Conditions for the Absence of Infinite Renorrnalization in a General Model

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We work with a general model with interacting scalar, pseudoscalar and spin 1/2 charged particles. Perturbative calculations are developed up to one loop and some relations are found between the characteristic constants of the system which guarantee (up to the considered order) absence of infinite renormalization of masses and coupling constants.

I. Introduction

Certain difficulties are found in quantum field theory, when we treat problems related to the interaction of particles through perturbative theory[1]. The main question is the presence of divergences (infinities) in successive approximations, as well as their elimination in order to obtain convergent results. Without considering the traditional solution[2] (to redefine the characteristic constants of the system in order to incorporate the infinities in the new definition), we attack the question in another way, by looking for compensations of these infinities in appropriate models. The difference with respect to the earlier point of view begins with the simultaneous introduction of particles with different spin in such a way that most of the divergences (which appear in the perturbative expansion) cancel mutually between the bosonic and the fermionic sectors of the system.

The fact that these cancelations have been studied for the first time in the context of supersymmetric theories currently associates this solution of the problems with those theories. Nevertheless, there are bosonic and fermionic sectors which can be combined in a non-supersymmetric model (the corresponding particles cannot be included in the same supersymmetric multiplet).

In this way, we arrive at the central idea of this work, that is, to develop a model with particles of different spin (one real scalar, one real pseudoscalar and one Dirac spinor field) and to establish the relations (between the characteristic constants of the system) necessary for the ocurrence of cancelations of divergences similar to the observed in those theories.

II. The model

The starting point is the Lagrangian of the system:

$$\begin{aligned} L &= \frac{1}{2}(\phi(\Box - m_2^1)\phi + \frac{1}{2}\eta(\Box - m_1^2)\eta + \\ &+ f\phi^3 + f'\phi\eta^2 + \lambda\phi^4 + \lambda'\eta^4 + \lambda''\phi^4\eta^2 \\ &+ \bar{\psi}(i\partial\!\!\!/ - M)\psi + \bar{\psi}(g\phi - g'\gamma_5\eta)\psi, \qquad (1) \end{aligned}$$

where all ten constants ($m_1; m_2$, M, f, f', λ, A', λ”, g and g') are initially supposed to be arbitrary and independent of each other. The three fields 4, η and ψ (real scalar, real pseudo scalar and spinorial respectively) have non-zero masses, and the spinor ψ, is a Dirac spinor. Instead of directly constructing a perturbative series from this Lagrangian, we preferred to rewrite it using auxiliary fields, in order to avoid the various kinds of overlapping divergences appearing in these calculations.

The equivalence of the two Lagrangians is a consequence of the equivalence of the corresponding partition functions. The equivalent Lagrangian is given by:

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$$
\begin{aligned}
L &= \frac{1}{2}(\phi(\Box - m_1^2)\phi + 2m_1\phi F + F^2) \\
&+ \frac{1}{2}(\eta(\Box - m_2^2)\eta + \\
&+ 2m_1\eta G + G^2) + \frac{N^2}{2} + \bar{\psi}(i\not{\partial} - M)\psi \\
&+ (\phi^2 c^2 + \eta^2 c'^2)F + (\phi^2\tilde{c}^2 + \eta^2\tilde{c}'^2)N + 2\phi\eta G d \\
&+ \bar{\psi}(g\phi - g'\gamma_5\eta)\psi, \qquad (2)
\end{aligned}
$$

where we introduce (for convenience) a new set of coupling constants (c,c', $\tilde{c}$, $\tilde{c}'$ and d), whose relation with the old set, is given by

$$
\begin{aligned}
f &= m_1 c \\
f' &= m_1 c' - 2m_2 d \\
\lambda &= -\frac{1}{2}(c^2 + \tilde{c}^2) \\
\lambda' &= -\frac{1}{2}(c'^2 + \tilde{c}'^2) \\
\lambda" &= -cc' - \tilde{c}\tilde{c}' + 2d^2 \\
\lambda &< 0 \qquad \lambda' < 0 .
\end{aligned} \qquad (3)
$$

