Gap Properties of Semiconductor Alloys

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Features of experimental interest in $Al_x Ga_{1-x}As$ and $In_x Ga_{1-x}P$ alloys are studied through a new theoretical treatnieiit, based on the *small crystal* approach. Different alloy configurations accommodated ill a relatively large basic cluster are numerically generated and solved independently. Periodic boundary conditions are imposed. The alloy's properties are identified to an *ensemble* average over those calculated for a large number of *small crystal* configurations generated according to the overall composition and degree of order. For randomly disordered $Al_x Ga_{1-x}As$, we focus on the direct-to-indirect transition in the nature of the optical gap, which occurs at $x_c \simeq 0.4$. Physical aspects of the transitioni are discussed and explored in analogy with order-clisorder transitions in statistical mechanics. For $I_{1x}Ga_{1-x}P$, ordering into the experimeitally observed monolayer superstructure along the [11] direction is considered. The band-gap energy and structural properties of partially ordered alloys are determined. The dependence of the calculated properties on the loig-range order parameter is found to follow simple functional relationships. Structural aniisotropies scale accurately with the square of the order parameter. Our results are ill good quantitative agreement with the experimentally reported order-induced baild-gap narrowing effect.

I. Introduction

 $Al_x Ga_{1-x} As$ alloys have been extensively studied during tlie last years due to tlieir primary importance in high-speed electronic and optoelectronic devices. Tliis development was only possible because these alloys exhibit distinct electronic properties as the composition is changed, but are essentially lattice matched over the while composition range. The artificial growth of different kinds of structures witli controlled electronic and optical properties is tlius facilitated. Optimized use of these materials must be based on accuiate knowledge of tlie alloy's properties as a function of composition. Particularly relevant cliaracteristics in this respect are related to tlie band gap and its optical nature. GaAs i,; a direct-gap material, wliile tlie main energy gap of AlAs is indirect. Therefore, as observed experimentally^[1], progressive substitution of Ga by Al must cliange the nature of the gap ill the alloy towards an indirect-gap behavior. Usual tlieoretical pictures describing tlie direct-to-indirect gap transition in $Al_xGa_{1-x}As$ are based on interpolation schemes connecting the band structures of the binary constituents. The top of the valeice band is expected to remain of Γ symmetry for all x, while a Γ -X conduction-band-edge crossover is presumed to occur at a transition composition x_c , above which the gap becomes indirect^[1,2]. This is strictly not tlie case, since alloys are not translationally invariaiit, and therefore \vec{k} is not a good quantum number except for x = 0 or 1. This crossover must be understood within a theory which does not rely on zinc-blende symmetry points in reciprocal space, and which allows for 2, and other properties related to the trainisition to be calculated.

In_xGa_{1-x}P alloys are also very convenient light emitting materials for device applications, due to their large direct band-gap and to a close lattice match to GaAs at x = 0.5. Special attentioii lias been given lately to the experimental observation of spontaneous ordering of this alloy iiito a metastable structure, and to the consequences of this ordering in the electronic properties of the material, particularly the order-induced band-gap narrowing. Relative to the random alloy at x = 0.5, a band-gap narrowing of over 100 meV lias been observed in ordered samples^[3-5]. Accurate determination of the long-range order (LRO) parameter is essential in the investigation of the mechanisms leading to ordering as well as to establish the effect of differeiti growth parameters in ordering^[6].

Conventional alloy approximations are inadequate for the tlieoretical treatment of tlie problems mentioned above. Such techniques^[2] usually involve approximating tlie system's Hamiltonian or Green's function by an effective operator representing the corresponding configurational average over tlie alloy, and from whicli alloy's properties are obtained. The averaging procedure casts tlie problem into a higher-symmetry environment, which for $A^{III}B^{III}C^{V}$ seiniconductor alloys is commonly

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taken to be zinc-blende. The wave-function symmetry is accordingly constrained, and therefore a poor or incomplete description results for related properties, such as the nature of the gap in $Al_xGa_{1-x}As$. Also, full or partial ordering of $In_xGa_{1-x}P$ can not be mapped iiito a zinc-blende symmetry problem, since the primitive cell for the observed ordered structure does not fit into the zinc-blende primitive cell.

