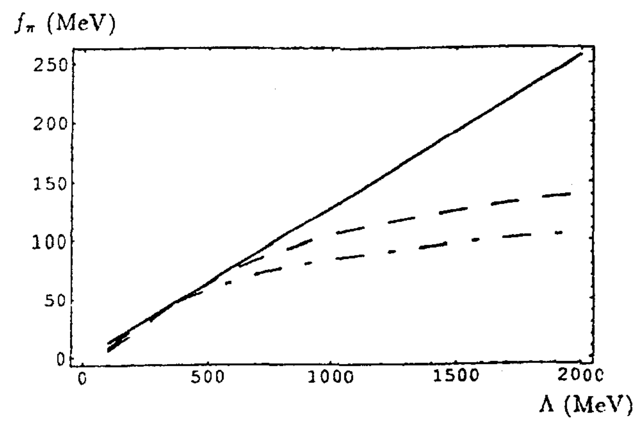


Figure 3: Tlie δ-expansion result (continuous line) for f_π compared to the H-F resuts of Ref. [4] for $M_q = 300$ MeV (dashed line) and $M_q = 200$ MeV (dot-dashed line).

We then estimate

$$< \bar{q}q >_0 = -(210MeV)^3 \ . \tag{26}$$

In Fig. 4 we show tlie dimensionless quantity

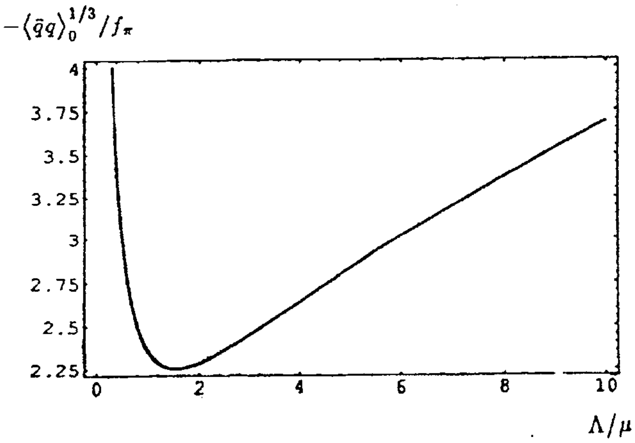


Figure 4: The dimensionless quantity $- < \bar{q}q >_0^{1/3} \times f_\pi^{-1}$ plotted as a function of Λ/μ.

$$-\frac{< \bar{q}q >_0^{1/3}}{f_\pi} ,$$

whose empirical value lies between 2.68 and 2.15 as a function of Λ/μ. It is evident that the PMS variational method places us at the stationary point $\Lambda/\bar{\mu} \approx 1.55$, for which tlie ratio is 2.25.

Next we calculate tlie quark inass, which at $O(\delta^0)$ is given by

$$M_q = \mu' \tag{27}$$

At $O(\delta)$ it becoines

$$M_q = \mu' - \delta\mu + \delta M_{\rm dyn}(\mu') ; \tag{28}$$

with $M_{\rm dyn}(\mu')$ given by the diagram of Fig. 5. which yields

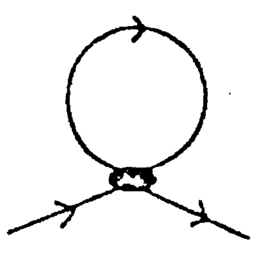


Figure 5: The $O(\delta)$ diagrams contributing to $M_{\rm dyn}(\mu')$.

