A Generalized Scalar Model

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A model including several scalar fields rotating under the same symmetry group is studied. It suggests that the same physics can be described by different field parametrizations. One consequence is that a given symmetry will take different representations depending on the field basis where it is developed.

I. Introduction

Scalar fields are regarded, at least formally, as the most primitive manifestation of matter. A modern framework to characterize such an abundance of scalar excitations would be through superstring-inspired models. The appearance of families of scalars in addition to the charged matter scalars is a feature of models derived from superstrings upon compactification on 6-dimensional Calabi-Yau spaces[1]. These models display in their spectrum the so called dilaton field (a scalar present in the supergravity multiplet) along with further scalars, known as moduli fields, that account for the shape of the compact internal space[1,2].

The fact is that there is an experimental reality expressing the existence of various scalar particles. This evidence is always challenging the creation of theoretical models to describe it. In this way, the main effort in this text is to organize such a variety of scalar flavours in a model based on the invariance under certain global transformations. It means to consider the presence of different flavours rotating under the same group through the following global transformation:

$$\Phi_a \rightarrow \Phi'_a = e^{i\alpha q_a}\Phi_a. \qquad (I.1)$$

It yields the following Lagrangian

$$\mathcal{L} = \Phi^+ \Box \Phi - \Phi^+ m^2 \Phi, \qquad (I.2)$$

that reproduces the historical meson-π case[3] for

$$\Phi^+ = (\pi^+, \pi^0, \pi^-). \qquad (I.3)$$

Nevertheless there is a more general possibility which is to consider a global phase rotation involving different fields

$$\varphi_i(x) \longrightarrow \varphi'_i(x) = \left(e^{i\alpha\widetilde{Q}}\right)_{ij} \varphi_j(x), \qquad (I.4)$$

where Q is an N-dimensional charge matrix, not necessarily diagonal. It yields a more general global invariant Lagrangian which includes non-diagonal kinetic and mass terms,

$$\mathcal{L} = \varphi^+ \Box K \varphi - \varphi^+ M^2 \varphi, \qquad (I.5)$$

where φ is an N-dimensional vector. From the reality condition, one concludes that K and M^2 must be N-dimensional Hermitean matrices. Thus it yields a total of N(N+1) parameters involved in eq. (1.5). Calculating the corresponding canonical momenta

$$\pi_i[\varphi_N] = K_{ij}\dot{\varphi}_j \qquad (I.6)$$

one concludes that when K is invertible there are N dynamical fields, i. e., the velocities $\dot{\varphi}_i$ can be written in terms of the canonical momenta π_{φ_i}.

For global gauge invariance to be achieved, the following relationships must hold:

$$K\widetilde{Q} = \widetilde{Q}^+ K$$
$$M\widetilde{Q} = \widetilde{Q}^+ M \qquad (I.7)$$

However the interesting fact is that Lagrangeans (1.5) and (1.2) are related. For this, let us define the following R transformation between fields

$$\varphi = \Omega\Phi \qquad (I.8)$$

which yields

$$\Omega^+ K \Omega = \mathrm{I},$$
$$\Omega^+ M^2 \Omega = \mathrm{m}^2(\text{diagonal}). \qquad (I.9)$$

Notice from (1.9) that R is not necessarily unitary (unless the kinetic vector is a multiple of the identity). Substituting (1.8) in (I.1), one also gets

$$\widetilde{Q} = \Omega Q \Omega^{-1}, \qquad (I.10)$$

where Q is the diagonal charge expressed in (1.1)

$$Q_{ab} = q_a \delta_{ab}. \qquad (I.11)$$

Consequently transformations (I.8) - (1.10) are showing that Lagrangians (1.2) and (1.5) are just different parameterizations that describe the kinematics of a same family of scalar quanta. In order to differentiate them, fields φ with non diagonal terrns in the Lagrangian are entitled constructor fields, while fields Φ are considered as the physical fields.

In the following section, an extended scalar model which includes interaction is proposed. Section III studies the properties of a R matrix which is responsable for rotations on the field basis. Finally section IV deals with initial steps for a future renormalization program to be developed. Two appendices are left for complementing the text.

II. Generalized Scalar Model

For articulating the model the next task should be to calculate the R matrix which relates the diagonal and non-diagonal parameterizations. We will follow a constructive process. This means by starting in (1.5) to define (1.2). So the first step is to diagonalize the kinetic term, matrix K. Considering that it is hermitean there is an unitary transformation S such that

$$\widetilde{K} = S^t K S = \widetilde{K}_i \delta_{ij}, \qquad (II.1)$$

where we assume that the eigenvalues $\widetilde{K}_i$ are all non-zero. Then rewriting the Lagrangian, one obtains a mass term

$$\mathcal{M}^2 = \widetilde{K}^{-\frac{1}{2}} S m^2 S^t \widetilde{K}^{-1/2}, \qquad (II.2)$$

which is also hermitean and then can be diagonalized by another unitary transformation

$$m^2 \equiv R\mathcal{M}^2 R^t = m_{ij}^2 \delta_{ij}. \qquad (II.3)$$

Thus one derives

$$\Omega = S^t \widetilde{K}^{-1/2} R^t. \qquad (II.4)$$

One should nstice that in order to perform the R calculation it is necessary to use algebraic computation techniques which depend on the number of involved field[4].

Taking the equations of motion one notices that this N scalar complex model contains N-Klein Gordon equations. For Φ-basis they are

$$(\Box + m^2)\Phi = 0. \qquad (II.5)$$

Using (1.8) and (I.9), we have

$$(K\Box + M^2)\varphi = 0. \qquad (II.6)$$

Thus, from (11.5) and (I.1), one concludes that a defined mass and charge are associated to every field $\Phi_i(x)$. Now it is more clear why they were defined as physical fields, while $\varphi_i(x)$ will be called as original or constructor fields. Considering Yukawa interpretation, Φ_i can be identified as the meson fields associated to a correspondent nuclear source, $\rho_i = g_i \delta(\overrightarrow{x})$. Thus, equation (11.5) is representing the usual situation where particles have a mass m_{ii}, and the field is significant in size only out to a range of force r $\sim 1/m_{ii}$.

Analysing the associated conserved currents

$$\frac{\partial P_i}{\partial t} + \operatorname{div} \overrightarrow{S}_i = 0, \qquad (II.7)$$

where the probability $P_i(\overrightarrow{r}, t)$ and the current $S_i(\overrightarrow{r}, t)$ are described as

$$P_i(\overrightarrow{r}, t) = \frac{i}{2m_{ii}} \left(\Phi_i^* \frac{\partial \Phi_i}{\partial t} - \Phi_i \frac{\partial \Phi_i^*}{\partial t} \right),$$

$$S_i(\overrightarrow{r}, t) = \frac{-i}{2m_{ii}} \left(\Phi_i^* \overrightarrow{\nabla} \Phi_i - (\nabla \Phi_i^*) \Phi_i \right). \qquad (II.8)$$

We see that the old question about negative probability still exists, as expected. But the important fact to be noticed here is that by changing the parameterization system the relative sign question remains unchanged. Physics should not change under R rotation.

