The Role of the $\Delta N\pi$ Vertex as Part of the A N Interaction

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We evaluate the one pion exchange contribution due to the $\Delta N\pi$ vertex to the isospin one $^5S_2$ and $^5P_3$ A N amplitudes, and discuss the effect of its addition to the short-range part of the A N interaction which was evaluated previously from an analysis of nd elastic scattering.

I. Introduction

In a series of papers[1-3] we were able to show that the inclusion of a residual A N interaction yields substantial and very specific improvements in the results of Faddeev calculations[4] for nd elastic scattering. These contributions lead to an almost perfect description of all measured observables in this process. The extracted values for these parts of the A N amplitudes show a very smooth energy dependence and can be well described by reasonable potentials[5]. However, our extraction of the parameters of the A N interaction is based on theoretical calculations of Faddeev type[4], which include the ANn– vertex via the πN scattering amplitude, contributions of which are already included in the background amplitude, and thus the A N interaction parameters extracted through our procedure are only the residual ones. We must be aware that a large contribution of the ANn-vertex would interfere non-linearly with the residual interaction. This would not be of large importance for the overall description of the elastic πd data, since the main contribution to it is given by the simple impulse approximation, and the A N interaction is only a correction to that, but it may influence the extracted parameter values severely.

In the present note we evaluate the one-pion-exchange graph containing the $\Delta N\pi$ vertex shown in fig. 1, and project out the two A N partial waves whose contributions have been shown to be decisive in the analysis of πd elastic scattering, namely the isospin $\mathbf{I} = 1$, $^5S_2$ and $^5P_3$ waves.

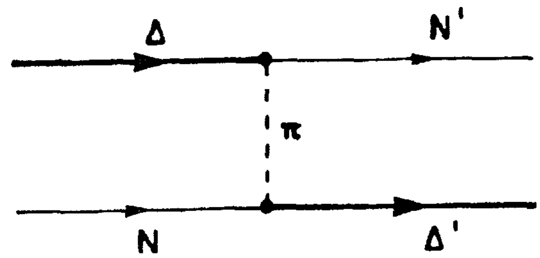

Figure 1: One pion exchange diagram for the A N interaction due to the $\Delta N\pi$-vertex.

It is here important to recall the question of the definition of the scattering amplitude for unstable states. The A instability manifests itself in singularities of the Born term on the physical sheet. The denominator in the propagator of the n-exchange graph, $1/(u - m_\pi^2 + i\epsilon)$, where u is the squared 4-momentum transfer from the initial **A** to the final nucleon, may vanish, thus leading to an imaginary part for the Born amplitude.

We have dealt with this problem in two different ways:

1) - We just ignore the imaginary part of the Born amplitude and proceed with the real part only. Then the real part of the partial wave amplitude still shows logarithmic singularities in the energy, but these disppear as cne smears over the mass of the A, taking into account its finite width.

2) - We give the resonance mass a value below the N a thresholtl.

Although being independent in principle, these two methods lead to similar results for the amplitudes, in the range of momenta of our interest.

II. Born Amplitude and Partial Waves

The Born amplitude corresponding to the diagram of fig. 1 for the A N interaction in the $\mathbf{I} = 1$ state is given by

$$\begin{aligned} M &= (-1/3)g^2_{\Delta N\pi}\ \bar{u}^\mu(\Delta')u(N)\cdot(\Delta'-N)_\mu\frac{1}{(\Delta-N')^2-m_\pi^2+i\epsilon} \\ &\quad \bar{u}(N)u^\sigma(\Delta)(\Delta-N')_\sigma(i^3)(2\pi)^4(-1)\delta(N'+\Delta'-N-\Delta)\ . \end{aligned} \tag{1}$$

Here A, A', **A'**, N' denote the four-momenta of the in and outgoing A-resonance and nucleon respectively and m_π is the picn mass. $\bar{u}^\mu(\Delta)$ and $u(N)$ are respectively the Rarita-Schwinger and Dirac spinors and $g_{\Delta N\pi}$ is the $\Delta^{++}p\pi^+$ coupling constant, with $g^2_{\Delta N\pi}/4\pi = 20.4$ GeV^{-2}. The factor (–1) is due to the exchange of the **A'** and N' ir the final state.