$$M_q = \mu' - \delta\mu + \delta \frac{G' N_c N_f}{2\pi^2} \mu' \left[\Lambda^2 - \mu'^2 \ln\left(1 + \frac{\Lambda^2}{\mu'^2}\right)\right] , \tag{29}$$

or, in tlie chiral limit, we have

$$M_q = \mu - \delta\mu - 2\delta G' N_f < \bar{q}q >_0 \tag{30}$$

where

$$G' = G\left(1 + \frac{1}{4N_c}\right) , \tag{31}$$

$N_c = 3$ and $N_f = 2$. Eq. (30) shows the relation between tlie $O(\delta)$ M_q and the $O(\delta^0) < \bar{q}q >_0$. For $m_c = 0$, it is clear that the $\bar{\mu}$ which optmises the quantity M_q is given by Eq. (15) independently of G. Note that G' contains an excliange term of order $1/N_c$ which does not appear in the large-N_c calculation. This illustrates tlie fact that diagrams wliich contribute to a given order in the 6-expansion appear at different orders in a $1/N_c$ type of calculation.

In order to estimate M_q we need to fix the parameter G. This can be done by evaluating the propagator of the dynamically generated pseudoscalar boundstate (pion) and requiring it to have a pole at $q^2 = m_\pi^2 = (138MeV)^2$. Because this calculation is done away from the chiral limit, we also need to know the value of the current quark mass (m,). Using the Gell-Mann-Oakes-Renner relation

$$m_c = -\frac{m_\pi^2 f_\pi^2}{2 < \bar{q}q >_0} , \tag{32}$$

and our Eq. (26) for $< \bar{q}q >_0$ we obtain

$$m_c = 8.9 MeV . \tag{33}$$

Diagrams representing the exchange of pion propagator to $O(\delta^2)$ are shown in Fig. 6. where the bubble represents tlie $O(\delta)$ pion self energy.

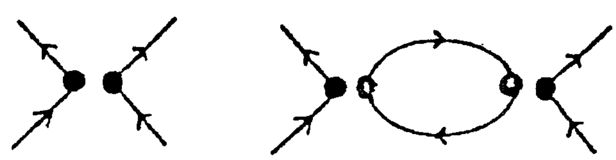


Figure 6: The $O(\delta^2)$ diagrarns contributing to $\Gamma_P(q^2)$.

Adding the diagrams of Fig. 6, we get

$$\Gamma_P(q^2) = -2\delta G[1 + 2\delta G J_P(q^2) + O(\delta^2) + ...] . \quad (34)$$

Using a Padé approximant we can write this last equation in a form capable of expressing the poles needed for the identification of the pion mass

$$\Gamma_P(q^2) = \frac{-2\delta G}{1 - 2\delta G J_P(q^2)} , \quad (35)$$

where

$$J_P(q^2) = tr \int_\Lambda i d^4 p \gamma_5 S(p) \gamma_5 S(p-q) . \quad (36)$$

After taking the trace and integrating the expression (36) me can write tlie denominator of Eq. (35) as

$$1 - 2\delta G J_P(q^2) = 1 - \delta \frac{M_{\rm dyn}(\mu')}{\mu'} - \delta G \frac{3}{\pi^2} I(q^2, \Lambda, \mu') , \quad (37)$$

where

$$\frac{M_{\rm dyn}(\mu')}{\mu'} = G\frac{3}{\pi^2}\left[\Lambda^2 - \mu'^2 \ln\left(1 + \frac{\Lambda^2}{\mu'^2}\right)\right] , \quad (38)$$

and

$$\begin{aligned} I(q^2, \Lambda, \mu') = & \ln\left(1 + \frac{\Lambda^2}{\mu'^2}\right)^{1/2} - \left(\frac{4\mu'^2}{q^2} - 1\right)^{1/2} \arctan\left(\frac{4\mu'^2}{q^2} - 1\right)^{-1/2} \\ & - \frac{\left[1 - \frac{4\mu'^2}{q^2}\left(1 + \frac{\Lambda^2}{2\mu'^2}\right)\right]}{\left[\frac{4\mu'^2}{q^2}\left(1 + \frac{\Lambda^2}{\mu'^2}\right) - 1\right]^{1/2}} \arctan\left[\frac{4\mu'^2}{q^2}\left(1 + \frac{\Lambda^2}{\mu'^2}\right) - 1\right]^{-1/2} \end{aligned} \quad (39)$$