Now let us explore some consistencies of this scalar extended model. For this we have to study some basic aspects as a positive free Hamiltonian as the presence of tachyons. We know that in order to undertake the perturbation theory program, a first condition is to have a positive Hamiltonian corresponding to the free sector. Then, deriving the generalized energy-momentum tensor through the method of coupling with gravitation, it gives

$$T_{\mu\nu}[\varphi] = 2K_{ij} \partial_\mu \varphi_i^* \partial_\nu \varphi - \eta_{\mu\nu} L[\varphi] \qquad (II.9)$$

Expressing the corresponding Hamiltonian in terrns of canonical momenta,

$$H[\varphi] = K_{ij}^{-1} \pi_i \pi_j^* + K_{ij} \overrightarrow{\nabla} \varphi_i^* \nabla \varphi_j + M_{ij}^2 \varphi_i^* \varphi_i. \qquad (II.10)$$

However eq. (11.10) does not give information about the Harniltonian positivity. So, as a first advantage of being able to express the physical model under different parameterizations it appears that by rotating to the well known Φ-basis we get the information that the scalar Harniltonian is positively defined.

For analysing the possible presence of tachyons we are going to take as an example a case involving just two fields

$$K = \begin{pmatrix} k_{12}^* & k_{22} \end{pmatrix} \quad ; \quad M^2 = \begin{pmatrix} \mu_{11} & \mu_{12}\mu_{12}^* & \mu_{12} \end{pmatrix}$$

It gives the following physical masses,

$$m_{1,2}^2 = \frac{1}{2(k_{11}k_{22} - k_{12}k_{12}^*)} [k_{22}\mu_{11} - t(k_{12}\mu_{12} + k_{12}^*\mu_{12}^*) + k_{11}\mu_{12} \pm \Delta] \qquad (II.11)$$

wliere

$$\begin{aligned}\Delta &= [(k_{22}\mu_{11} + (k_{12}\mu_{12} + k_{12}^{*}\mu_{12}) + k_{11}\mu_{22})^2 \\ &- 4(k_{11}k_{22} - k_{12}k_{12}^{*})(\mu_{11}\mu_{22} - \mu_{12}\mu_{12}^{*})]^{1/2}.\end{aligned} \qquad \text{(II.12)}$$

Consequently eq. (11.12) shows that by parametrizing the free coefficients given by matrices K and M elements space-like poles could be avoided.

After tliese two very minimal conditions are achieved, one can switch on tlie interaction part. For tlie Φ-basis, it

$$\mathcal{L} = \partial_\mu \Phi^+ \partial^\mu \Phi - \Phi^+ m^2 \Phi - \frac{\Lambda}{4}\left(\Phi^+\Phi\right)^2, \qquad (II.13)$$

and for φ-basis

$$\mathcal{L} = \partial_\mu \varphi^+ K \partial^\mu \varphi - \varphi^+ M^2 \varphi - \frac{1}{4}\varphi^+\varphi^+\lambda\varphi\varphi. \qquad (II.14)$$

Thus eqs. (11.13) and (11.14) will correspond to the so-called generalized scalar model which this work analyze.

III. R Matrix

Section II provides us with the information that a Lagrangian containing N scalar flavours can be rewritten through different field parameterizations. Then, an R – matrix naturally emerges. It contains N(N+1) parameters. Its task is to connect the various parameterization descriptions. Nevertheless, for tlie validity of such R-matrix it will be necessary to prove that physics does not depend on it. This is tlie effort of tliis section.

Thus, R is basically an abstract entity coordinating the field transformations. Eqs. (1.9) are the first expression relating an R existence. Subsequently, through a constructive analysis, an explicit expression was developed in eq. (11.4). Tlien it follows two basic properties for R which are that it is not generally unitary (unless kinetic terms are diagonal) and tliat tlie set of R's do not form a group. Being a matrix it contains the identity and associative properties; the condition for being invertible is obtained by avoiding zero eigenvalues for $\widetilde{K}$ matrix in (11.1); however eq. (11.4) does not allow the closure property. Nevertheless, although R does not constitute a group, it should be noted that its transformations generate tensors.

The first two basic theoretical entities preserved by R are the S-matrix and the number of degrees of freedom. Borscher[5] states that any S-matrix is preserved under field parameterizations which does not violate locality, a statement that eq. (1.7) satisfies. The condition for the number of degrees of freedom be preserved is the K-matrix be non singular, as eq. (1.6) says.

Putting into a quantum field theory tlie physical fields and their conjugate momenta must satisfy equal time commutation relations:

$$[\Phi_i(x), \Phi_j(y)]_{x^0=y^0} = [\pi_{\Phi_i}(x), \pi_{\Phi_j}(y)]_{x^0=y^0} = 0, \qquad (III.1)$$

$$[\Phi_i(x), \pi_{\Phi_j}(y)]_{x^0=y^0} = i\delta_{ij}\delta(\vec{x} - \vec{y}). \qquad (III.2)$$

Then, considering that

$$\pi_\varphi = (\Omega^{-1})^t \pi_\Phi, \qquad (111.3)$$

one obtains,

$$[\varphi_i(x), \varphi_j(y)]_{x^0=y^0} = [\pi_{\varphi_i}(x), \pi_{\varphi_j}(y)]_{x^0=y^0} = 0, \qquad (111.4)$$

$$[\varphi_i(x), \pi_{\varphi_j}(y)]_{x^0=y^0} = i\delta_{ij}\delta(\vec{x} - \vec{y}). \qquad (111.5)$$

Observe tliat (111.5) is consistent with (1.8) in (111.2).

Analysing from the kinematics viewpoint, tlie minimal action principle is also preserved under R transformation:

$$\frac{\delta S}{\delta \varphi}[\varphi] = \Omega^{-1}\frac{\delta S'}{\delta \Phi}[\Phi] = 0, \qquad (III.6)$$

which is confirmed by working out tlie Euler Lagrange equation

$$\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \partial} = \Omega^{-1}\left[\frac{\partial \mathcal{L}}{\partial \Phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \Phi}\right] = 0. \qquad (III.7)$$

Thus such change on field basis does not affect the on-sliell information. This information can also be obtained from Hamilton's canonical form,

, = —SH
h ~ ;

$$\pi_\varphi = -\frac{\delta H}{h,,} \qquad \text{(III.8)}$$

whicli gives under Ω transformation

$$\begin{aligned}\dot{\Phi} &= \frac{\delta H}{\delta \pi_\Phi} \\ \dot{\pi}_\Phi &= -\frac{\delta H}{\delta \Phi}\end{aligned} \qquad \text{(III.9)}$$

Thus an important aspect tliat eqs. (111.2) and (III.5), (111.8) and (111.9) are showing is that the proposed R rotation is a canonical transformation.

As a next step we should study tlie symmetry properties under R transformations. For this we will prefer first to understand the internal symmetries. Given a global symmetry the corresponding conserved current is

$$J^\mu[\Phi] = \frac{\partial \mathcal{L}}{\partial \partial_\mu \Phi_j}\delta \Phi_j. \qquad (III.10)$$

Similarly for the φ-basis,

$$J^\mu[\varphi] = \frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi_j}\delta \varphi_j \qquad (III.11)$$

Substituting (1.8) in (III.10) one obtains,

$$J^{\mu}[\varphi] = J^{\mu}[\varphi], \qquad (III.12)$$

a consistent result to the fact that to every symmetry in the Lagrangian must be associated only one conserved current. The difference here is that this current can be written through different functional formulae.

Assuming that the symmetry takes the form

$$\Phi \longrightarrow \Phi' = U(\alpha)\Phi, \qquad (III.13)$$

where

$$U = e^{i\alpha^a ta} \qquad U^+U = 1, \qquad (111.14)$$

and

$$[t_a, t_b] = if_{abc}t_c\ , \qquad (III.15)$$

a charge Q^a follows,

$$Q^a[\Phi] = \int d^3x J_0^a(x), \qquad (III.16)$$

with tlie following properties

$$[Q^a[\Phi], \Phi_i] = t_{ij}^a \Phi_j(x), \qquad (III.17)$$

and

$$[Q^a[\Phi], Q^b[\Phi]] = if^{abc}\Phi_c[\Phi]. \qquad (III.18)$$

Thus the appearance of these charges and their associated currents is an important consequence of symmetry. It is the basis for physical states to be prepared. Therefore it is important to understand how far such symmetry property is preserved under field parameterizations.

