The Quantum Mechanics of Faraday's Law

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The quantum approach to the electric and magnetic fluxes is reconsidered[1]. These quantities are treated as operators, and we discuss how this approach takes care of the magnetic flux quantization and of the magnetoresistance phenomena. We also discuss the derivation of the electromagnetic flux quantization, and make it compatible with Gauss' law. We also claim that this quantization rule can be found from an elementary analysis of an electric oscillator: the L-C circuit.

I. Introduction

The research in magnetic flux quantization (M.F.Q) appeared in the 50's, with a comment of London about the properties of the wave function phase[2]. It led to the experimental discovery of the M.F.Q [3,4]. The usual way to understand it makes use of the fact that the wave function phase is given by an integration of the vector potential along a path with arbitrary initial point, and end point exactly where we have the argument of the wave function[5]. In a ring this approach gives the magnetic flux quantization

$$\phi_B = \frac{2\pi}{e} n \qquad n = 0, \pm 1, \pm 2 .... \tag{1}$$

There are many phenomena where the M.F.Q acts as a head-stone. The most inquisitive one is the Aharonov-Bohm effect[6], where two-slit interference patterns of electrons show the action of the electromagnetic field in regions where it is apparently absent. The clear interpretation of this phenomenon was controversial, although it was evident that it furnishes us, at least, two choices; to accept the "reality" of the vector potential[7] or to be challenged by the non local character of the electromagnetic interaction[6,7].

Another class of phenomena built upon the M.F.Q is the magnetic resistivity oscillations, where we have observation of oscillations in the resistance of a multiply connected conductor, with period determined by the magnetic quantum flux through it. There have been an increasing interest in magnetoresistance since the observation of oscillations with period of just one quantum flux, in one dimensional rings where the ratio of the ring diameter to width is large[9,10]. This discovery, for normal conductors, revives the problem of interpretation of M.F.Q[11].

Another phenomenum where the M.F.Q also plays an important role is the quantum Hall effect. In this case note that the Laughlin argument claims just for the quantization of rnagnetic flux[12].

By what have been said, there are no doubt that the study of M.F.Q has been important by at least two reasons. At first, its understanding can shed light on fundamental aspects of the electromagnetic field quantization, like the function of the vector potential, or its non local nature. Also, it can be helpful in the understanding and designing of experiments and devices, like the ones related to magnetoresistance.

We will begin this paper by facing a fundamental question: what can be the meaning of equation (1)? Since the M.F.Q acts as a rule that assign the possible numbers of quantum flux in a supercondutor ring, it resembles an eigenvalue problem in quantum mechanics, in which the spectrum of an operator is given as a multiple of a fundamental state. So we are led to guess: there could be a magnetic flux operator that acting in a Hilbert space, with the appropriated boundary conditions, affords equation (1) as its eigenvalues? This approach was defended in recent works,[1,13,14] which adopt the point of view that the electromagnetic flux quantization is a natural consequence of the principles of quantum electrodynamics. It was suggested that the magnetic flux may be considered as an operator $\hat{\phi}_B$, whose canonical pair is the electric flux $\hat{\phi}_E$, and that both are linked in a ring by the commutation rule

$$[\hat{\phi}_E, \hat{\phi}_B] = i. \tag{2}$$

Using this equation and applying a boundary condition on ϕ_E that reflects the charge quantization one arrives at Eq. (1)[1].

We divide this paper into two sections. In the first one we show how to derive equation (2) from the quantization of a classical electric oscillator. In doing so, we try to avoid the immediate question concerning the interpretation of this relation in terms of quantum electrodynamics, which is left to the second section. We also develop, in the first section, aspects of the phenomenology implied by (2), in such a way to obtain a scope of the applicability of the electromagnetic flux quantization (E.F.Q). In the second section we derive the E.F.Q from quantum electrodynamics.

II. Formulation of the Problem

A - The L-C circuit

Consider a closed electrical device which can be analyzed in terms of capacitive and inductive elements, such as the classical L-C circuit. When we separate the capacitive C and the inductive L elements of the device we can say that the energy stored in C is given by

$$U_c = \frac{Q^2}{2C}, \tag{3}$$

where Q is the charge in the capacitor. But as we know, $Q = \phi_E \epsilon$, where ϕ_E is the electric flux in the capacitor, and ϵ is the dielectric constant characterizing the capacitor medium. Therefore,

$$U_c = \frac{\epsilon^2}{2C}{\phi_E}^2. \tag{4}$$

In the same way, the energy stored in L is given by

$$U_L = \frac{L.i^2}{2}. \tag{5}$$

However, from $\phi_B = L.i$, where i is the current in the circuit, we have

$$U_L = \frac{1}{2L}{\phi_B}^2. \tag{6}$$

Adding (4) to (6), we get the total energy stored in the device

$$H = \frac{\epsilon^2}{2C}{\phi_E}^2 + \frac{1}{2L}{\phi_B}^2. \tag{7}$$

Using the energy conservation we are led to

$$\frac{\epsilon^2}{C}\phi_E\dot{\phi}_E + \frac{1}{L}\phi_B\dot{\phi}_B = 0, \tag{8}$$

which can be solved by

$$\dot{\phi}_E = \frac{\partial H}{\partial \phi_B} \qquad and \qquad \dot{\phi}_B = -\frac{\partial H}{\partial \phi_E}. \tag{9}$$