In tlie $1/N_c$ H-F calculations tlie fiist two terms on the right hand side of Eq.(37) cancel in the chiral limit due to tlie gap given by Eq.(9). Away from tlie chiral liinit tliese two terms are replaced by m_c/M_q. In our approach me have either to apply the PMS directly to $\Gamma_P(q^2)$ or t3 require it to have a pole at $q^2 = m_\pi^2$, isolate G and then apply PMS to it. However, as it stands Eq. (35) does not have an extremum and the PMS condition, Eq. (4), fails to fix μ. We can, however, use Eq.(28) to write Eq.(37) as

$$1 - 2\delta G J_p(q^2) = 1 - \frac{M_q(\mu')}{\mu'} + \frac{m_c}{\mu'} + \frac{(\mu - \delta\mu)}{\mu'} - q^2 \delta G \frac{3}{\pi^2} I(q^2, \Lambda, \mu, m_c) . \quad (40)$$

Requiring that Eq.(35) has a pole at $q^2 = m^2\pi = (138MeV)^2$, we have

$$1 - \frac{M_q(\mu')}{\mu'} + \frac{m_c}{\mu'} + \frac{(\mu' - \delta\mu)}{\mu'} - m_\pi^2 \delta G \frac{3}{\pi^2} I(m_\pi^2, \Lambda, \mu, m_c) = 0 . \quad (41)$$

It can be seen from this last equation that in the chiral liinit the existence of a Goldstone boson ($m_\pi^2 = 0$) implies tlie gap equation, fixing $M_q = \mu$. Thus, in this case the Goldstone theorem imposes a much stronger constraint in the determination of μ than the PMS itself. But this is not tlie case away from the chiral liinit. The term which causes Eq.(41) not to have an extremum is $M_q(\mu, m_c)/(\mu + m_c)$. This quantity is equal to unity in the chiral limit. However, away from the chiral limit, we will assume a value ≈ 1; we then have

$$\frac{M_c}{(\mu' + m_c)} + \frac{(\mu - \delta\mu)}{(\mu' + \mu_c)} = m_\pi^2 \delta G \frac{3}{\pi^2} I(m_\pi^2, \Lambda, \mu, m_c) \quad (42)$$

Setting $\delta = 1$, applying tlie PMS condition to

$$G = m_c \frac{\pi^2}{3}[m_\pi^2(\mu + m_c) I(m_\pi^2, \Lambda, \mu, m_c)]^{-1} , \quad (43)$$

and using our inputs we get

$$G = 8.545 \times 10^{-6} MeV^{-2} \quad (44)$$

at $\bar{\mu} = 171.4MeV$. Going back to Eq. (29). setting $6 = 1$ and applying PMS to the quark mass, me obtain

$$M_q = 343.05MeV \ , \qquad (45a)$$

at $\bar{\mu} = 538.37MeV$ in tlie chiral limit, and

$$M_q = 351.95MeV \ , \qquad (45b)$$

at $\bar{\mu} = 529.47MeV$ away from the chiral limit. These results are close to tlie accepted empirical value for the constituent masses of the up and down quarks ($M_q \sim M_n/3 \sim 313MeV$). Fig. 7 displays M_q as a function of μ in the chiral limit for different values of $G\Lambda^2$. Curve 1 (continuous line) corresponds to $G\Lambda^2 = 10.53$, curve 2 (dot-dashed line) was obtained assuming $G\Lambda^2 = 7.27$ and curve 3 (dashed line), corresponds to $G\Lambda^2 = 4.5$. Note that all curves intercept the straight line $M_q = \mu$ at two points. The interception point $M_q = \mu \neq 0$ is the one at which we recover the conventional gap equation in tlie form of Eq.(9). Curve 1 represents the firinly broken regime (with $\bar{\mu}$ to the left of tlie intersection); curve 3 represents the barely broken regime with $\bar{\mu}$ to the right of the intersection and in curve 2 the intersection occurs at $M_q = \bar{\mu} = 538.37MeV$. Note that, for a given value of GA^2, tlie δ-expansion prediction for tlie quark mass (stationary points) is always higher than predicted by other methods (intersection points).