Substituting (1.8) in (III.13), one obtains a new transformation

$$\varphi' = T(\alpha)\varphi, \qquad (111.19)$$

where $T(\alpha)$ are new N $\times$ N matrices given by

$$T(\alpha) = e^{i\alpha^a H_a}, \qquad (III.20)$$

with

$$\alpha^a H_a = \Omega \alpha^a t_a \Omega^{-1}. \qquad (111.21)$$

Then, (111.21) in (111.14) gives the general condition

$$T^+(\alpha)T(\alpha) \neq 1, \qquad (III.22)$$

which shows that a same symmetry can be represented by a unitary matrix in some field parameterization and by a non-unitary matrix in another field basis. In terms of generators it is expressed as

$$t^a = t^{a+},$$

and

$$H^a \neq H^{a+}. \qquad (111.23)$$

Observe that when the kinetic matrix is a multiple of identity H^a is herrnitean.

Considering tlie above classical transformations as acting on field operators, the transformations now read:

$$\Phi'(x) = e^{i\alpha^a Q_a[\Phi]}\Phi(x)e^{-i\alpha^a Q_a[\Phi]},$$

and

$$\varphi'(x) = e^{i\alpha^a \widetilde{Q}_a[\varphi]}\varphi(x)e^{-i\alpha^a Q_a[\varphi]}, \qquad (III.24)$$

with the corresponding infinitesimal transformations

$$\delta\Phi = i\alpha^a[t_a, \Phi] = i\alpha^a t_a \Phi,$$

and

$$\delta\varphi = i\alpha^a[H_a, \varphi] = i\alpha^a H_a \varphi. \qquad (III.25)$$

Thus, although these field parameterizations change the generators shape it is important to show that the symmetry message is not lost under these transformations. In order to analyse this crucial question we have to study tlie generators and their algebra, and the corresponding maximum Abelian set. The observables definitions must be the same.

Under such field parameterization the number of generators is obviously preserved, and also the corresponding Lie algebra

$$[H_a, H_b] = if_{abc}H_c. \qquad (III.26)$$

From the invariance of Lagrangian (11.14) under (111.19) follow the relationships

$$\begin{aligned} H_a^+ K &= K H_a, \\ H_a^+ M^2 &= M^2 H_a, \\ H_a^+ H_a^+ \lambda &= \lambda H_a H_a, \end{aligned} \qquad (III.27)$$

where the above equations are not imposed.

Working on the φ-basis, we get

$$[\widetilde{Q}(t), \varphi_i(t, \overrightarrow{x})] = H_{ij}\varphi_j(t, \overrightarrow{x}), \qquad (III.28)$$

and

$$[\widetilde{Q}_a[\varphi], [\widetilde{Q}_b[\varphi]] = if_{abc}[\widetilde{Q}_c[\varphi], \qquad (III.29)$$

which are also indicative that the charges algebra is independent on Ω transformation. This result can be confirmed by remembering from (111.12) that the Hilbert operators $Q^a[\varphi]$ and $Q^a[\varphi]$ are equal

$$Q^a[\Phi] = \widetilde{Q}^a[\varphi]. \qquad (111.30)$$

Eq. (111.29) reaffirrns that the canonical commutation rules are preserved under fields parameterizations.

More explicitly, calculating the conserved charges corresponding to for the physical and original basis, we get

$$Q^a[\Phi] = -i\int d^3\overrightarrow{x}(\Phi^+ t_a \dot{\Phi} - \dot{\Phi}^+ t_a \Phi), \qquad (III.31)$$

and

$$Q^a[\varphi] = -i\int d^3\overrightarrow{x}(\varphi^+ KH_a\dot{\varphi} - \dot{\varphi}^+ KH_a\varphi). \quad (III.32)$$

Then, the determination of how eqs. (111.31) and (111.32) act on fields Φ_i and φ_i respectively, yields that eqs. (111.17) and (111.28) are reproduced. Similarly for eqs. (111.18) and (111.29).

Finally we have to understand the consequences of Ω rotations on the physical states. Observables must belong to a maximum Abelian set derived from the algebra of charges. Considering as example the SU(3) group, we can take $f_{380} = 0$ the Q_3 and Q_8 generators are adopted as members of this maximum Abelian set. Consequently one derives that the corresponding $\widetilde{Q}_3$ and $\widetilde{Q}_8$ generator will also commute. This shows that from Ω invariance for charges algebra we have that observables are independent under field parameterizations. Concluding, we would note that symmetry can change shape (generators expressions) and type (be unitary or not) but their physics preserved. For this we have studied how the Lie algebra, currents and charges, current algebra and the Cartan subalgebra, are invariants under Ω transformations. Thus, through the introduction of more fields in a same group one can learn that symmetry expression can change but this fact does not mean that the involved quantum numbers for preparing the physical states will change.

As for space-time transformations, the R matrix is more transparent to them than to internal symmetries. From literature[6] the corresponding conservation laws for the scalar case are for translational invariance, Lorentz rotations, dilation transformations, conformal boosts, respectively:

$$\partial_\mu \theta^\mu_\nu[\Phi] = 0.$$

with

$$\theta^\mu_\nu[\Phi] = \frac{\partial\mathcal{L}}{\partial\partial_\nu\Phi}[\Phi]\partial^\mu\Phi - \delta^\mu_\nu\mathcal{L}[\Phi]\ , \quad (III.33)$$

$$\partial_\mu L^\mu_{\nu\rho}[\Phi] = 0$$

with

$$L^\mu_{\nu\rho} = x_\nu\theta^\mu_\rho[\Phi] - x_\rho\theta^\mu_n u[\Phi]\ , \quad (111.34)$$

$$\partial^\mu D_\mu[\Phi] = 0$$

with

$$D_\mu[\Phi] = x^\nu\theta_{\mu\nu}[\Phi] - \frac{\delta\mathcal{L}[\Phi]}{\delta\partial\mu\Phi}L\Phi\ , \quad (III.35)$$

and

$$\partial^\mu K_{\mu\nu}[\Phi] = 0$$

with

$$K_{\mu\nu}[\Phi] \equiv x_\nu\mathcal{D}[\Phi] + x^\lambda L_{\lambda\mu\nu}[\Phi],$$

where

$$\mathcal{D}[\Phi] = x^\rho\theta_{\rho\mu}[\Phi] \quad (111.36)$$

and $\theta^\mu_\nu[\Phi]$, $L^\mu_{\nu\rho}[\Phi]$, $\mathcal{D}_\mu[\Phi]$, $K_{\mu\nu}[\Phi]$ are respectively the improved energy-momentum tensor, angular momentum tensor, dilation current, and the conserved tensor associated to the special conformal transformation.

Considering the p-basis, we have

$$\theta^\mu_\nu[\varphi] = \frac{\partial\mathcal{L}}{\partial\partial_\mu\varphi}[\varphi]\partial_\nu\varphi - \delta^\mu_\nu\mathcal{L}[\varphi], \quad (III.37)$$

and

$$D^\mu[\varphi] = x^\nu\theta_{\mu\nu}[\varphi] - \frac{\partial\mathcal{L}}{\partial\partial_\mu\varphi}[\varphi]L\varphi. \quad (III.38)$$

Then, substituting eq. (I.8), yields

$$\theta^\mu_\nu[\Phi] = \theta^\mu_\nu[\varphi] \quad (III.39)$$

and

$$D^\mu[\Phi] = D^\mu[\varphi]. \quad (111.40)$$

Thus all space-time charges are preserved under R transformation.