We are interested in the amplitudes at cm momenta below 0.4 GeV, so that we may keep only the leading non-relativistic terms. Furthermore, for the cases of our interest (the $^5S_2$ and $^5P_3$ A N partial waves), we only consider statrs with total spin $\mathbf{S} = 2$, hence the A and N spins can be aligned, and we obtain for the $S = 2$ part of the amplitude $\mathcal{T}$, defined through

$$\mathrm{M} = i(2\pi)^4\mathcal{T}\cdot\delta^4(\Delta + \mathrm{N} - \mathrm{A'} - N')\quad , \tag{2}$$

the expressicn

$$\begin{aligned} \mathcal{T}^{(S=2)} &= (-1/3)\, g^2_{\Delta N\pi}\epsilon_+^{*\mu}(\Delta')N_\mu N'_\sigma\epsilon_+^\sigma(\Delta) \\ &\cdot\ \frac{1}{\beta-\gamma p^2-2p^2\hat{p}\cdot\hat{p}'}\ . \end{aligned} \tag{3}$$

Here $\vec{p}$ and $\vec{p}\,'$ are the A and **A'** momenta, respectively, in the A N cm frame,

$$|\vec{p}|^2 \equiv p^2 = \{(s + m_N^2 - m_\Delta^2)^2 - 4sm_N^2\}/4s\quad , \tag{4}$$

with

$$\hat{p} = \vec{p}/|\vec{p}|\quad ,$$

$$\gamma = m_\Delta/m_N + m_N/m_\Delta \tag{5}$$

and

$$\beta = (m_\Delta - m_N)^2 - m_\pi^2 + i\epsilon\quad . \tag{6}$$

In the denominator in eq.(3), we have kept only terrns of lowest order in (p^2/m^2). We remark that $\gamma \geq 2$ identically, and for the physical masses $m_\Delta = 1.211$, $m_N = 0.94$, its value y = 2.06 is only slightly larger than 2.

In a nonrelativistic approximation the polarization four vector $\epsilon^\mu(\Delta)$ can be expressed through the polarization vector Eof the A in its rest frame in the form

$$\epsilon^\mu(\Delta) = \left(\vec{\epsilon}\cdot\vec{p}\,\frac{1}{m_\Delta},\ \vec{\epsilon}\right)\quad . \tag{7}$$

We then obtain, with

$$\vec{\epsilon}_+\cdot\hat{p} = \sqrt{\frac{4\pi}{3}}Y_{11}(\hat{p})\quad , \tag{8}$$

the expression for the amplitude is written as

$$\begin{aligned} \mathcal{T}^{(S=2)} &= -\frac{4\pi}{9} g^2_{\Delta N\pi} p^2 \left\{ \frac{m_N}{m_\Delta} Y_{11}(\hat{p}) Y^*_{11}(\hat{p}) + \frac{m_N}{m_\Delta} Y_{11}(\hat{p}') Y^*_{11}(\hat{p}') \right. \\ &+ \left. \frac{m_N^2}{m_\Delta^2} Y_{11}(\hat{p}) Y^*_{11}(\hat{p}') + Y_{11}(\hat{p}') Y^*_{11}(\hat{p}) \right\} \frac{1}{\beta - \gamma p^2 - 2p^2(\hat{p}\cdot\hat{p}')} \quad . \end{aligned} \tag{9}$$

In order to facilitate the angular projection, we use the identity

$$\frac{1}{\beta - \gamma p^2 - 2p^2(\hat{p}\cdot\hat{p}')} = (-i)\int_0^\infty d\alpha \ \exp(i\alpha(\beta - \gamma p^2)) \cdot 4\pi \sum_{m\ell} (-i)^\ell j_\ell(2\alpha p^2) \cdot Y_{\ell m}(\hat{p}) Y^*_{\ell m}(\hat{p}') \tag{10}$$