As can be seen, the new Lagrangian has six fields-three physical fields, ϕ, η and ψ, and three auxiliary fields, of which two are real scalar, N and F, and one is a real pseudo-scalar, G. In the classic limit, they satisfy the equations:

$$
\begin{aligned}
\frac{\partial L}{\partial N} &= 0 = N + \tilde{c}\phi^2 + \tilde{c}'\eta^2 \\
\frac{\partial L}{\partial N} &= 0 = F + m_1\phi + c\phi^2 + c'\eta^2 \\
\frac{\partial L}{\partial G} &= 0 = G + m_2\eta + 2\phi\eta d \qquad (4)
\end{aligned}
$$

III. The propagators

The propagators are calculated by considering the functional integral of the system coupled to external currents,

$$Z[\phi, \mathcal{J}] = \frac{1}{\mathcal{N}} \int \mathcal{D}\Phi \; e^{i \int d^4x (L_0 + \mathcal{J}\Phi)} \qquad (5)$$

where $\mathcal{J}$ and Φ are column matrices containing the external currents and all the fields. Taking the derivatives with respect to the components of $\mathcal{J}$, we obtain, in momentum space,

$$
\begin{aligned}
\Delta_\phi(p) &= \frac{1}{p^2 - m_1} \qquad \Delta_\eta(p) = \frac{1}{p^2 - m_2} \qquad \Delta_\psi(p) = \frac{\not{p} + M}{p^2 - m_2}, \\
\Delta_F(p) &= \frac{p^2}{p^2 - m_1} \qquad \Delta_G(p) = \frac{p^2}{p^2 - m_2} \qquad \Delta_N(p) = 1, \\
\Delta_{\phi F}(p) &= \frac{-m_1}{p^2 - m_1} \qquad \Delta_{\eta G}(p) = \frac{-m_2}{p^2 - m_2}. \qquad (6)
\end{aligned}
$$

The presence of crossed propagators is a consequence of the crossed terms appearing in the free part of the Lagrangian, while the absence of a crossed term involving the N field makes the N propagator equal to one. In fact, this N field was introduced only to allow us to work with the same number of independent coupling constants we started with, namely seven. Note that this would be impossible using only the F and G fields.

IV. Infinite part of the Green's functions up to one loop

The calculation of the infinite parts is done by using dimensional regularization[3] because of its formal advantages, like Lorentz and Gauge covariance, which allows for later extensions of the work. All the two legged Green's functions of the model were calculated up to one loop and the obtained result for the infinite part is:

$$
\begin{aligned}
&: L(x_1)L(x_2) :_{\text{[infinite part]}}= \qquad (7)\\
&\left(: \bar{\psi}^a(x_1)\bar{\psi}^b(x_2) : \left(g^2(i\partial\!\!\!/_x - 2M)^{ab} + g'^2(i\partial\!\!\!/_x + 2M)^{ab}\right)\right)\\
&: \phi(x_1)\phi(x_2) : 8(-3c^2m_1^2 - 3d^2m_2^2 + \tilde{c}^2m_1^2 + 3g^2M^2 - \frac{1}{2}g^2\Box_x)\Big)\\
&: \eta(x_1)\eta(x_2) : 8\left(-(c'^2 + d^2)(m_1^2 + m_2^2) - 2c'm_1m_2d + \tilde{c}'^2m_2^2 + g'^2M^2 - \frac{1}{2}g'^2\Box_x\right)\\
&- : F(x_1)F(x_2) : 4(c^2 + c'2) - : N(x_1)N(x_2) : 4(\tilde{c}^2 + \tilde{c}'^2) - : G(x_1)G(x_2) : 8d^2\\
&+ : F(x_1)\phi(x_2) : 16(c^2m_1 + c'm_2d) + : N(x_1)\phi(x_2) : 16(\tilde{c}cm_1 + \tilde{c}'m_2d)\\
&+ : G(x_1)\eta(x_2) : 16(d^2m_2 + c'm_1d) - \;\; F(x_1)N(x_2) : 8(\tilde{c}c + c'\tilde{c}'))\,\frac{i\pi^2}{(2\pi)^4\epsilon}\delta^4(x_1 - x_2). \qquad (8)
\end{aligned}
$$

Here, ϵ is the dimensional regularization parameter and the above expression diverges when $\epsilon \to 0$.