Deriving these charges explicitly for this extended scalar model (11.13) yields the expression in (11.9) and the subsequent relations. Calculating the dilation current and considering that the scalar case has dimension L = −1, one gets for the massless case

$$\partial^\mu\mathcal{D}_\mu[\Phi] = 0. \quad (III.41)$$

For the massive case, we have the expected scale invariance breakdown

$$\theta^\mu_\mu = m_i^2\Phi^{i2}. \quad (111.42)$$

The main object of a quantum field theory are the quanta. Thus, the most relevant aspect for a R matrix to be accepted is to prove the quanta invariance under its rotations. Considering that a given quanta is characterized through entities such as spin, mass, and charges (space-time, internal, discrete), the task here should be to prove that all these entities are preserved under (1.8) transformation.

The tensor spin density[6],

$$S^{\alpha\beta\nu} = -\frac{1}{2}\frac{\partial\mathcal{L}}{\partial\partial_\nu\Phi}\sum\nolimits^{\alpha\beta}\Phi, \quad (III.43)$$

is obviously an invariant for the scalar case.

For the physical masses, the similarity transformation

$$\Omega^{-1}(K^{-1}M^2)\Omega = \mathrm{m}^2, \quad (111.44)$$

show that the poles are preserved under a R rotation.

The space-time charges invariances were already proved in (111.33) - (111.40). Then we get that the Harniltonian, linear and angular momentum can be equally described by any parameterization system satisfying Borscher theorem. The instruction is that a given Lie Algebra for the Poincaré, Weyl, Conforma1 group will be preserved under R transformations.

A next discussion is on the internal charges that label the quanta. We will study just cases with global invariance. They are the possibilities $U(N)$, $[U(1)]^N$, $U(N-M)\otimes[U(1)]^M$, and $U(N_1)\otimes\ldots\otimes U(N_M)\otimes[U(1)]^P$. Considering the physical

(i) For $U(N): U(\alpha)=e^{iw^a t_a}$

The conditions are

$$m^2 = m^2 11_{N\times N},$$

$$\Lambda_{ijkl} = \Lambda\delta_{ik}\delta_{jl} \quad \text{or} \quad \Lambda\delta_{il}\delta_{jk}, \qquad (III.45)$$

yielding the following conserved current

$$J_a^\mu[\Phi] = -i\Phi^+ t_a \overleftrightarrow{\partial}\Phi. \qquad (III.46)$$

(ii) For $[U(1)]^N : U(\alpha) = e^{i\alpha_1 y_1}\ldots e^{i\alpha_N y_N}$ where y_i are the charges associated to different $U(1)$'s. The conditions are

$$m_1^2 \neq m_2^2 \neq \ldots \neq m_N^2,$$

$$\lambda_{ijkl} = \Lambda\delta_{ik}\delta_{jl} \quad \text{or} \quad \Lambda\delta_{il}\delta_{jk}, \qquad (III.47)$$

associated to N conserved currents

$$J_i^\mu[\Phi] = -iq_i\Phi_i^+\overleftrightarrow{\partial}\Phi_i. \qquad (III.48)$$

(iii) For $U(N-M)\otimes[U(1)]^M : [U(\alpha) = e^{iw_a t a}e^{iw_\alpha y^\alpha}$. The conditions are a mix between above cases. Similarly there are the (N − M) currents J_a^μ, and M currents J_α^μ.

(iv) For $U(N_1)\otimes\ldots\otimes U(N_M)\otimes[U(1)]^P$, with $N_1+N_2+\ldots+N_P=N$. The conditions are physical masses with degeneracy $N_1,\ldots,N_M$ and P differents masses.

Discrete symmetries under different field basis also need be studied. Parity will be the first case to be considered. For simplicity, one should first analyse it on the diagonal basis

$$\Phi(x)\overset{P}{\longmapsto}\Phi'(\overrightarrow{x'},t') = \eta_P\Phi(-\overrightarrow{x},t)$$

$$\eta_P^2 = 1, \qquad (III.49)$$

which reproduces the usual Klein Gordon situation. Then, transforming into φ-basis we have

$$\varphi(x)\overset{P}{\longmapsto}\varphi'(\overrightarrow{x'},t') = P\varphi(-\overrightarrow{x},t), \qquad (111.50)$$

where the matrix P which represents space inversions

$$P = \Omega\eta_P\Omega^{-1},$$

with

$$P = P^{-1}. \qquad (111.51)$$

Notice that the fields φ_i and φ_j can have different intrinsic parities and the Lagrangian P-invariance remains unbroken.

The charge invariance is

$$\Phi(x)\overset{C}{\longmapsto}\Phi^C(x) = \eta_C\Phi^*(x),$$

with

$$(\eta_C)_{ij} = \eta_i\delta_{ij}. \qquad (111.52)$$

Then,

$$\varphi(x)\overset{C}{\longmapsto}\varphi^C(x) = C\varphi^*,$$

where

$$C = \Omega\eta_C(\Omega^{-1})^*. \qquad (111.53)$$

Time inversion is

$$\Phi(x)\overset{T}{\longmapsto}\Phi'(x') = \eta_T\Phi(x), \qquad (III.54)$$

which one reads on φ-basis as

$$\varphi(x)\overset{T}{\longmapsto}\varphi'(x') = \mathcal{T}\varphi(x),$$

where

$$\mathcal{T} = \Omega^*\eta_T\Omega^{-1}. \qquad (111.55)$$

Now taking (III.51), (III.53), (111.55) transformations in the φ-basis Lagrangean one can explicitly show that the kinetic, mass, and interaction terms preserve such invariances separately. Consequently no constraints on matrices K, M^2, λ are necessary. It is important to note that every discrete symmetry when analysed in terms of Φ_i fields is well-defined. Because of this precise aspect they can be called physical fields.

Considering that the Lagrangean is local, Hermitean, and invariant under restrict Poincaré group we obtain the expected CPT invariance

$$\Phi(x)\overset{CPT}{\longmapsto}\Phi'(x') = \eta_{CPT}\Phi^*(x),$$

$$\eta_{CPT} = \eta_C\eta_P\eta_T = \pm 1, \qquad (III.56)$$

and

$$\varphi(x)\overset{CPT}{\longmapsto}\varphi'(x') = \mathcal{N}\varphi^*(x),$$

$$\mathcal{N} = \Omega^*\eta_{CPT}(\Omega^{-1})^*. \qquad (III.57)$$

The currents associated to this extended scalar model transform

$$J_a^\mu[\Phi]\longmapsto^{CPT} - J_a^\mu[\Phi], \qquad (III.58)$$

and

$$j_a^\mu[\varphi]\longmapsto^{CPT} - j_a^\mu[\varphi]. \qquad (III.59)$$

Eqs. (111.58) and (111.59) show that Ω transformation also preserve the particle-antiparticle description.

A final aspect would be to analyse the annhilation and creation operators in terms of R transformations. For the physical basis, fields $\Phi_i(x)$ are Hermitean operators whose Fourier expansion may be writen

$$\Phi_i(x) = \frac{1}{(2\pi)^{3/2}}\int d^3 \quad [A_i(k)e^{-ikx} + A_i^+(k)e^{ikx}] \qquad (III.60)$$

with $w_i = (\overrightarrow{k} + m_i^2)^{1/2}$. The A_i^+ and A_i operators create or annihilate a single scalar particle with energy w_i. A scalar particle is thus an excitation of one particular oscillator mode.

