Minijets and Inelasticity in High Energy Collisions

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We study the energy dependence of inelasticity in hadron-hadron collisions in the framework of the Interacting Gluon Model (IGM). It is shown that the introduction of a minijet component in this model will lead to increasing inelasticities at higher energies. Leading particle spectra are also presented.

I. Introduction

The concept of inelasticity plays an important role in cosmic rays and accelerator physics. It is usually defined as the fraction K of the available energy $\sqrt{s}$, in a given interaction, effectively employed for multiparticle production The energy dependence of inelasticity is a problem of great interest both for the interpretation of cosmic ray data and also for quark-gluon plasma (QGP) physics since inelasticity decreasing with energy would make the formation of QGP more difficult. Experimentally the situation is not clear and many authors have proposed different behaviours of the average inelasticity $\langle K\rangle$ as a function of $\sqrt{s}$.

One of the models which in a natural way leads to (K) decreasing with energy is the Interacting Gluon Model (IGM)[1]. It included originally only soft gluonic interactions and used the phenomenological soft gluon-gluon cross section as an input. However, it was claimed recently that semi-hard QCD interactions (which produce the so called minijets) represent an important fraction ($\sim$ 25%) of the total cross section already at the CERN collider energies and are expected to be even more important at higher energies[2]. In this paper we discuss therefore the effect of the inclusion of such semi-hard component to the original IGM.

II. Modified Interacting Gluon Model

In the framework of the IGM, in a first approximation, valence quarks do not interact at all but instead form leading particles. The interaction is supposed to come entirely from the gluonic contents of the colliding hadrons via the formation of gluonic fireballs (clusters). The originally predicted *decrease* of $\langle K\rangle$ with energy can be traced to the assumption that the phenomenological behaviour of gluon-gluon cross section $\sigma_{gg}(\hat{s})$ is limited to $1/\hat{s} < \sigma_{gg} <$ const to the $1/x$ form of the gluonic structure functions for small x (see below for details) and to the assumed constancy with energy of the percentage p of the energy-momentum of the projectile allocated to gluons. Here we shall relax the first condition by allowing the QCD semi-hard interaction mechanism which leads to σ_{gg} increasing with energy.

The probability to deposit in the central region of reaction fractions x and y of the energy momenta of the incoming hadrons by means of the gluon-gluon interactions is given by the following formula[1]:

$$\chi(x,y) = \frac{\chi_0}{2\pi\sqrt{D_{xy}}} \times$$
$$\times \exp\left\{-\frac{1}{2D_{xy}}\left[\langle y^2\rangle(x-\langle x\rangle)^2 + \langle x^2\rangle(y-\langle y\rangle)^2 - 2\langle xy\rangle(x-\langle x\rangle)(y-\langle y\rangle)\right]\right\}, \tag{1}$$

where

$$D_{xy} = \langle x^2\rangle\langle y^2\rangle - \langle xy\rangle^2, \qquad \langle x^n y^m\rangle = \int_0^1 dx\, x^n \int_0^1 dy\, y^m\, w(x,y), \tag{2}$$

and χ_0 is a normalization constant defined by the condition that

$$\int_0^1 dx \int_0^1 dy\, \chi(x,y)\, \theta(xy - K^2_{\min}) = 1,$$

with $K_{\min}$ being the minimal inelasticity,

$$K_{\min} = \frac{m_0}{\sqrt{s}}, \tag{3}$$

which is defined by tlie mass m_0 of tlie lightest possible produced state.

Tlie function $w(x,y)$ (called "spectral function") contains all tlie dynamical input of the model and is proportional to tlie mean number of gluon-gluon interactioiis with given x and y. It reads

$$w(x,y) = w_S(x,y) + w_H(x,y), \tag{4}$$

wliere

$$w_S(x,y) = A\,\frac{\sigma^S_{gg}(\hat{s})}{\sigma^{in}_{hN}(s)}\, G_h(x)\, G_N(y)\ \theta\left(xy - K^2_{\min}\right)\theta\left(\xi - xy\right), \text{ and} \tag{5}$$

$$w_H(x,y) = A\,\frac{\sigma^H_{gg}(\hat{s})}{\sigma^{in}_{hN}(s)}\, G_h(x)\, G_N(y)\, \theta\left(xy - \xi\right) \tag{6}$$