Up to this point all the physical quantities have been calculated with the bare propagator of Eq. (12). As we now have tlie quark mass (as a function of μ) expanded to $0(\delta)$ as in Eq. (20) and also an order-δ numerical value for the quark mass Eq. (45), it would be interesting to evaluate a physical quantity employing different forms of the quark propagator and to compare the results. We will do this by calculating the quark condensate involving $0(\delta)$ corrections in three different ways. First, proceeding as before, we calculate ali the diagrams of Fig. 2 with the bare propagator given by Eq. (12). This gives

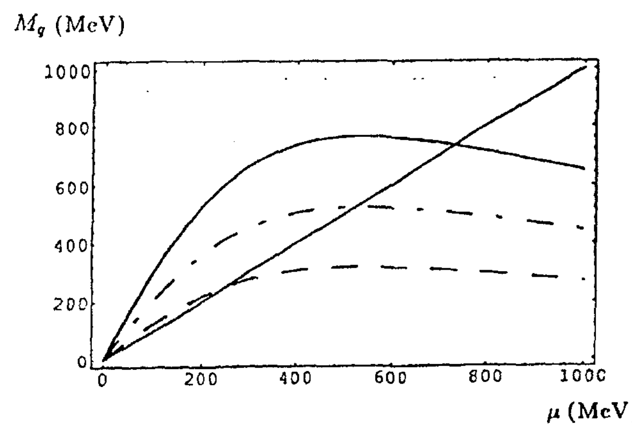


Figure 7: M_q at $O(\delta)$ plotted as a function of μ for different values of Gh^2, as described in tlie text.

$$\langle \bar{q}q \rangle_0 = -\frac{3}{4\pi^2}\mu\left[\Lambda^2 - \mu^2 \ln\left(1 + \frac{\Lambda^2}{\mu^2}\right)\right] + \frac{3\delta}{4\pi^2}\mu\left[\Lambda^2 + 2\Lambda^2\left(1 + \frac{\Lambda^2}{\mu^2}\right) - 3\mu^2 \ln\left(1 + \frac{\Lambda^2}{\mu^2}\right)\right]\left(1 - \frac{M_{\rm dyn}}{\mu}\right) \ , \qquad (46)$$

where

$$M_{\rm dyn} = \frac{13}{4\pi^2}\mu G\left[\Lambda^2 - \mu^2 \ln\left(1 + \frac{\Lambda^2}{\mu^2}\right)\right] \ . \qquad (47)$$

As usual, we set 6 = I and apply PMS to Eq. (46) to obtain $<\bar{q}q>_\phi = -(220.4MeV)^3$ at $\bar{\mu} = 1018MeV$. Note that the $0(\delta)$ result depends in both tlie original parameters of the theory and that $\bar{\mu}$ is not a linear function of Λ as it was at $0(\delta^0)$. Fig. 8 shows the $0(\delta^0)$, the $0(\delta)$ and the H-F result of Ref. [4] for the quark condensate as a function of Λ.