This yields the convenient expression

$$\begin{aligned} \mathcal{T}^{(S=2)} &= i\frac{4\pi}{9} g^2_{\Delta N\pi} p^2 \int_0^\infty d\alpha \ \exp(i\alpha(\beta - \gamma p^2)) \times \\ & (4\pi) \sum_{\ell m} (-i)^\ell j_\ell(2\alpha p^2) Y_{\ell m}(\hat{p}) Y^*_{\ell m}(\hat{p}') \times \\ & \left\{ \frac{m_N}{m_\Delta} Y_{11}(\hat{p}) Y^*_{11}(\hat{p}) + \frac{m_N}{m_\Delta} Y_{11}(\hat{p}') Y^*_{11}(\hat{p}') \right. \\ & \left. + \frac{m_N^2}{m_\Delta^2} Y_{11}(\hat{p}) Y^*_{11}(\hat{p}') + Y_{11}(\hat{p}') Y^*_{11}(\hat{p}) \right\} \quad . \end{aligned} \tag{11}$$

We now perform the angular projection of the $\mathcal{T}^{(S=2)}$ amplitude, to build the $\mathcal{T}^{J,S}_{\ell\ell}$ partial amplitudes for the $^5S_2$ and $^5P_3$ states, respectively, through

$$\begin{aligned} \mathcal{T}^{2,2}_{00}(s) &= \int \mathcal{T}^{(S=2)} Y_{00}(\hat{p}) Y_{00}(\hat{p}') d\Omega_{\hat{p}} d\Omega_{\hat{p}'} \\ &= i(4\pi/9) g^2_{\Delta N\pi} p^2 \int_0^\infty d\alpha \ \exp(i\alpha(\beta - \gamma p^2)) \times \\ & \left\{ \frac{2m_N}{m_\Delta} j_0(2\alpha p^2) - i j_1(2\alpha p^2) \frac{m_\Delta^2 + m_N^2}{m_\Delta^2} \right\} \end{aligned} \tag{12}$$

and

$$\begin{aligned} \mathcal{T}^{3,2}_{11}(s) &= \int \mathcal{T}^{(S=2)} Y^*_{11}(\hat{p}) Y_{11}(\hat{p}') d\Omega_{\hat{p}} d\Omega_{\hat{p}'} \\ &= i(4\pi/9) g^2_{\Delta N\pi} p^2 \int_0^\infty d\alpha \ \exp(i\alpha(\beta - \gamma p^2)) \times \\ & \left\{ -i\frac{12}{5}\frac{m_N}{m_\Delta} j_1(2\alpha p^2) + \frac{m_N^2}{m_\Delta^2} j_0(2\alpha p^2) - j_2(2\alpha p^2) \left[\frac{m_N^2}{5m_\Delta^2} + \frac{6}{5} \right] \right\} \end{aligned} \tag{13}$$

The integrations over α give

$$I_0(p^2) \equiv ip^2 \int_0^\infty \exp(i\alpha(\beta - \gamma p^2)) j_0(2\alpha p^2) \ d\alpha = \frac{1}{4} \ln \frac{\beta - \gamma p^2 - 2p^2}{\beta - \gamma p^2 + 2p^2} \ , \tag{14}$$

$$I_1(p^2) \equiv p^2 \int_0^\infty \exp(i\alpha(\beta - \gamma p^2)) j_1(2\alpha p^2) \ d\alpha = B I_0 + \frac{1}{2} \tag{15}$$

and

$$I_2(p^2) \equiv ip^2 \int_0^\infty \exp(i\alpha(\beta - \gamma p^2)) j_2(2\alpha p^2) \ d\alpha = \frac{1}{2}(1 - 3B^2) I_0 - \frac{3}{4} B \quad , \tag{16}$$