We treat disorclered alloys in the small crystal $approach^{[7-9]}$, and show that the difficulties mentioned above are properly handled in this context. Specific alloy properties are identified to configurational averages of tliese properties calculated for tlie ordered compounds accommodated witliin a given basic cluster. Tlie accuracy of this assumption increases with the basic cluster size, becoming exact in the limit of an infinite cluster. In order to simulate an infinite solid, periodic boundary conditions are imposed. Occupational correlations within the basic cluster size are preserved. In Section II we discuss the crossover in the optical nature of the gap of $Al_x Ga_{1-x} As$ alloys. Results for the electric dipole moment associated to the inain gap transitioii, obtained witli basic cluster sizes ranging from 16 to 216, are presented and compared to the virtual crystal approximation results. Structural properties and the energy gap of partially ordered $\ln_r \operatorname{Ga}_{1-r} P$ alloys are considered in Section III, where we discuss the possibility of inferring tlie order parameter from measurements of tliese properties.

II. Direct-to-Indirect gap transition in $Al_x Ga_{1-x} As$

The small crystal approach^[10] is equivalent, for ordered systems, to a sampling over a small number of points in the Brillouin zone of the infinite, periodic lattice. A basic cluster of sites is chosen, and different coifigurations are defined according to their specific occupation, with periodic boundary conditions imposed. For a 16-site basic cluster in the diamond lattice, the inequivalent structures for $A_n^{III}B_{8-n}^{III}C_8^V$ compounds are systematically classified in Ref. [7]. Disordered $A_x^{III}B_{1-x}^{III}C^V$ alloy's properties are approximated by configurational averages over the ordered structures accommodated in a given cluster size. Averages may thus be exactly performed for a 16-site cluster, namely

$$P(x) = \frac{\sum_{\xi} x^n (1-x)^{8-n} D(S_{\xi}) P(S_{\xi})}{\sum_{\xi} x^n (1-x)^{8-n} D(S_{\xi})} , \qquad (1)$$

where **P** is any alloy property and $P(S_{\xi})$ is the value of this property calculated for the ordered compound structure S_{ξ} . The sums are performed over the inequivalent structures $\{S_{\xi}\}$, $D(S_{\xi})$ is the degeneracy of configuration S_{ξ} , and *n* is the number of A^{III} atoms in the basic cluster for this configuration.

Inproved alloy approximations are obtained by increasing the basic cluster size C. However, the number of possible configurations increases exponentially with C. and a systematic classification of the inequivalent ones becomes impractical as tlie basic cluster size increases. For example, changing C from 16 to 64 leads to a Sactor of over 7 orders of magnitude increase in tlie number of possible configurations. This difficulty is overcome by the standard statistical procedure of replacing the configurational average by an ensemble average performed over a representative set of structures generated numerically according to the occupation probabilities of the sites of the group-III sublattice. For a randoin $Al_x Ga_{1-x} As$ alloy, these are $p_{Al} = x$ and $p_{Ga} = 1 - 2$. Randoin Al_xGa_{1-x}As is thus simulated by a number N of small crystal structures $\{S_{\ell}\}$, whose basic cluster configurations provide a realistic sampling of C-atom cells extracted froin the macroscopic system.

We describe electronic properties of $Al_n Ga_m As_{m+n}$ orderecl compounds within the tight-binding formalism, witli Hamiltonian matrix elements taken from tlie parametrization suggested by Vogl *et al*^[11] for the binary constituents GaAs and AlAs. Such parametrization reproduces the major features of conduction and valence bands of these compounds. The basis set is of type sp^3s^* , where s^* refers to an excited effective s orbital, and the Hamiltonian for each small crystal configuration is obtained according to tlie sitcs occupatioii. Iii order to account for the valence band-edge discontinuity, tlie on-site elements of AlAs are shifted downward with respect to those for GaAs by 0.47 eV. Environmental disorder at the group-V sublattice is dealt with by taking for the on-site As matrix elements the average of the parameters for As in the binary compounds weiglited by tlie number of Ga and As atoms among its four neighbors.