Then, considering reparameterization (I.8) the new field can be written as

$$\varphi_i(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3\overrightarrow{k}}{\sqrt[3]{2w_i}}$$
$$[a_i(\overrightarrow{k},t)e^{i\overrightarrow{k}\overrightarrow{x}} + b_i^+(\overrightarrow{k},t)e^{-i\overrightarrow{k}\overrightarrow{x}}] \text{(III.61)}$$

where

$$a_i(\overrightarrow{k},t) \equiv \Omega_{ij} A_j(\overrightarrow{k}) e^{-iwjt},$$
$$b_i^+(\overrightarrow{k},t) \equiv \Omega_{ij} B_j^+(\overrightarrow{k}) e^{iwjt}. \qquad (III.62)$$

Notice that $a_i(\overrightarrow{k},t)$ and $b_i^+(\overrightarrow{k},t)$ do not correspond to states with a well defined mass. This result is a consequence from the equation of motion

$$(\Box K_{ij} + M_{ij})\varphi_j = 0. \qquad (III.64)$$

Where the φ-basis operators show an expansion which mix frequencies. This fact constitutes as another proof that basis-φ is associated to non-physical quanta.

However the physical information is preserved under (III.61) and (III.62). Although the respective commutation rules, counting operator, and physical entities as charge, Hamiltonian and momenta can change their shape, the model consistency is preserved. For instance, for Φ-basis the charge is

$$Q[\Phi] = \sum_i Q_i[\Phi],$$

where

$$Q_i[\Phi] = \int d^3\overrightarrow{k} [A_i^+(k)A_i(k) - B_i^+(k)B_i(k)] \,, \quad (III.65)$$

while for φ-basis

$$Q[\varphi] = \int d^3\overrightarrow{k} [b^+Kb - a^+Ka] \,, \qquad (III.66)$$

However, as we know, (III.65) and (III.66) are equal.

IV. On the Renormalizability

As an introduction to the renormalizability of the model, one intends to study propagators, power counting, Feynman graphs, and to discuss a possible modulation of the infinities of the theory.

Propagators are the first step to be analysed in order to understand the health of a theory. Through their poles one understands the structure of the physical masses; through the residue corresponding to each pole one detects the associated degrees of freedom and the presence of eventual non-physical states. Finally, through the analytical structure of the loop-corrected propagators, one can understand about processes leading to quanta production.

Deriving the propagators corresponding to Lagrangian (I.5) one gets, in momentum space, the following matrix expression

$$\langle T\varphi\varphi*\rangle = \frac{i}{Kk^2 - M^2} = \frac{i}{det(Kk^2 - M^2)} cof(Kk^2 - M^2). \qquad (IV.1)$$

Eq.(IV.1) contains diagonal and non-diagonal propagators. As the cofactor is a Hermitean matrix (symmetric in the real case), it gives a total number of $\frac{N(N+1)}{2}$ propagators. In components, one writes

$$\langle T\varphi_i\varphi_j^*\rangle = \frac{i}{det(Kk^2 - M^2)} cof(Kk^2 - M^2)_{ij}, \quad (IV.2)$$

which gives

$$\langle T\varphi_i\varphi_j^*\rangle = \frac{i}{A(k^2 - m_1^2)\dots(k^2 - m_N^2)}$$
$$[B_{ij}^{(1)}(k^2)^{(N-1)} + \dots + B_{ij}^{N-1}(k^2) + B_{ij}^{(N)}], \qquad \text{(IV.3)}$$

where m_p^2 are the physical masses and $A, B^{(f)}$ are coefficients expressed in terms of the parameters of the free Lagrangian (p and f varie from $1, ..., N$).

Thus, equations (IV.1) and (IV.3) bring four relevant characteristics:

(i) the roots of

$$det(Kk^2 - M^2) = 0, \qquad (IV.4)$$

are the poles of the propagators;

(ii) each propagator carries the same pole structure. This means that the association between fields and poles is no longer one-to-one;

(iii) the cofactor matrix is symmetric and can be factorized in a polynomial of maximum power $(k^2)^{N-1}$;

(iv) the asymptotic behaviour of each propagator is, in principle, of form $1/k^2$, since $det(Kk^2 - M^2)$ is a polynomial of power $(k^2)^N$ which is compatible with the diagonalized parameterization, and so, it is expected a healthy power counting.

After the physical masses, m_p^2 , stipulated by eq. (IV.4) a further step would be to infer on the physicity of these poles. For this, we have to read off the residue matrix, $R^{(p)}$, corresponding to each pole

$$R^{(p)}(k^2 = m_p^2) = \frac{i}{A\prod\limits_{\substack{i=1\\i\neq p}}^{N}(m_p^2 - m_i^2)} cof(Km_p^2 - M^2).$$

Consequently, to every simple pole there corresponds an $N \times N$ residue matrix. However, from the above expression the physicity question cannot be interpreted so easily. Thus, in order to derive the degrees of freedom associated to the physical excitations, we have to decouple the propagators in terms of N simple poles. This procedure brings about another way of expressing $R^{(p)}$:

$$\langle T(\varphi\varphi)\rangle = \frac{i}{A}\left[\frac{1}{k^2-m_1^2}r^{(1)}+\ldots+\frac{1}{k^2-m_N^2}r^{(N)}\right], \qquad (IV.6)$$

where

$$r^{(1)}+\ldots+r^{(N)} = B^{(1)}. \qquad (IV.7)$$

Thus the residue matrix, $R^{(p)}$, corresponding to every single pole is

$$\underset{k^2=m_p^2}{Res}\langle\varphi\varphi\rangle = \frac{1}{A}r^{(p)}(k^2=m_p^2) = R^{(p)}(k^2=m_p^2). \qquad (IV.8)$$

One expects that the information on degrees of freedom and on the presence of ghosts can be read off from the eigenvalues of $R^{(p)}$. For this, we should explore its properties. Observing the LHS of (IV.6) , one notes that the matrices $R^{(p)}$ are symmetric which ensures that their respective eigenvalues are real. Thus there are only positive, negative and zero eigenvalues to inform about the physical conditions.

Considering that physics does not depend on field parameterization, we should study about d. f. and ghosts in the so-called physical basis, Φ_i. There, it follows that there are N d. f. and it also says that for the norm of a state to be positive (negative), it depends whether the kinetic term has a positive (negative) sign. Therefore, a given physical state carrying just one d. f. is defined as

$$\underset{k^2=m_p^2}{Res}\langle\Phi\Phi\rangle = i. \qquad (IV.9)$$

Then, since physics does not change under R parameterization, it is expected from d. f. arguments that (N-1) eigenva'ues of $R^{(p)}$ must be zero and only one will assume a real value. It is also expected that physical quanta, ghosts and non-dynamical quanta will correspond to positive, negative and zero eigenvalues respectively.

Thus, it is still necessary to express $R^{(p)}$ more properly. Since the propagator in both parameterization basis are related by R

$$\langle T(\varphi\varphi^*)\rangle = \Omega\langle T(\Phi\Phi^*)\rangle\Omega^t, \qquad (IV.10)$$

one gets

$$\langle T(\varphi_i\varphi_j^*)\rangle = \sum_{p=1}^{N}\frac{i}{k^2-m_p^2}\Omega_{ip}\Omega_{jp}, \qquad (IV.11)$$

i.e., for a p-pole the associated residue matrix is

$$R_{ij}^{(p)} = i\Omega_{ip}\Omega_{jp}, \qquad (IV.12)$$

without summing over p.