where

$$B = \frac{\beta}{2p^2} - \frac{\gamma}{2} \quad . \tag{17}$$

These quantities $I_j(p^2)$ may develop imaginary parts as determined by

$$\begin{aligned} I_0(p^2) &= \frac{1}{4}\ln\frac{\beta-\gamma p^2-2p^2}{\beta-\gamma p^2+2p^2} = \frac{1}{4}\left\{\ln\left|\frac{\beta-\gamma p^2-2p^2}{\beta-\gamma p^2+2p^2}\right|\right. \\ &+ \left.\frac{i\pi}{2}\left[\mathrm{sign}(\beta-\gamma p^2-2p^2)-\mathrm{sign}(\beta-\gamma p^2+2p^2)\right]\right\} \quad . \end{aligned} \tag{18}$$

As mentioned in the introduction, we deal with the instability problem in two ways. In one procedure, we ignore the existence of the imaginary part of the Born amplitude, while in the other one we displace the Δ-resonance mass to a value below the πN threshold so that $\beta-\gamma p^2+2p^2<\beta<0$, in which case there is no imaginary part to begin with (we recall that $\gamma>2$).

The explicit forms taken by the real parts of the two partial wave amplitudes are, for both cases,

$$\begin{aligned} \mathrm{Re}\mathcal{T}_{00}^{2,2}(s) &= \frac{4\pi}{9}g_{\Delta N\pi}^2\frac{1}{2m_\Delta^2}\left\{\frac{1}{4}\ln\left|\frac{\beta-\gamma p^2-2p^2}{\beta-\gamma p^2+2p^2}\right|\times\right. \\ &\times\left.\left[4m_Nm_\Delta+2\left(\frac{\beta}{2p^2}-\frac{\gamma}{2}\right)(m_N^2+m_\Delta^2)\right]+m_N^2+m_\Delta^2\right\} \end{aligned} \tag{19}$$

and

$$\begin{aligned} \mathrm{Re}\mathcal{T}_{11}^{3,2}(s) &= \frac{4\pi}{9}g_{\Delta N\pi}^2\frac{3}{10m_\Delta^2}\left\{\frac{1}{4}\ln\left|\frac{\beta-\gamma p^2-2p^2}{\beta-\gamma p^2+2p^2}\right|\times\right. \\ &\times\left[(m_N^2+6m_\Delta^2)\left(\frac{\beta}{2p^2}-\frac{\gamma}{2}\right)^2+8m_Nm_\Delta\left(\frac{\beta}{2p^2}-\frac{\gamma}{2}\right)+3m_N^2-2m_\Delta^2\right] \\ &\left.+4m_Nm_\Delta+\frac{1}{2}(m_N^2+6m_\Delta^2)\left(\frac{\beta}{2p^2}-\frac{\gamma}{2}\right)\right\} . \end{aligned} \tag{20}$$

For the case $\beta>(\gamma-2)p^2$, the amplitudes show logarithmic singularities for $(\gamma+2)p^2>\beta$. If we take into account the mass spread of the A-resonance and integrate the amplitude over the m_Δ-mass, we obtain regular results for any value of β. We thus evaluate in this case the averaged amplitudes

$$\begin{aligned} \mathrm{Re}\bar{\mathcal{T}}_{\ell\ell}^{J,S}(s) &= \int_{m_N}^{\infty} dm_\Delta \mathrm{Re}\mathcal{T}_{\ell\ell}^{J,S}(m_\Delta,s) \\ &\frac{\Gamma/(2\pi)}{(m_\Delta-m_R)^2+\Gamma^2/4} \quad , \end{aligned} \tag{21}$$

with s fixed, $m_R=1.211$ GeV and $\Gamma=0.11$ GeV.

Near the AN threshold, the finite mass spread of the A-resonance influences strongly the value of $|\vec{p}|$, and we use the same procedure as employed in the analysis of the data: we evaluate $|\vec{p}|^2$ using the complex mass $m_R+i\Gamma/2$ in eq. (4) and identifying $|\vec{p}|$ with $|\sqrt{p^2}|$.