The nature of the gap of a compound with structure S_{ℓ} is related to the electric dipole moment, $\vec{M}(S_{\ell}) = \langle v_{\ell} | \vec{p} | c_{\ell} \rangle$, which vanishes for indirect-gap materials. The transition inatrix element above is related to the states $|c_{\ell}\rangle$, at the bottom of the conduction band, and $|v_{\ell}\rangle$, at the transition in the valence band. This quantity is easily calculated in the tight-binding approach, since in the basis set $\{\mathbf{li}, \mu\rangle\}$, with i representing the orbital^[8],

$$\langle i, \mu | \vec{p} | j, \nu \rangle \simeq \frac{im}{\hbar} \langle i, \mu | H | j, \nu \rangle \vec{r}_{ij}$$
 (2)

The dipole inoment $\vec{M}(S_{\ell})$ is thus obtained from the Hamiltonian matrix elements and from the eigenfunctions calculated in the tight-binding basis set. Within the small crystal treatment, $M^2(x)$ for an $Al_x Ga_{1-x} As$ alloy is approximated by the ensemble average of $|\vec{M}(S_{\ell})|^2$, with $Al_n Ga_m As_{m+n}$ structures S_{ℓ} defined in a C = 2(n+m)-site basic-cluster.

In Fig 1 we present results for $M^2(x)$ calculated for basic cluster sizes C = 16, 64 and 216. Note that

for C = 16, M^2 is a smoothly decreasing function of x, with no ind cation of a crossover from direct-to-indirect gap regimes^[8]. Results for this basic cluster size are obtained from exact configurational averages, as indicated in Eq.(1). For C = 64, ensembles with N = 400 configurations were generated for eacli value of x. Statistical error bars are given for the data points (triangles). An indirect-gap region is identified above $x \simeq OG$. The direct-to-indirect gap crossover is even clearer from tlie C = 216 results (squares), calculated from N = 40 configurations for each value of x. The indirect-gap region appears above $x \simeq 0.5$, while a sharp decrease in M^2 occurs in the composition range from 0.35 to 0.45. Results for $M^2(x)$ calculated within the virtual crystal approximation (VCA) are also presented. Matrix elements are linearly nterpolated from the binary constituents, and a zinc-blende symmetry Hamiltonian results for the whole composition range. The calculated M² presents a negligible decrease with x (less than 1%) as x increases from 0 to 0.3, where it drops discontinually to zero. At the composition $x_c^{VCA} = 0.3$, a conduction-band-crossin; occurs. leading to a strictly indirect-gap situation for $x > x_c^{VCA}$.



Figure 1: Dipo e moment squared for the main gap transition of $Al_x Ga_{1-x} As$, normalized to the GaAs value, vs alloy composition x. Calculations within the small crystal approximation and basic cluster sizes C = 16, 64 and 216 are given by the dotted line, triangles and squares respectively. The solid line is the result obtained in the virtual crystal approximation.

Comparison of the results for different levels of the small crystal approximation show a tendency towards sharper direct-to-indirect gap transitions as the basic cluster size C increases. At the transition region, larger

statistical error bars are obtained, another indication tliat for $C \rightarrow \infty$, a well defined critical concentration x_c may be found at which M² goes to zero. From the present results, a value of x_c between 0.4 and 0.5 is to be expected, which is in good agreement witli experiments^[1]. The analytic behavior of the transition is hard to be inferred from the present results. Note that tliis problem may be studied in analogy with orderdisorder transitions in statistical meclianics, where M² naturally defines tlie associated order parameter. Formally, it is important to establish whetlier the order parameter goes continually (second order transition) or discontinually (first order transition) to zero. Effectivepotential approximations lead to first order transitions, as illustrated by the VCA calculations in Fig 1. However, this result is implied by the underlying approximations and by tlie related band-crossing picture criterion for the transition. The small crystal approach provides, for increasing values of C, a liierarchy of approximations for alloy properties from whicli infinite-system values may be extrapolated through Monte Carlo statistical mechanics techniques^[12,13]. Calculations under way, for larger values of C and aiming at these questions, shall be reported elsewhere.

III. Structural and gap properties of partially ordered In_{0.5}Ga_{0.5}P

The band-gap energy (E_q) of $In_{0.5}Ga_{0.5}P$ alloys grown on (001) GaAs substrates by metalorganic vapor phase epitaxy^[3-6,14] and atomic layer epitaxy^[15] is found to depend strongly on the growth conditions. According to the growth parameters, values for E_g ranging from 1.78 eV to 1.92 eV at 300 K^[3,4,15] and from 1.882 eV to 1.987 eV at 13 K^[5] have been measured. Gomvo *et al.*^[3,4] qualitatively related this gap narrowing to the degree of ordering in the alloy's group-III sublattice. Samples producing stronger superstructure spots in the electron-diffraction patterns also presented narrower gaps^[4]. Kanata et $al^{[5]}$ assumed that E_g decreases linearly witli tlie LRO parameter in these alloys. On tlie other hand, Kurtz et al.[16] pointed out tliat the correlation between the degree of long-range order (LRO) in In_{0.5}Ga_{0.5}P and clianges in the band gap properties is not yet established in detail. They find samples with almost "normal" band gaps but still showing a significant degree of order in X-ray diffraction patterns.