Tlie cross sections σ^S_{gg} and σ^H_{gg} are tlie gluon-gluon cross sections in the non-perturbative (soft) and ill tlie perturbative (liard) regime respectively. For tlie former we take tlie previously used phenomenological ansatz[1] and for tlie latter tlie lowest-order perturbative QCD results (see for example refs. 3 and 4); σ^{in}_{hN} is the inelastic hadron-liadron cross section, A is a constant parameter and tlie $G_{h,N}$ are the effective number of gluons wliich we approximate by tlie gluonic structure functions of corresponding hadrons normalized to the percentage p of hadronic momentum allocated to the glue

$$\int_0^1 dx\, x\, G_{h,N}(x) = p_{h,N}. \tag{7}$$

Tlie soft gluon-gluon cross section is chosen to be

$$\sigma^S_{gg} = \frac{\alpha}{\hat{s}}, \tag{8}$$

where a is a parameter. Tlie hard gluon-gluon cross section is given by[3]

$$\sigma^H_{gg} = \frac{\pi}{18 p^2_{T\min}}\left[\alpha_S(Q^2)\right]^2 H, \tag{9}$$

with

$$H = 16T + \frac{4\xi}{xy}\ln\left[\frac{(1-T)}{(1+T)}\right] + \frac{2\xi}{xy}T,$$

$$T = \left(1 - \frac{\xi}{xy}\right)^{1/2},$$

$$\xi = \frac{p^2_{T\min}}{S},$$

$$\alpha_S(Q^2) = \frac{12\pi}{25\ln\left(\frac{Q^2}{\Lambda^2}\right)},$$

where $p_{T\min}$ is a cutoff parameter and $\Lambda = 0.2$ GeV.

The gluon distribution is taken to be the same as before[1], i.e.,

$$G(x) = p\,\frac{(1+n)}{x}\,(1-x)^n \tag{10}$$

III. Results and Discussion

Tlie new element introduced in this work with respect to ref. [1] is tlie inclusion of w_H in the spectral function. It was introduced iiere in tlie same way as tlie semi-hard coinponent of the eikonal function was introduced by Durand and Pi[4] in their diffraction-scattering formalism for total cross sections.

Tlie QCD parameters are fixed to their most accepted values namely $\Lambda = 0.2$ GeV aiid $p_{T\min} =$

2 GeV. Tlie scale is chosen to be $Q^2 = p^2_{T\,\mathrm{min}}$. Since we want to compare our results witli those obtained previously in ref. [1] we keep $m_0 = 0.35$ GeV, $n = 5$ and $p = 0.5$, tlie only modification being tlie introduction of tlie semi-hard spectral function, w_H. We have tlien only two parameters to adjust, A and a, which will be fixed by two experimental constraints. Tlie first one is tliat for p-$\bar{p}$ reactions at $\sqrt{s} = 540$ GeV tlie following relation holds[2]:

$$\frac{\sigma^{\mathrm{minijets}}}{\sigma^{\mathrm{in}}_{pp}} \cong \frac{\sigma^H_{gg}}{\sigma^S_{gg} + \sigma^H_{gg}} \cong \frac{1}{4} \quad . \tag{11}$$

This fixes tlie value of a. Tlie second constraint is given by tlie requirement[1] that for protoii-proton collisions at $\sqrt{s} = 16.5$ GeV tlie mean inelasticity (K) $\cong$ 0.50. Tliis condition fixes tlie value of A. We have checked tliat at 16.5 GeV tlie product Aa is equal to the old value of a found in ref. (1) as it should be since at such low eiiergies minijets have no importance.