In Table 1 we present numerical values for $\mathcal{T}_{00}^{2,2}$ and $\mathcal{T}_{11}^{3,2}$, both for $m_\Delta=1.211$ GeV and for aficticious stable A with mass value $m_\Delta=m_N+$ m, x $(1-10^{-6})$. In the latter case the expressions have been evaluated for the same values of p^2 (thus not the same values of s) as used for the experimental mass case. In the same table we also present the values of the phases obtained for the two cases.

Table 1. Born amplitudes of the one pion exchange diagram for the $^5S_2$ and $^5P_3$ states. The unprimed quantities are evaluated with central mass values $m_\Delta = 1.211$ GeV, and amplitudes smeared over the physical finite width of the A, while the primed quantities are evaluated using a ficticious value $m_\Delta = m_N + m_\pi(1-10^{-6})$ for the **A** mass. In the last two columns we give the total AN phases for these waves, i.e. the combination of the residual phases of ref.[3] and an interpolation of the δ_ℓ and δ'_ℓ values. It should be emphasized that for the S-wave this value should only serve as a rough estimate, since in this case an addition of the phases is not justified.

$\sqrt{s}$ [GeV]	$\frac{9}{4\pi g^2_{\Delta N\pi}}\times$ Re $\mathcal{T}^{(2)}_{00}$	$\frac{9}{4\pi g^2_{\Delta N\pi}}\times$ Re $\mathcal{T}^{(3)}_{11}$	$\frac{9}{4\pi g^2_{\Delta N\pi}}\times$ Re $\mathcal{T}^{(2)'}_{00}$	$\frac{9}{4\pi g^2_{\Delta N\pi}}\times$ Re $\mathcal{T}^{(3)'}_{11}$	δ_0 (degrees)	δ'_0	δ_1	δ'_1	δ^{tot}_0	δ^{tot}_1
2.15	0.422	0.102	0.822	0.101	6	12	1.5	1.4	31	30
2.16	0.444	0.118	0.820	0.101	8	14	2.0	1.8	36	30
2.18	0.498	0.134	0.817	0.106	12	19	3.2	2.6	42	33
2.20	0.573	0.136	0.813	0.109	17	23	4.1	3.3	51	33
2.22	0.620	0.125	0.813	0.111	21	26	4.4	3.9	53	33
2.24	0.663	0.108	0.812	0.112	25	30	4.3	4.5	56	32
2.26	0.693	0.089	0.811	0.113	28	32	3.9	5.0	57	30
2.28	0.705	0.071	0.811	0.114	31	34	3.4	5.5	58	25

The evaluated amplitudes are related to the Argand amplitudes through

$$T^J_{\ell\ell} = \frac{\exp(2i\delta^J_\ell)-1}{2i} = \frac{|\vec{p}|}{8\pi^2}\cdot\frac{m_N\cdot m_\Delta}{m_N+m_\Delta}\cdot\mathcal{T}^{J,S}_{\ell\ell}(s) \quad , \tag{22}$$

so that the phases are given by

$$\delta^J_\ell \equiv \tan^{-1}\left\{\frac{|\vec{p}|}{8\pi^2}\,\frac{m_N\cdot m_\Delta}{m_N+m_\Delta}\,\mathcal{T}^{J,S}_{\ell\ell}(s)\right\}. \tag{23}$$

III. Discussion and Comments

The S-wave amplitude is, except for very small values of p^2, dominated by the large constant term $(m_N^2+m_\Delta^2)/2m_\Delta^2 \approx 0.8$, while in the p-wave amplitudes we have for the constant term the value $3\left[16m_Nm_\Delta - \gamma(m_N^2+6m_\Delta^2)\right]/40m_\Delta^2 \approx -0.06$, which is an order of magnitude smaller. These constant terms disappear if a $\Delta N\pi$ form factor is introduced. Introducing a parameter p, with dimensions of mass, to replace the $\Delta N\pi$-vertex $g_{\Delta N\pi}$ by