The observed ordered structure is the $(GaP)_1/InP_1$ monolayer superstructure along the [111] direction. We define the LRO parameter in terms of site occupation probabilities in the group-III sublattice^[17]. This sublattice is divided into two sublattices, α and β , which are occupied respectively by In and Ga in the perfectly ordered [111] monolayer siiperlattice. Four site occupatioii probabilities, p_K^{σ} , give the fraction of sites in sublattice $\sigma = \alpha$ or β which are occupied by species K = Iiior Ga respectively. In terms of these, the alloy composition is $x = (p_{In}^{\alpha} + p_{In}^{\beta})/2 = 1 - (p_{Ga}^{\beta} + p_{Ga}^{\alpha})/2$, and the LRO parameter is defined as $\mathbf{S} = p_{In}^{\alpha} - p_{In}^{\beta} = p_{Ga}^{\beta} - p_{Ga}^{\alpha}$. The fully ordered structure correspoids to x = 0.5and $\mathbf{S} = 1$ while, for partially ordered configurations, 0 < S < 1.

Alloys are treated in the small crystal approximation, using a 64-site cubic basic cluster, which contains eight conventional cubic cells of the diamond lattice^[9]. Partially ordered $\text{In}_x \text{Ga}_{1-x} P$ alloys are simulated through small crystal configurations numerically generated according to the occupation probabilities of the 32 sites of the group-III sublattice: $p_{In}^{\alpha} = (2x + S)/2 = 1 - p_{Ga}^{\alpha}$, $p_{In}^{\beta} = (2, -S)/2 = 1 - p_{Ga}^{\beta}$. For the present study, x = 0.5, aiid 400 structures are generated for each value of S in the range [0,1].

The binary constituents of $In_x Ga_{1-x}P$ present a bond-length mismatch of 7.6%, which results ill a strained alloy. The equilibriuin atoinic positions for eacli small crystal in the ensemble of configurations was determined^[17] assuming the elastic eiergies are described by a Keating-type valence force field (VFF) model^[18,19]. Tlie elastic energy was minimized usiiig a molecular-dynamics algorithm^[20], allowing for full unconstrained relaxation of each configuratioii. Results froin tlie equilibrium configurations are suinmarized in Fig 2. Tliere, tlie average nearest-neighbor distances along tlie [111] ordering direction (0) and along tlie lateral directions (L) are plotted versus S^2 for Ga-P and In-P bonds. Of course tlie notation O and L is to be taken as a general indication of the bonds orientation, silice in equilibrium the alloy bonds are liot strictly parallel to any special directioii. Average values for the 4 types of bonds sliow an excelleiit quadratic fit for the dependence witli tlie order parameter. In particular, tlie O - L bond shift may be written as

$$\Delta b(S) = \Delta b_{\max} S^2 , \qquad (3)$$

witli $\Delta b_{\text{max}} = -0.050$ Å for Ga-P bonds and 0.057 Å for In-P bonds. Bond lengtlis for the ordered (S = 1) system are in agreement with previous calculations^[19]. O-bonds are very close to the ideal unstrained length value for each bond type, while L-bonds are shifted by Δb_{max} with respect to those. As S decreases from 1 to 0, average O and L bond lengths merge into values which depend only on the species: 2.512 Å for In-P and 2.383 Å for Ga-P.



Figure 2: Average calculated lengths of O-boncls (solid symbols) and L-bonds (open symbols) for (a) In-P, and (b) Ga-P bonds, versus the order parameter squared. Statistical error bars are at most equal to symbol size. The corresponding unstrained bond length is given by an horizontal dashed line. Dotted lines indicate the quadratic dependence of average bond lengths with S.