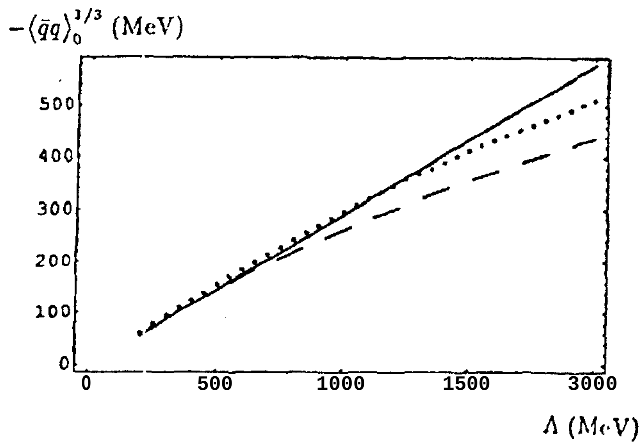


Figure 8: Tlie $O(\delta^0)$ (continuous line), $O(\delta)$ (dotted line) and H-F (dashed line) results for $<\bar{q}q>_0$ as a function of A. Tlie H-F result was obtained in Ref.[4] using $M_q = 300$ MeV as input.

In the second way, we evaluate only the first diagram of Fig. 1 but using the full non-optimisecl propagator

$$iS^{-1}(p) = \not{p} - M_q(\mu) , \qquad (48)$$

with $M_q(\mu)$ expanded to $0(\delta)$ as in Eq. (29). At $6 = 1$ the result is

$$\langle\bar{q}q\rangle_0 = \frac{3}{4\pi^2} M_{\rm dyn}\left[\Lambda^2 - M_{\rm dyn}^2 \ln\left(1 + \frac{\Lambda^2}{M_{\rm dyn}^2}\right)\right] , \qquad (49)$$

with M_{dyn} given in by Eq. (47). Applying PMS to Eq. (49) yields $\langle\bar{q}q\rangle_0 = -(204.18MeV)^3$ at $\bar{\mu} = 538.37$ MeV.

Finally, we evaluate tlie first dlagram of Fig. 1 using the full optirnized propagator

$$iS^{-1}(p) = \not{p} - M_q(\mu) , \qquad (50)$$

with $M_q(\bar{\mu}) = 343.05MeV$ to to obtain $\langle\bar{q}q\rangle_0 = -(204.18MeV)^3$, which is exactly the result obtained with Eq. (49). In Fig. 9 we compare these three ways of calculating $\langle\bar{q}q\rangle_0$. Although the final results are fairly close in the region 700 MeV < Λ < 900 MeV we advocate the first way as being more in line with the linear 6-expansion and PMS philosophy. More than that, it is the first way that gives best result for $\langle\bar{q}q\rangle_0$ at $\Lambda = 725.7MeV$.

The second way employs the full propagator to of Eq. (43) with the self-energy truncated to $O(\delta)$; this propagator is equivalent to an *infinite* sum of reducible two-point Green function containing only one-loop contributions (Fig. 5) to the self-energy. For example, the geometric series expansion of Eq. (48) contains a $O(\delta^2)$ two-point diagram formed by joining two diagrams like the one of Fig. 5, but does not include an irreducible two-point diagram with a two-loop self-energy that would appear at the same order. This corresponds of doing an approximation (6-expansion) within another approximation (one-loop or large $-N_c$). Finally, the third way to calculate $\langle\bar{q}q\rangle_0$ employs a μ-*independent* propagator, in which the self- energy has been truncated at $O(\delta)$ and optimized, corresponding to weaker version of PMS.

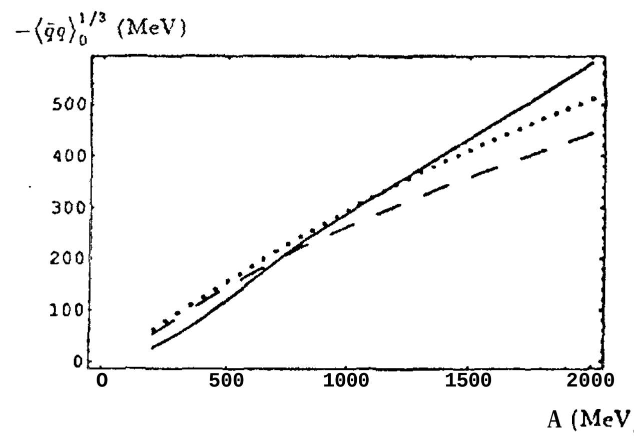


Figure 9: The three different results involving $O(\delta)$ corrections to $<\bar{q}q>_0$. The dotted line represents the result obtained with the bare propagator; the continuous line represents the results obtained with the full non-optimized propagator and the dashed line result was obtained with the full optimized propagator.