In this way, based on the fact that the kinetic matrix is positive-definite, one arrives at eq. (IV.12) which shows that the residue matrix can be formulated as a tensorial product of a vector by itself. It yields

$$\frac{1}{i}R^{(p)} = v^{(p)}\otimes v^{(p)t}, \qquad (IV.13)$$

where $v^{(p)}$ is the p-th. row of the matrix R. Then, finally we have arrived in a expression to study the eigenvalues. Defining

$$v^{(p)} = \begin{pmatrix} v_1^{(p)} \\ v_2^{(p)} \\ \vdots \\ v_N^{(p)} \end{pmatrix}, \qquad (IV.14)$$

one gets

$$\frac{1}{i}R_{ij}^{(p)} = v_i^{(p)}v_j^{(p)}, \qquad (IV.15)$$

working out the secular equation

$$\det\left(R^{(p)}-\lambda_{11}\right)=0, \qquad (IV.16)$$

it gives,

$$(-\lambda)^N+(-\lambda)^{N-1}\left(v_1^{(p)}+v_2^{(p)}+\ldots+v_N^{(p)}\right)=0, \qquad (IV.17)$$

which shows:

$$\begin{aligned}\lambda &= 0\ ,(N-1)\text{ times}\\ \lambda &= \|v^{(p)}\|^2>0\ . \end{aligned} \qquad (IV.18)$$

Concluding, eq. (IV.18) says that each residue matrix informs that it is associated to one only d. f. and the ghost question is controlled by having a K matrix positive-definite.

A necessary relationship for deriving the power counting analysis is to compare eqs. (IV.3) and (IV.11). It yields,

$$\begin{aligned}
B_{ij}^{(1)} &= \sum_{p=1}^{N}\Omega_{ip}\Omega_{jp}\\
B_{ij}^{(2)} &= -\sum_{p=1}^{N}\Omega_{ip}\Omega_{jp}\sum_{\substack{i=1\\ i\neq p}}^{N}m_i^2,\\
B_{ij}^{(3)} &= \sum_{p=1}^{N}\Omega_{ip}\Omega_{jp}\sum_{\substack{i,j=1\\ i\neq j\neq p}}^{N}m_i^2m_j^2,\\
B_{ij}^{(N)} &= \sum_{p=1}^{N}\Omega_{ip}\Omega_{jp}\sum_{\substack{i,j,\ldots,N=1\\ i\neq j\neq\ldots\neq N\neq p}}^{N}m_i^2\ldots m_N^2.
\end{aligned} \qquad (IV.19)$$

Notice that the sign in the above expression is positive or negative depending whether the number of physical masses is even or odd respectively.

We should now understand the condition for the coefficient $B_{ij}^{(1)}$ to be zero. For $B_{ii}^{(1)} = 0$, one gets a singular matrix R. However the non diagonal propagators can be modulated because their highest term in numerator, $B_{ij}^{(1)}$, can be taken to be zero without violating the model physicity. Thus the asymptotic behaviours of the involved propagators are

$$\lim_{k^2\to\infty} \langle T(\varphi_i\varphi_i)\rangle \sim \frac{1}{k^2},$$

$$\lim_{k^2\to\infty} \langle T(\varphi_i\varphi_j)\rangle \sim \frac{1}{(k^2)^{d_{ij}}}. \qquad (IV.20)$$

where $d_{ij} \geq 1$. Thus tlie corresponding superficial divergence for this extended scalar model described by eq. (11.14) is

$$\delta_{\text{graph}} = 4 - \sum_i E_{\varphi_i} + \sum_{i,j}(1-d_{ij})I_{\varphi_i\varphi_j}, \qquad (IV.21)$$

E_{φ_i} and $I_{\varphi_i\varphi_j}$ are the externa1 and interna1 lines of the corresponding graph.

An interesting consequence derived from gauge models which include more than one field rotating in the same group is the possibility of modulating the theory infinities. Then, considering the classical sector one has to study the Pauli-Jordan correlation functions (in configuration space) and the asymptotic behaviour of the propagators (in momentum space). Switching on the interactions, we would have to understand the quantum corrections to the counter terms and tlie vanishing of some β-functions. This work has only classical purposes.

In field theory, the singularities are independent of the Lagrangian. The field is an operator-valued distribution which carries singularities, detected in the Pauli-Jordan commutator functions and in the consequent propagator expression. Thus, for this extended scalar model, one has to calculate in the physical parameterization basis the following expression

$$D_{ij}(z,m_i^2) = i[\Phi_i(x),\Phi_j(y)] = \delta_{ij}D(z,m_i^2), \qquad (IV.22)$$

where $z^\mu = (x-y)^\mu$ and $D(z,m_i^2)$ is the Pauli- Jordan correlation function associated to a scalar field $\Phi_i(x)$. From [7],

$$D(z,m_i^2) = \frac{1}{2\pi}\epsilon(z^0)\delta(z^2) - \frac{m_i}{4\pi\sqrt{z^2}}\epsilon(z^0)\theta(z^2)J_1(m_i\sqrt{z^2}) \qquad \text{(IV.23)}$$

Eq. (IV.23) shows that there are two types of singularities in the light-cone. Rotating for the p-basis we get $\frac{N}{2}(N+1)$ correlation functions

$$i[\varphi_i(x),\varphi_j(y)] = \Omega_{ik}\Omega_{kj}\frac{1}{2\pi}\epsilon(z^0)\delta(z^2) - \Omega_{ik}\Omega_{kj}\frac{m_K}{4\pi\sqrt{z^2}}\theta(z^0)\epsilon(z^0)J_1(m_k\sqrt{z^2}). \qquad \text{(IV.24)}$$

Thus, eq. (IV.24) confirms eq. (11.19). Both show that only off-diagonal propagators can be modulated.

All the canonical information is in tlie Pauli-Jordan expression. The microcausality is preserved

$$[\Phi_i(x),\Phi_i(y)]_{z^2<0} = 0, \qquad (IV.25)$$

$$[\dot\Phi_i(x),\Phi_j(y)]_{x^0=y^0} = -i\delta_{ij}\delta^3(\vec{x}-\vec{y}). \qquad (IV.26)$$

For the φ-basis, the microcausality property between the fields is mantained, but the commutation rules change

$$[\dot\varphi_i(x),\varphi_j(y)]_{x^0=y^0} = -i(K^{-1})_{ij}\delta^3(\vec{x}-\vec{y}). \qquad (IV.27)$$

The two-point Green functions are

$$\Delta_F(z)_{ii} = i\langle T(\Phi_i(x)\Phi_i(y))\rangle = \frac{1}{4\pi}\delta(z^2) - \frac{m_i}{8\pi\sqrt{z^2}}\theta(z^2)J_1(m_i\sqrt{z^2}) - iN_i(m_i\sqrt{z^2}) + \frac{im_i}{4\pi\sqrt{-z^2}}\theta(-z^2)K_1(m_i\sqrt{-z^2}). \qquad \text{(IV.28)}$$

Observe that all singularities lie on the light-cone. Rotating, we get

$$i\langle(\varphi_i(x),\varphi_j(y))\rangle = (K^{-1})_{ij}\Delta_F(z)_{ii} \qquad (IV.29)$$

which confirms the asymptotic behaviour in (IV.23). Concluding, we notice that the introduction of more fields does not change the structure of infinities of tlie theory, as expected. However it allows to modulate the infinities in a certain basis-φ.

V. Conclusion

Gauge theory's argument is symmetry, from which theoretical tools and experimental predictions are developed. Therefore a natural road in the context of such theories is to look for new possibilities on extending symmetries. Thus the effort in this work was to observe the consequences on syrnrnetry considerations from a scalar model which includes several fields in a same group.