Tlie gluons deposited in the central region are supposed to form of a fireball (gluonic cluster) of mass M $= \sqrt{xy \cdot s}$. Tlie inelasticity variable K is defined tlien as

$$K = \frac{M}{\sqrt{s}} = \sqrt{xy}, \tag{12}$$

and tlie inelasticity distribution $\chi(K)$ can be obtained from $\chi(x,y)$ by a simple change of variables

$$\chi(K) = \int_0^1 dx \int_0^1 dy \, \delta\left(\sqrt{xy} - K\right) \chi(x,y) \quad . \tag{13}$$

Finally we can calculate tlie average inelasticity as

$$\langle K \rangle = \int_0^1 dK \, K \, \chi(K) \tag{14}$$

and leading particle spectra ($x_L \in (0, 1 - K^2_{\mathrm{min}})$):

$$f(x_L) = \int_0^1 dx \int_0^1 dy \, \theta\left(xy - K^2_{\mathrm{min}}\right) \delta\left(1 - x - x_L\right) \chi(x,y). \tag{15}$$

One can easily see tliat for symmetrical (e.g. proton-proton) collisions $\langle K \rangle \sim \langle x \rangle$ and tlie width of the K and x_L distributions is controled by (x^2). In order to investigate qualitatively tlie energy dependence of $\langle K \rangle$ it is tlien enlightening to consider what happens to (x) and (x^2). Approximating $G(x)$ by its most singular terin, $G(x) = 1/x$, we can calculate (x), (x^2) aiid $\langle xy \rangle$ analytically, considering tlie effect of the soft and hard components separately. In tlie high energy limit ($s \to \infty$) we obtain

$$\begin{aligned}
&\langle x \rangle_S \sim \frac{1}{m_0^2} \cdot \frac{\alpha}{\sigma^{\mathrm{in}}_{hN}(s)}; \qquad
\langle x^2 \rangle_S \sim \frac{1}{2m_0^2} \cdot \frac{\alpha}{\sigma^{\mathrm{in}}_{hN}(s)}; \qquad
\langle xy \rangle_S \to 0; \\
&\langle x \rangle_H \sim \frac{1}{\sigma^{\mathrm{in}}_{hN}(s)} \cdot \ln\left(\frac{s}{p^2_{T_{\mathrm{min}}}}\right); \quad
\langle x^2 \rangle_H \sim \frac{1}{\sigma^{\mathrm{in}}_{hN}(s)} \cdot \ln\left(\frac{s}{4p^2_{T_{\mathrm{min}}}}\right); \quad
\langle xy \rangle_H \sim \frac{1}{\sigma^{\mathrm{in}}_{hN}(s)}
\end{aligned} \tag{16}$$

where $\langle x^n y^m \rangle_S$ ($\langle x^n y^m \rangle_H$) were calculated witli w_S (w_H). It is tlien clear that the soft component contribution to energy deposition decreases witli tlie reaction energy and tlierefore $\langle K \rangle$ will be asymptotically dominated by the semi-hard component. Wlietlier tlie total average inelasticity will increase or not will depend on the exact form of the hadron-hadron cross section.

As one can see from eq. (16) both $\langle x \rangle_S$ and $\langle x^2 \rangle_S$ decrease witli energy whereas $\langle x \rangle_H$ and $\langle x^2 \rangle_H$ remain essentially constant (one can easily check that tlie c $ & (~) we have used , cf. below, essentially cancels tlie log term there). Tliis implies that tlie soft contribution will produce distributions for K and x_L narrowing witli energy (as already observed in ref.[1]) while tlie semi-hard component will lead to spectra broadening with energy. Tlie numerical evaluation of $\langle K \rangle$, (as given by eq. (14)) as a function of $\sqrt{s}$ is shown in the Figure 1 (for the proton-proton cross section we have used the following form for $\sigma^{in}_{hN}(s)$[5]: $\sigma^{in}_{hN}(s) = 39.5s^{-0.38} + 21.7s^{0.08}$ (mb)). As can be seen in Fig. 1 the inclusion of minijets reverses the trend of decreasing inelasticities found in the previous calculations with the IGM. It seems that the value of $\langle K\rangle$ tends to a saturation point (as it is suggested by the full line in Fig. 1), its precise value depending on the asymptotic behaviour of σ^{in}_{hN}. This is the main result of this paper. The idea that minijets are responsible for increasing $\langle K\rangle$ was already advanced by some authors[6] and here it was brought to the IGM. One can therefore argue that here we provide a *model* for the parameter κ appearing in the formula for inelasticity presented in ref. [7]. In tliis sense the remarks made in ref. [8] about the expected limiting asymptotic behaviour of inelasticity K as being caused by the assumed energy independence of the amounts of the energy-momenta p of the projectiles allocated to gluons are valid also here. Althougli we did not attempt to make a detailed analysis of existent data our values of $\langle K\rangle$ are very close to those found in cosmic ray studies[5,7].