Tlie electronic structure of eacli small crystal is obtained ill tlie tight-binding approximation^[11] by directly solving for the Hamiltonian spectrum^[17]. Distortions due to stress are incorporated locally in each nearneighbor inatrix element through a $(b_0/b_{ij})^2$ scaling^[21], wliere b_{ii} is the calculated equilibrium distance between atomic sites i and j and b_0 is the unstrained value of tlie corresponding bond. Fig 3 gives tlie ensemble average values for the main energy-gap as a function of tlie order parameter S. As expected, E_g is a decreasing function of S, but, contrary to the structural parameters plotted in Fig 2, a quadratic dependence alone does iiot describe tlie calculated trend accurately. Symmetry implies that $E_q(S) = E_q(-S)$, therefore polyiomial corrections are restricted to even powers of S. Addition of a small quartic term yields a satisfactory fit, iiidicated by the dotted line in Fig 2. It correspoilds to the for $E_g(S) = E_g(0) - \Delta E_g(S)$, where

$$\Delta E_g(S) = (0.13 S^2 - 0.03 S^4) \text{ eV}$$
(4)

is the gap reduction of ordered samples with respect to the random alloy.



Figure 3: Calculated low temperature energy-gap of $In_{0.5}Ga_{0.5}P$ versus order parameter. Each data point gives the *ensemble* average over 400 64-sites small crystal structures. Statistical error bars are indicated. The dotted line is a fourth-order polynomial fit, described in the text.

All measured gap values given in Ref. [5] for ordered samples fall within the range of our calculation. Assuming our calculated $E_g(S)$ dependence, those would correspond to S between 0.5 and 0.9. The order parameter values estimated there are considerably smaller due to the presumed linear dependence of E_g with S. The lowest value for E_g reported in the literature is 1.868 eV (at T=4 K) for a sample grown by atomic layer epitaxy^[15]. From our calculation, this sample would essentially correspond to $5 \sim 1$.

The dependence of the gap with S obtained here explains the apparent contradiction described in Ref. [16] of samples with almost "normal" band gaps but still showing order in X-ray diffraction patterns. From Fig 2 we note that for S < 0.4 the gap reduction effect is almost negligible (less than 1%), while values of $S \sim 0.4$ would still show superlattice diffraction spots in X-ray experiments.

Our results indicate that local structural measurements provide essential information to establish the degree of ordering in $In_x Ga_{1-x}P$ alloys. Electron diffraction^[4] and X-ray diffraction^[16] experiments show great quantitative uncertainties, while photoluminescence spectra, from which E_g is usually obtained, present a strong and anomalous temperature dependence^[16]. Therefore diffraction spots intensities and band-gap values alone can not give accurate estimates for the LRO parameter. On the other hand, the simple quaditatic dependence of the O – L bond shifts with S obtained here may be used as a reliable measure for the degree of ordering of these alloys. Of course O – L bond shift measurements are also subject to uncertainties, so that trying to infer the absolute value of the order parameter from these experiments is subject to such inaccuracies, and must also be based on a precise knowledge of Δb_{max} . We estimate our value of Δb_{max} is reliable within 0.005 to 0.01 Å, which is comparable to EXAFS experimental accuracy.

Previous studies have mainly dealt with the composition dependence of elastic and electronic properties of semiconductor random alloys. Near-neighbor distances are usually found to follow a linear dependence with x [22,23]. Band gaps, however, show distinct "bowing" effects, and both linear and quadratic terms in xare present in fits to $E_q(x)$ [2]. In the case of order, the simplest functional dependence of physical quantities on any order parameter S for which the system has $S \rightarrow -S$ symmetry is quadratic: no linear terms are allowed. For the particular system considered here, this is an excellent approximation for average near-neighbor distances, while gap properties require higher order corrections. The situation is quite analogous to the composition dependence of the these quantities, only that expansions are performed in powers of x in one case and in powers of S^2 in the other.

IV. Conclusions

The small crystal formalism presented here is of general application to disordered or partially ordered alloys, and gives direct information about the dependence of different alloy properties on statistical parameters related to composition, long or short range order. Specific properties focused here, namely the change in nature of the optical gap in $Al_x Ga_{1-x} As$, equilibrium bond lengths and order-induced gap reduction effects in $In_{x}Ga_{1-x}P$ alloys, illustrate the flexibility of this scheme. Previous applications of this method to semiconductor systems include the formation energy of ordered compounds^[7], and pressure effects in $Al_x Ga_{1-x} As$ [9,24]. It is interesting to note that the small crystal approach was originally developed for the study of many-body effects in exactly soluble models^[10]. We have shown that it may be extended to couple statistical mechanics concepts and simulation techniques to standard solid-state methods in a variety of problems involving bulk materials. Future extensions include the study of heterostructures and doping in semiconductor materials.

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