V. Conclusions

We have applied the δ-expansion and the PMS to investigate the breaking of chiral symmetry in the Nambu-Jona-Lasinio model. The first interesting result occurrecl in the lowest order calculation of tlie quark condensate and the pion decay constant, two quantities directly related to CSB. Usually these quantities turn out to be a function of Λ/M_q [4]. In our approach, because of the PMS, they became a function of **11** only, emphazing the role of this parameter as the scale of CSB. Using the empirical value of f_π we have fixed Λ = 725.7 MeV, that is, of tlie order of the hadron scalc. We faced *a* problem when trying to apply the PMS to the quark-antiquark scattering amplitude, Eq. (35), in order to fix G, because this quantity does not have an extremum for a. finite value of μ. We then assumed that a quantity $(M_q(\mu, m_c)/(\mu + m_c))$ which has tlie unity value in the chiral limit does not have its value changed significantly away from this limit. This is particularly true for tlie quark mass. The need to impose this constraint in our approach can be compared with the imposition of further constraints in the same type of calculation carried out with otlier methods, where the value of the quarlí mass is restricted within certain limits[15].

Having fixecl G, we calculated the constituent quark mass to $O(\delta)$ including diagrams which would not be taken into account in a $1/N_c$ calculation. The final result agrees with tlie einpirical data. We then calculated the quark condensate involving order-δ corrections in three different ways. It was shown that tlie calculation which adheres most rigidly with tlie δ-expansion and the PMS gives the best numerical result.

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References

1. W. A. Bardeen, C. T. Hill and M. Lindener, Phys. Rev. **41**, 1647 (1990).
2. Y. Nambu aiid G. Jona-Lasinio, Phys. Rev. 122, 345 (1961).
3. T. Hatsuda and T. Kunihiro, Phys. Lett. **B198**, 126 (1987).
4. V. Bernard, Phys. Rev. **D34**, 1601 (1986).
5. C. Bender, K/ Xlilton, M. Moshe, S. S¿ Pinsky aiid L. M. Simmons, Phys. Rev. Lett. 58, 2615 (1987); Phys. Rcv. D37, 1472 (1988).
6. A. Duncan and M. Moshe, Phys. Lett. **B215**, 325 (1988).
7. P. M. Stevenson, Phys. Rev. **D23,** 2916 (1981).
8. H. F. Jones aiid M. Monoyios, Int. J. of Mod. Phys. A4, 17 (1989).
9. H. F. Jones and M. Monoyios, Imperial College preprint (IMPERIAL 89/92).
10. I. Buckley, A. Duncan and H. F. Jones, Phys. Rev. D47, 2554 (1903).
11. A. Duncan and H. F. Jones, Phys. Rev. **D47**, 2560 (1993).
12. A. Okopińiska, Pliys. Rev. D35, 1835 (1987); 36, 2415 (1987).
13. S. K. Gandhi, H. F. Jones and M. B. Pinto, Nucl. Phys. **B359**, 429 (1991).
14. S. K. Gandhi and M. B. Pinto, Phys. Rev. D46, 2570 (1992).
15. V. Bernard, R. I,. Jaffe and U.-G. Meissner, Nucl. Pliys. **B308**, 753 (1988).
16. V. Bernard aiid U.-G. Meissner, Commun. Nucl. Part. Phys. 19, 67 (1989).
17. H. Pagels and S. Stokar, Phys. Rev. **D20,** 2947 (1979).
18. H. Georgi and A. Manoliar, Nucl. Phys. 234, 189 (1984).