$$g_{\Delta N\pi}\times\frac{-\rho^2}{u-\rho^2+i\epsilon}\ , \tag{24}$$

where $u = (\Delta - N)^2$, we can incorporate this modification analytically in the evaluation of the amplitudes. In the approximation $m_\pi^2 \ll \rho^2$, we have

$$\frac{1}{u-m_\pi^2+i\epsilon}\cdot\frac{\rho^4}{(u-\rho^2+i\epsilon)^2} = \frac{1}{u-m_\pi^2+i\epsilon} - \frac{1}{u-\rho^2+i\epsilon} + \rho^2\,\frac{\partial}{\partial\rho^2}\,\frac{1}{(u-\rho^2+i\epsilon)}\ , \tag{25}$$

and the S-wave amplitude with $\Delta N\pi$ form factor then becomes, in the place of eq. (19),

$$\mathcal{T}^{2,2;FF}_{00}(s) = \frac{4\pi}{9}\,g^2_{\Delta N\pi}\,\frac{m_N^2+m_\Delta^2}{8m_\Delta^2}\left\{\frac{\beta}{p^2}\left[\ln\left|\frac{\beta-\gamma p^2-2p^2}{\beta-\gamma p^2+2p^2}\right| - \ln\left|\frac{\beta-\gamma p^2-\rho^2-2p^2}{\beta-\gamma p^2-\rho^2+2p^2}\right|\right] + \frac{4\rho^2}{(\rho^2-\beta)+(\gamma+2)\,p^2}\right\} \tag{26}$$

We have also neglected $(m_\Delta - m_N)^2$ compared to ρ^2. An analogous expression can be easily written for the pwave.

As mentioned at the beginning, the contributions of the $\Delta N\pi$ vertex must be added to our (short range) residual amplitudes in order to describe the full interaction. For the $^5P_3$ wave, both the residual amplitudes and the π-exchange amplitude are small enough in order to justify a simple addition of the phases. There is another parametrization[6] of the A N interaction based on a two channel analysis of the NN, AN system. The phases there obtained are, in the limit of weak $NN \to AN$ iiiteraction, the full AN scattering phases. The inclusion of the contribution of one pion exchange increases sliglitly the difference between our phases[3] and those of ref. 6.

The situation is more involved in the case of the $^5S_2$ channel. Her: the Born term is large and there is no justification for just adding the phases. However the qualitative result is clear, namely that the contribution of the $\Delta N\pi$ certex is attractive. Hence the full phases should be stronger than the residual ones.

In order to obtain a more qualitative insight in the influence of the $\Delta N\pi$-vertex on the $^5S_2$ channel, we may construct a potential local in the distance r, representing the contribution of diagram 1 to this wave. Identifying the Born term in potential scattering with the result obtained above we have

$$2\pi^2 V_{00}^{2,2}(r) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathcal{T}_{00}^{2,2}(p)\exp(-2ipr))\, p^2 dp \quad (\text{for } r \neq 0)\ . \quad (27)$$

The dominant part in the expression for $\mathcal{T}_{00}^{2,2;FF}$ is the last term of eq. (26), which leads to the potential

$$V_{00}^{2,2;FF}(r) = -\frac{1}{72\pi}\, g_{\Delta N\pi}^2\, \frac{m_N^2+m_\Delta^2}{2M_\Delta^2}\, \rho^3\, e^{-\rho r}\ , \quad (28)$$

where we have taken $\gamma = 2$ and neglected β compared to ρ^2.

The other terms in the expression for $\mathcal{T}_{00}^{2,2}$ are small and negative itnd could be treated perturbatively.

The global picture that we obtain for the contribution of the $\Delta N\pi$ vertex to the A N $^5S_2$ potential is thus the following: there is a weak long range repulsion and a stronger short range attraction, the range of the latter being deterrnined by the $\Delta N\pi$ form factor.

Summarizing, we note that we get reliable full phases for the $^5P_3$ channel and some qualitative insights for the $^5S_2$ channel. We empliasizc, however, tliat a more consistent determination of the AN $^5S_2$ interaction must start with a background amplitude which should not include contributions due to virtual pion exchange at all.

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