Lagrangians (11.13) and (11.14) show the existence of different field parameterizations that describe the same N scalar quanta. This fact generates a R matrix whose properties were studied at chapter III. It creates scalars and tensors under field rotations. An interesting consequence from such change on field basis is that symmetry also can change its shape. This means that depending on the field basis, there appear different representations for a same symmetry. This fact is noticed in eqs. (111.13) and (111.19) where the respective generators t_a and H_a preserve the Lie Algebra but H_a is not necessarily unitary.

Another consequence of the introduction of more fields in a saine group is that the infinities can be modulated. Section III has analysed not only a health spectroscopy wh ch avoids tachyons and ghosts, but also about a partial theory finiteness which can be modulated by manoeuvring the free coefficients organized in the initial Lagrangian.

Finally we should discuss about some advantages of working with different parameterization system. The standard case is to consider the Φ-basis as eq. (11.13). However the existence of a Lagrangian (11.14) describing the same physics, opens a panorama that $\mathcal{L}[\Phi]$ does not offer. This means that while C[@] kinetic part is build up by N mass parameter $\mathcal{L}[\varphi]$ will have N(N + 1)/2 Consequently a same symmetry can be studied in more detail in the original φ-basis. Thus, as general conclusion, we would observe that a change on field basis can be more propitious for detecting physics under more primitive coefficients.

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Appendix A. A Practical Example

Consider the following Lagrangian involving just two fields

$$\mathcal{L} = -\varphi^{+}\Box K\varphi - \varphi^{+}M^{2}\varphi - \frac{\lambda}{4}(\varphi^{+}\varphi)^{2} - \frac{\Lambda}{4}\left(|\varphi_1|^2|\varphi_2|^2\right)$$

where

$$\varphi = \begin{pmatrix} \varphi \\ \varphi_2 \end{pmatrix} \quad , \quad K = \begin{pmatrix} \frac{\alpha}{2} & i\frac{\beta}{2} \\ -i\frac{\beta}{2} & \frac{\alpha}{2}, \end{pmatrix}$$

$$M^2 = \gamma\begin{pmatrix} \alpha & -i\beta \\ i\beta & \alpha \end{pmatrix} \quad , \quad \gamma = \frac{1}{4}(m^2+\mu^2) \tag{A.1}$$

(λ and Λ are non negative parameters.)

Working out the diagonal Lagrangian, it yields

$$\mathcal{L} = -\Phi^{+}\Box\,\Phi - \Phi^{+}m^{2}\Phi - w\left(\frac{|\Phi_1|^2}{\lambda_+} + \frac{|\Phi_2|^2}{\lambda_-}\right) - -\frac{\Lambda}{16\lambda_+\lambda_-}(\Phi_1^*\Phi_2 + \Phi_2^*\Phi_1)^2, \tag{A.2}$$

where

$$\Omega = \frac{\sqrt{2}}{2}\begin{pmatrix} \frac{1}{\sqrt{\lambda_+}} & \frac{1}{\sqrt{\lambda_-}} \\ \frac{-i}{\sqrt{\lambda_+}} & \frac{i}{\sqrt{\lambda_-}} \end{pmatrix},$$

and

$$\lambda_+ = \frac{1}{2}(\alpha+\beta) \quad , \quad \lambda_- = \frac{1}{2}(\alpha-\beta). \tag{A.3}$$

The physical masses are

$$m_1^2 = 2\gamma\frac{\lambda_-}{\lambda_+} \quad , \quad m_2^2 = 2\gamma\frac{\lambda_+}{\lambda_-}. \tag{A.4}$$

The interaction parameter w is

$$w = \frac{\lambda}{4} + \frac{\Lambda}{16}. \tag{A.5}$$

A next aspect is to study the behaviour of a certain symmetry under reparameterization. For this we will choose the $SU(2)$ group. Imposing that our diagonalized model has this syrnrnetry we have the following restrictions to the free parameters

$$\beta = 0 \quad , \quad \Lambda = 0 \quad , \quad w = \frac{\lambda}{4},$$

and

$$\mathrm{m}^2 = 2\gamma 1\!\!1_{2\times 2}. \tag{A.6}$$

Thus under the above conditions **(A.2)** is invariant under the rotations

$$\Phi' = e^{iw^a t_a}\Phi,$$

where t_a are the Pauli-matrices. Calculating, as example, the corresponding conserved current $J_2^\mu[\Phi]$ associated to t_2:

$$J_2^\mu[\Phi] = -\{(\partial^\mu\Phi_2^*)\Phi_1 - (\partial^\mu\Phi_1^*)\Phi_2 - \Phi_2^*\partial_\mu\Phi_1 + \Phi_1^*\partial_\mu\Phi_2\}. \tag{A.7}$$

Now we should calculate one of the proposals of this work which is to observe modifications on the syrnrnetry shape under field parameterizations. Applying (A.3) in eq. (III.21), one gets

$$H_1 = t_3 \quad , \quad H_2 = -t_1 \quad , \quad H_3 = -t_2. \tag{A.8}$$

Thus (A.8) shows how generators can change but preserving the Lie algebra. Notice that this example is a very particular case where the Ω matrix are unitary. Calculating the conserved current $J_2^\mu[\varphi]$ related to H_2

$$J_2[\varphi] = -i\alpha\{(\partial^\mu\varphi_1^*)\varphi_2 + (\partial^\mu\varphi_2^*)\varphi 1 - \varphi_1^*\partial^\mu\varphi_2 - \varphi_2^*\partial^\mu\varphi_1\} \tag{A.9}$$

(A.9) is showing another expression for $SU(2)$ symmetry.

Considering the discrete transformations

$$\begin{pmatrix} \Phi_1' \\ \Phi_2' \end{pmatrix} = \begin{pmatrix} \eta_1^P & 0 \\ 0 & \eta_2^P \end{pmatrix}\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix},$$

$$\begin{pmatrix} \Phi_1' \\ \Phi_2' \end{pmatrix} = \begin{pmatrix} \eta_1^C & 0 \\ 0 & \eta_2^C \end{pmatrix}\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix},$$

$$\begin{pmatrix} \Phi_1' \\ \Phi_2' \end{pmatrix} = \begin{pmatrix} \eta_1^T & 0 \\ 0 & \eta_2^T \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix},$$

$$\begin{pmatrix} \Phi_1' \\ \Phi_2' \end{pmatrix} = \begin{pmatrix} \eta_1^{CPT} & 0 \\ 0 & \eta_2^{CPT} \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}. \quad (A.10)$$

$$\mathcal{P} = \frac{1}{2} \begin{pmatrix} \eta_1^P + \eta_2^P & i(\eta_1^P - \eta_2^P) \\ -i(\eta_1^P - \eta_2^P) & \eta_1^P + \eta_2^P \end{pmatrix}$$

$$C = \frac{1}{2} \begin{pmatrix} \eta_1^C + \eta_2^C & -i(\eta_1^C - \eta_2^C) \\ -i(\eta_1^C - \eta_2^C) & -(\eta_1^C + \eta_2^C) \end{pmatrix},$$

$$\mathcal{T} = \frac{1}{2} \begin{pmatrix} \eta_1^T + \eta_2^T & i(\eta_1^T - \eta_2^T) \\ i(\eta_1^T - \eta_2^T) & -(\eta_1^T + \eta_2^T) \end{pmatrix}$$

$$\mathcal{N} = \frac{1}{2} \begin{pmatrix} \eta_1^{CPT} + \eta_2^{CPT} & -i(\eta_1^{CPT} - \eta_2^{CPT}) \\ i(\eta_1^{CPT} - \eta_2^{CPT}) & \eta_1^{CPT} + \eta_2^{CPT} \end{pmatrix} \quad (A.11)$$