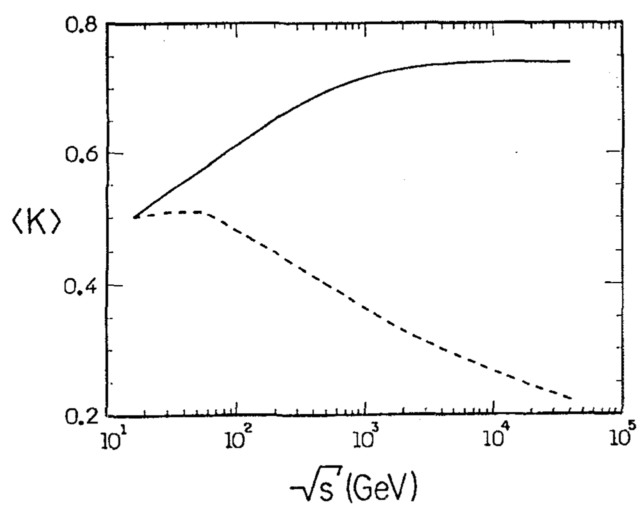

Figure 1: Average inelasticity as a function of $\sqrt{s}$ in proton-proton collisions. The dashed line represents previous results with w_s alone and the solid curve shows $\langle K\rangle$ calculated with both contributions, i.e., with $w = w_S + w_H$.

Figure 2 shows inelasticity distributions for three different energies $\sqrt{s}$ = 16 (fig.2a), 540 (fig.2b) and 1800 GeV (fig.3c). The total distribution (solid line) is at lower energies strongly dominated by the soft component (dotted line) but at higher energies the semi-hard component (dashed line) becomes increasingly important.

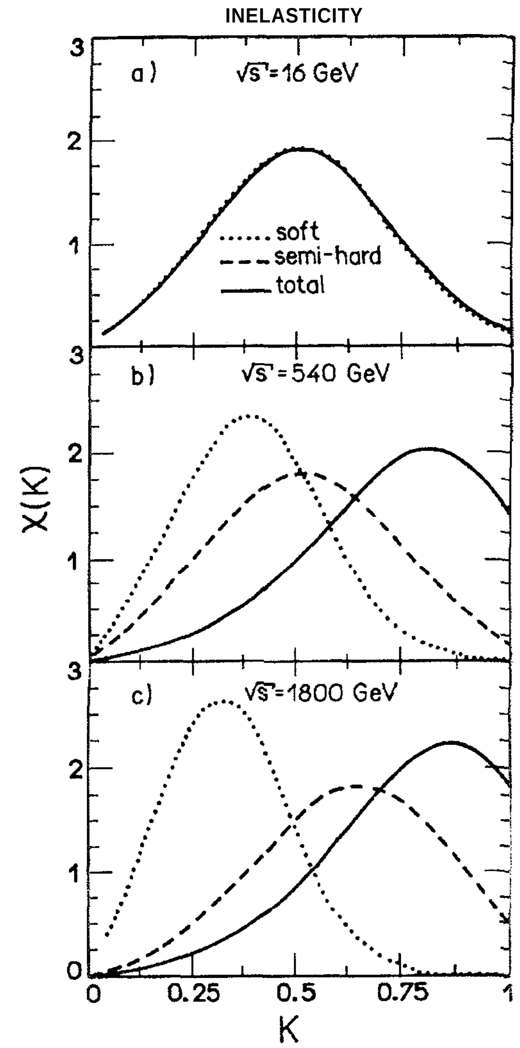

Figure 2: (a) Inelasticity distribution for proton-proton collisions at $\sqrt{s} = 16$ GeV. The dotted line represents eq.(13) with $w = w_S$, the dashed line is the same with $w = w_H$ and the solid curve includes both soft and semi-hard contributions $w = w_S + w_H$. (b) The same as (a) for $\sqrt{s} = 540$ GeV . (c) The same as (a) for $\sqrt{s} = 1800$ GeV.

