Counterexamples to Schiff's Conjecture*

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It is shown that gravitational non-minimally coupled theories, in general, are compatible with the weak equivalence principle, which directly implies that the former are counterexamples to Schiff's conjecture. The proof is model-independent.

The weak equivalence principle (WEP), i.e., the universality of free-fall trajectories, has played a crucial role in the development of gravitational theory. Newton regarded this principle as such a cornerstone of mechanics that he devoted the opening paragraphs of the ***Principia*** to a thorough discussion of it. Thus, it is not surprising that the considerable accuracy with which the Eötvös experiments confirmed WEP led Schiff[1] by 1960 to the view that any theory of gravity that embodies WEP necessarily embodies the Einstein equivalence principle (EEP), i.e., the minimal coupling principle. Therefore, according to Schiff's conjecture the following relation between the two equivalence principles should hold:

$$WEP \rightleftarrows EEP \, .$$

Around 1977, Wei-Tou Ni[2] produced evidence against Schiff's point of view: a pseudoscalar field ϕ that couples to electromagnetism in a Lagrangian term of the form $\phi\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$, where $\epsilon^{\mu\nu\rho\sigma}$ is the completely antisymetric Levi-Civita symbol[3]. Ni's counterexample to Schiff's speculation is a simple example of what is currently called a gravitational non-minimally coupled theory. In the following we shall show that gravitational non-minimally coupled theories, in general, are incompatible with Schiff's conjecture. Consequently, Ni's result is not a fortuitous one.

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Let $L_I = L_I(\psi^A, g_{\mu\nu})$ be the Lagrangian density for some field non-minimally coupled to gravity. The field is described by the variables ψ^A , where A represents any tensor indices. The action for the gravitational non-minimally coupled theory is

$$S(x) = \int d^4x\mathcal{L}(x) \; , \tag{1}$$

where

$$\mathcal{L}(x) = \mathcal{L}_E + \mathcal{L}_I + \mathcal{L}_M \; , \tag{2}$$

and

$$\begin{aligned} \mathcal{L}_E &= \frac{R\sqrt{-g}}{k} \; , \\ \mathcal{L}_I &= \sqrt{-g}L_I(\psi^A, g_{\mu\nu}) \; , \\ \mathcal{L}_M &= \sqrt{-g}L_M \; , \end{aligned} \tag{3}$$

where k is the Einstein constant and L_M is the Lagrangian density for the usual matter. We assume that L_M does not contain ψ^A. Since S is a scalar, it is unchanged under the infinitesimal coordinate transformation

$$\delta_c x^\mu \equiv \bar{x}^\mu - x^\mu = \xi^\mu \; . \tag{4}$$

Hence

$$\begin{aligned} 0 &= \bar{S}(\bar{x}) - S(x) \\ &= \int d^4x[\bar{\mathcal{L}}(x) - \mathcal{L}(x)] \\ &= \int d^4x\delta_c\mathcal{L}(x) \\ &= \int d^4x\delta_c\mathcal{L}_E + \int d^4x\delta_c\mathcal{L}_I + \int d^4x\delta_c\mathcal{L}_M \; . \end{aligned} \tag{5}$$

$\int d^4x\delta_c\mathcal{L}_E$, $\int d^4x\delta_c\mathcal{L}_I$ and $\int d^4x\delta_c\mathcal{L}_M$ are of course separately invariant, by the very nature of covariantization.

Let us then analyse what this invariance gives us in each case.

(i) $\int d^4x\delta_c\mathcal{L}_E = 0$;

$$0 = \int d^4x\delta_c\mathcal{L}_E = -\int d^4x\sqrt{-g}\frac{G^{\mu\nu}}{k}\delta_c g_{\mu\nu} \ . \qquad (6)$$

Since

$$\delta_c g_{\mu\nu} = -(\nabla_\mu\xi_\nu + \nabla_\nu\xi_\mu) \ ,$$

we get

$$0 = -\int d^4x\frac{2\sqrt{-g}}{k}(\nabla_\mu G^{\mu\nu})\xi_\nu \ . \qquad (7)$$

If we take into consideration that ξ_ν is arbitrary, equation (7) then gives

$$\nabla_\mu G^{\mu\nu} = 0 \qquad (8)$$

which is nothing but the contracted Bianchi identity.