Another case is to considere a $U(1)$ global transformation for

$$\begin{pmatrix} \Phi_1' \\ \Phi_2' \end{pmatrix} = \begin{pmatrix} e^{i\alpha y_1} & 0 \\ 0 & e^{i\alpha y_2} \end{pmatrix} \begin{pmatrix} (\Phi_1 \\ \Phi_2 \end{pmatrix}. \quad (A.12)$$

Applying eq.(III.21) we have a non diagonal expression

$$T = \frac{1}{2} \begin{pmatrix} i\alpha y_1 + i\alpha y_2 & i(e^{i\alpha y_1} - e^{i\alpha y_2}) \\ -i(e^{i\alpha y_1} - e^{i\alpha y_2}) & (e^{i\alpha y_1} + e^{i\alpha y_2}) \end{pmatrix}, \quad (A.13)$$

with

$$TT^+ = 11. \quad (A.14)$$

Appendix B. Propagator Expressions

The propagator expressions depend on the properties of the kinetic matrix K. Considering that $\mathbb{K}$ is Hermitean it can be diagonalized by a

$$\tilde{\varphi} = S\varphi, \quad (B.1)$$
$$\tilde{K} = S^t K S. \quad (B.2)$$

Substituting the above expresssions in (I.5), we get

$$\mathcal{L} = \frac{1}{2}\partial_\mu \tilde{\varphi}^t \tilde{K} \partial^\mu \tilde{\varphi} - \tilde{\varphi}^t \widetilde{M}^2 \tilde{\varphi},$$

where

$$M^2 = SM^2S^t. \quad (B.3)$$

In components,

$$\mathcal{L} = \frac{1}{2}\lambda_i \partial_\mu \tilde{\varphi}_i \partial^\mu \tilde{\varphi}^i - \frac{1}{2}\widetilde{M}_{ij}^2 \varphi^i \varphi^j. \quad (B.4)$$

Thus, there are three situations for the propagator expressions to be analysed. They depend on the cases where the kinetic matrix K is positive-definite, invertible but non-positive-definite, and singular.

I. $K > 0$

Defining

$$\Lambda = \begin{pmatrix} \sqrt{\lambda_1} & & \\ & \ddots & \\ & & \sqrt{\lambda_n} \end{pmatrix}, \quad (B.5)$$

and normalizing the fields

$$\hat{\varphi} = \sqrt{\lambda_i}\tilde{\varphi}_i, \quad (B.6)$$

it gives

$$\mathcal{L} = \frac{1}{2}\partial_\mu \hat{\varphi}^t \partial^\mu \hat{\varphi} - \frac{1}{2}\hat{\varphi}^t \widehat{M}^2 \hat{\varphi},$$

where

$$\widehat{M}^2 = \Lambda^{-1}\widetilde{M}^2\Lambda^{-1}. \quad (B.7)$$

As $\widehat{M}^2$ is symrnetric it can be diagonalized through

$$\Phi_i = \sqrt{\lambda_j} U_{ij} R_{jk} \varphi_k \quad (B.8)$$

which finally yields

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi_i \partial^\mu \Phi^i - \frac{1}{2}m_i^2 \Phi_i^2. \quad (B.9)$$

Then, (B.9) shows that ghosts are avoided due to the fact that all kinetic terms have the same canonical sign. Tachyons will exist for $m_i^2 < 0$.

II.

$$K < 0$$

It is the case where the eigenvalues are positive and negative. This means p positive eigenvalues: $\lambda_1, \ldots, \lambda_p$; and (N-p) negative eigenvalues: $\lambda_{p+1}, \ldots, \lambda_N$.

Normalizing fields as in (B.6),

$$\Lambda = \begin{pmatrix} \sqrt{\lambda_1} & & & & & \\ & \ddots & & & & \\ & & \sqrt{\lambda_p} & & & \\ & & & \sqrt{\lambda_{p+1}} & & \\ & & & & \ddots & \\ & & & & & \sqrt{\lambda_N} \end{pmatrix}, \quad (B.1O)$$

it yields

$$\mathcal{L} = \frac{1}{2}\partial_\mu \hat{\varphi}^t \hat{K} \partial^\mu \hat{\varphi} - \frac{1}{2}\hat{\varphi}^t \widehat{M}^2 \varphi,$$

where

$$\widehat{K} = \begin{pmatrix} 1 & & & & \\ & \ddots & & & \\ & & 1 & & \\ & & & -1 & \\ & & & & \ddots \\ & & & & & -1 \end{pmatrix};$$

$$\widehat{M}^2 = \Lambda^{-1} R M^2 R^t \Lambda^{-1} \quad \text{(B.11)}$$

The difference when positive and negative eigenvalues are included is that the kinetic and mass terms will not be diagonalized simultaneously. Therefore (B.11) is the final expression. A particular case is when K and when $\widehat{M}$ matrices commute.

III. $K = 0$

To study this case we are going to exemplify with just one eigenvalue be zero. Thus det $K = 0$, and K is not invertible. From (B.4) we have that field $\widetilde{\varphi}_N$ associated to the eigenvalue $\lambda_N = 0$ is not a dynamical field because it does not show a kinetic term. However it propagates due to its mass term which mix with other dynamical fields. Then consider

$$\widetilde{K} = \begin{pmatrix} \widetilde{k} & 0 \\ 0 & 0 \end{pmatrix} \quad : \quad \widetilde{k} = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_{N-1} \end{pmatrix} \quad \text{(B.12)}$$

Consequently

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\partial_\mu \widetilde{\varphi}^t \widetilde{k} \partial^\mu \widetilde{\varphi} - \frac{1}{2}\widetilde{\varphi}^t m^2 \widetilde{\varphi} \\ &- \sum_{i=1}^{n-1} \widetilde{M}^2_{Ni} \widetilde{\varphi}_N \widetilde{\varphi}_i - \frac{1}{2}\widetilde{M}^2_{NN} \widetilde{\varphi}_N \widetilde{\varphi}_N, \end{aligned}$$

where

$$\widetilde{M}^2 = \begin{pmatrix} \widetilde{m}^2 & \widetilde{\mu}^2 \\ \widetilde{\mu}^{2t} & \widetilde{M}^2_{NN} \end{pmatrix} \quad ; \quad \widetilde{\mu}^2 = \begin{pmatrix} \widetilde{M}^2_{iN} \\ \vdots \\ \widetilde{M}_{(N-1)N} \end{pmatrix} \quad \text{(B.13)}$$

Calculating the propagators

$$\langle T \widetilde{\varphi}_i \widetilde{\varphi}_j \rangle = \delta, \quad (B.14)$$

$$\langle T \widetilde{\varphi}_i \widetilde{\varphi}_j \rangle = -\frac{1}{\widetilde{M}^2_{NN}} \widetilde{\mu}^{2t} \delta, \quad (B.15)$$

$$\langle T \widetilde{\varphi}_i \widetilde{\varphi}_j \rangle = -\frac{1}{\widetilde{M}^2_{NN}} (\Box \widetilde{k} + \widetilde{\widetilde{m}}^2) \delta, \quad (B.16)$$

where the pole structure is given by

$$\delta = \left[\Box + k^{-1} \left(\widetilde{m}^2 - \frac{1}{\widetilde{M}^2_{NN}} \widetilde{\mu}^2 \widetilde{\mu}^{2t} \right) \right]^{-1} \widetilde{k}^{-1}. \quad (B.17)$$

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