V. Conditions for absence of infinite renormalization of the masses

The relations we are looking for are precisely those that cancel all the coefficients of the individual Green's functions in the expression above. This, in turn, results in the absence of divergent renormalization for the masses. Thus, the system of algebraic equations to be solved is:

$$
\begin{aligned}
0 &= g^2 - g'^2,\\
0 &= m_1^2(3c^2 - \tilde{c}^2) + m_2^2 3d^2 - M^2 3g^2,\\
0 &= m_1^2(c'^2 + d^2) + m_2^2(c'^2 + d^2 - \tilde{c}'^2)\\
&\quad + m_1m_2 2c'd - M^2g'^2,\\
0 &= m_1c^2 + m_2c'd,\\
0 &= m_1c\tilde{c} + m_2\tilde{c}'d,\\
0 &= m_1c'd + m_2d^2, \qquad (9)
\end{aligned}
$$

The solution is given by:

$$
\begin{aligned}
\tilde{c} &= \tilde{c}' = 0,\\
c' &= \pm c,\\
g' &= \pm g,\\
d &= \mp c,\\
m_1 &= m_2 = |\frac{g}{c}|\frac{M}{\sqrt{2}}, \qquad (10)
\end{aligned}
$$

VI. Conditions for the absence of infinite renormalization of the coupling constants

In the same way as in the previous section, we calculate the infinite part of all the three legged Green's functions of the model up to one loop, obtaining a new system of equations for the characteristic constants:

$$
\begin{aligned}
0 &= c^3 = c'd^2,\\
0 &= c'^3 + cd^2,\\
0 &= d^2 + cc',\\
0 &= g^2 - g'^2,\\
0 &= g^3M + 2c^3m_1 + 2m_2d^3,\\
0 &= cc'^2m_1 + (d^2 + c'^2 + cc')m_2d\\
&\quad + m_1d^2(c + c') + \frac{g}{2}Mg'^2. \qquad (11)
\end{aligned}
$$

The solution of this system, in turn, guarantees the absence of divergent renormalization in the coupling constants up to the order considered. In solving this system, we found two more relations between the characteristic constants:

$$
c = -c' \quad \text{and} \quad g^2 = \frac{-4c|c|}{\sqrt{2}} \qquad (12)
$$

The first relation fixes the relative sign of c', while the second one gives the relation between the coupling constants and the masses of the fields and implies $c < 0$. Using eq. (9) and eq. (11) together, we found the relation between the masses of the model to be

$$m_1 = m_2 = \sqrt{2}M. \tag{13}$$

Finally, the system of algebraic equations which we obtain by trying to cancel the divergent corrections to the norm of the fields have no solution except the trivial one which eliminates the interaction terms of the model. Nevertheless, this seems to be a non-essential problem in qiiantum field theories. We note that it also occurs in supersymmetric models[4].

VII. Conclusion

We worked a general model with fields which cannot be enclosed in a supersymmetric multiplet. The infinite part of all the two-legged and three-legged Green's functions was calculated up to one loop. We found some relations between the characteristic constants which have the property of making null all these infinite parts, guaranteeing the absence of renormalization of the masses and of the coupling constants.

In this way, we conclude that it is possible to construct a physical model where bosonic and fermionic sectors take part, without conforming a supersymmetric multiplet and with renormalization properties such as those found in those theories. This fact opens the way in a new direction of models with absence of divergences, as there is some freedom in the choice of the fields of the system, which does not exist in supersymmetric models.

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