(ii) $\int d^4x\delta_c\mathcal{L}_I = 0$;

$$0 = \int d^4x\frac{\delta L_I}{\delta\psi^A}\delta_c\psi^A + \int d^4x\frac{\delta L_I}{\delta g_{\mu\nu}}\delta_c g_{\mu\nu} \ . \qquad (9)$$

But $\delta L_I/\delta\psi_A = 0$ are the ψ^A-equations, which directly implies that

$$\nabla_\mu T_I^{\mu\nu} = 0 \ . \qquad (10)$$

Therefore the energy-momentum tensor for the gravitational non-minimal coupling is covariantly conserved on ψ^A - mass-shell irrespective of any properties of the rest of the action. This is a very important result since, in practice, it saves us the tedious work of computing the divergence of the stress tensor.

(iii) $\int d^4x\delta_c\mathcal{L}_M = 0$.

Repeating the reasoning that led previously to equation (10) we now find that

$$\nabla_\mu T_M^{\mu\nu} = 0 \qquad (11)$$

which tells us that the exchange of energy between matter and the gravitational field is described exactly as in Einstein's theory. This amounts to one version of WEP. So we arrive at the following theorem:

Theorem - Any theory whose action is given by *(1)* do obey the weak equivalence *principle.*

This theorem encompasses the results previously found by Ni[2], Accioly[4] and Accioly and Wichoski[5]. It tells us that the universality of free-fall trajectories does not imply the validity of EEP. Hence the relation between the two equivalence principles must be given by

$$EEP \ \underset{\not\Leftarrow}{\Rightarrow} \ WEP$$

Using the above theorem, we can easily show that:

Corollary - Gravitational non-minimally coupled theories, are *in* general, counterexamples to *Schiff's conjecture.*

We have so far assumed that L_M does not include ψ^A. Let us then suppose that L_M depends, for instance, on a scalar field ϕ. Instead of (11) we now obtain

$$\nabla_\mu T_M^{\mu\nu} = -\rho J^\nu \ , \qquad (12)$$

where

$$\rho \equiv \frac{\delta\mathcal{L}_M}{\delta\phi} \ , \quad J^\nu \equiv \partial^\nu\phi \ . \qquad (13)$$

Since $\delta\mathcal{L}_I/\delta\phi + \delta\mathcal{L}_M/\delta\phi = 0$ equation (12) can be rewritten as

$$\nabla_\mu T_M^{\mu\nu} = \frac{\delta\mathcal{L}_I}{\delta\phi}\partial^\nu\phi \ . \qquad (14)$$

Clearly, $\nabla_\mu T_I^{\mu\nu} = -\frac{\delta\mathcal{L}_I}{\delta\phi}\ \partial^\nu\phi$. So, neither $T_M^{\mu\nu}$ nor $T_I^{\mu\nu}$ are covariantly conserved. However, *in* a region of space-time in which ϕ is not changing, *such* as *here* and now, we do have $\nabla_\mu T_I^{\mu\nu} = 0$ and $\nabla_\mu T_M^{\mu\nu} = 0$. These results are of course completely independent of the model used to describe the non-minimal interaction between ϕ and the curvature. Zee[6] and Accioly and Pimentel[7] have arrived at equation (12) making use of specific models to describe the gravitational non-minimal coupling. Equation (14) was firstly obtained by Deser[8] when studying the divergence of stress tensors in external fields. A last remark: The method we have used to obtain equation (12) is equally applicable to any other tensor field ψ^A.

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