Figure 3 shows leading particle spectra for the same ISR, SPPS-collider and Tevatron energies. As it can be seen, the distributions move to the left implying a softening of leading particles. This is consistent with increasing inelasticities. Apart from showing the effect of rninijet dynamics these results are interesting because leading barion spectra at such energies will be soon available[9]. We would like to rernind that results in botli Figs. 2a and 3a are tlie same as already presented in ref.[1] where they were shown to be in agreement witli ISR data.

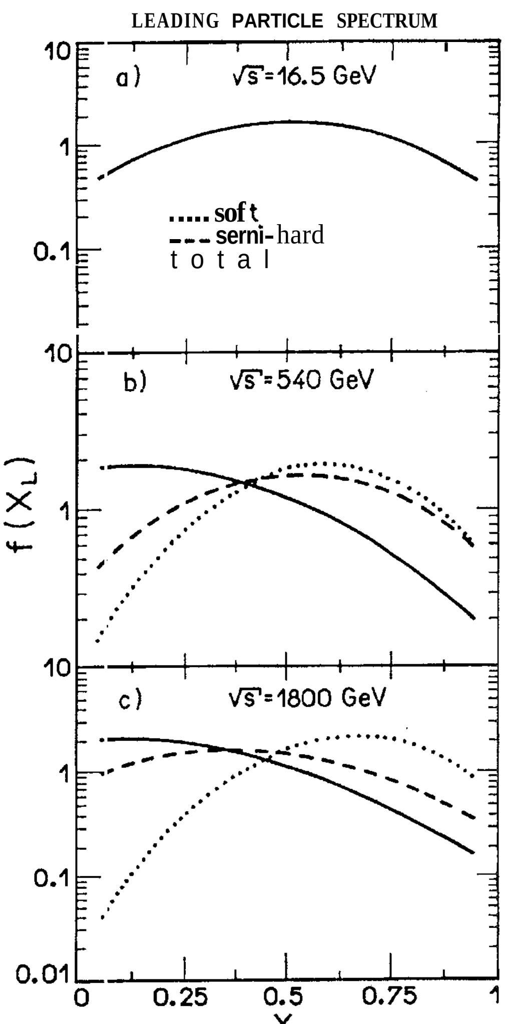

Figure 3: (a) Leading particle distribution for proton-proton collisions at $\sqrt{s} = 16$ GeV. Tlie dotted line represents eq.(15) with w $= w_S$, tlie daslied line is the same with w $= w_H$ and the solid curve includes botli soft and semi-hard contributions w $= w_S + w_H$. (b) Tlie same as (a) for $\sqrt{s} = 540$ GeV. (c) Tlie same as (a) for $\sqrt{s} = 1800$ GeV.

We have shown that contrary to soine claims[10] tlie IGM model can incorporate, in a quite natural way, also the inelast city (Ir')growing towards some limited value. However, it is quite clear from the present work (and was also discussed at length ill ref. [8]) tliat to get $\langle K\rangle$ increasing so fast as demanded by some otlier models (cf. ref. [8] again) one would either have to use $\sigma^{in}_{hN}(s)$ increasing very slowly with $\sqrt{s}$ (not faster than ln s) or to allow for tlie increase with the energy of tlie parameter p, i.e., the amount of energy-momenta allocated to gluons. In view of tlie above results we do not see a need for sucli scenario for tlie time being.

One slould be also aware of tlie fact tliat **(Ir')**as calculated above (i.e., containing botli *soft* and *hard* components) can be used as initial fractional energy ill statistical models only in tlie cases wliere one can expect thermalization of the produced fireball [11] (i.e., practically only in very liigli energy nuclear collisions). However, our inelasticity is perfectly usable for any cosinic ray applications[12].

Acknowledgments

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