This implies that the system described by (7) is Hamiltonian, and the variables ϕ_E and ϕ_B form a canonical pair. Since they have, in this system, a role analogous to the (p, q) variables in classical mechanics. Therefore, the quantization of this system requires

$$[\hat{\phi}_E, \hat{\phi}_B] = i, \tag{10}$$

where ϕ_E and ϕ_B are respectively the electric and magnetic flux operators. So, in fact, the quantization of the L-C circuit is consistent with the E.F.Q.

B - The E.F.Q in a superconductor ring[15]

We intend to use and apply heie the E.F.Q presented above before showing how it develops from quantum electrodynarnics. We think that its "deduction" from the quantization of a L-C circuit is convincing enough to carry out an immediate study of its applications.

From equation (10) we will build up an approach of the E.F.Q in which the states will be described in the electric flux representation. That means that the wave function will be given by

$$\Psi \equiv |\phi_E\rangle \tag{11}$$

and, in order to satisfy equation (10), we make

$$\hat{\phi}_B = -i\frac{\partial}{\partial \phi_E} \tag{12}$$

In this formulation, makes sense the question about the eigenstates and eigenvalues of the magnetic flux operator that is; what is the solution of the equation

$$\hat{\phi}_B|\phi_E\rangle = \phi_B|\phi_E\rangle \quad ? \tag{13}$$

Although this equation is similar in form to the equation that gives the momentum eigenstates in quantum mechanics, in at least one aspect the two equations are not equal. The electric flux ϕ_E is not arbitrary, like the position x of a particle in a free space. Given a surface S, the electric flux through it is not arbitrary; in a static situation it must reflect the charge quantization. To solve equation (13), under this condition, we must improve our approach with some considerations about the circuit in which the magnetic flux was measured.

Take a superconductor ring that it is described by the limit of the equation (7) when $\epsilon \to 0$. It means that there is not a capacitor in the circuit, which is solely inductive. Suppose also that there is just one free particle in the device[16]. In this situation we can say, using Gauss law, that the electric flux through one transversal section of the apparatus is in the interval

$$-\frac{e}{2} \leq \phi_E \leq \frac{e}{2} \tag{14}$$

With this condition on ϕ_E, and writing the solution of equation (13) as

$$|\phi_E\rangle = A.e^{i\phi_B\phi_E}, \tag{15}$$

we can assert that the situations in which $\phi_E = -\frac{e}{2}$ and $\phi_E = \frac{e}{2}$ must be indistinguishable. They represent infinitesimal displacements of the free particle at the right and at the left of the surface S. Therefore we have $\langle\frac{e}{2}|\frac{e}{2}\rangle = \langle-\frac{e}{2}|-\frac{e}{2}\rangle$, which yields:

$$|-\frac{e}{2}\rangle = |\frac{e}{2}\rangle e^{i\gamma}, \tag{16}$$

where y is an arbitrary phase. So (16) applied to (15) furnishes

$$e^{i\phi_B e} = 1, \tag{17}$$

where we made y = 0. The solutions of (17) are

$$\phi_B = \frac{2\pi}{e}.p \quad where \qquad p = 0, \pm1, \pm2, ... \tag{18}$$

So, the E.F.Q is able to reproduce the magnetic flux quantization. But in this formulation the magnetic flux $\hat{\phi}_B$ appears as an operator and the known result, equation (1), gives its possible eigenvalues. Furthermore we have now its canonical pair, the electric flux $\hat{\phi}_E$.

A fundamental aspect of this formulation is that we have a wave function characterizing the electromagnetic flux state, which is indexed by an integer number p. That is, putting (18) into (15) we get

$$|\phi_{E,p}\rangle \equiv |p\rangle = \frac{1}{\sqrt{e}} e^{i\frac{2\pi}{e}.p.\phi_E} \tag{19}$$

where the constant A was determined using the normalization condition $\langle p|p'\rangle = A^* A \int_{\frac{-e}{2}}^{\frac{e}{2}} d\phi_E e^{i\frac{2\pi}{2}(p-p')\phi_E} = \delta_{p,p'}$. So, the electromagnetic Aux state is given once we have determined the number of the magnetic quantum fluxes crossing the ring.

Much more can be learned about this system using this wave function. We can be interested, for example, in the average values of the electric flux in the ring. When the system is in an eigenstate of the magnetic flux, it is easy to see that

$$\langle\hat{\phi}_E\rangle = \langle p|\hat{\phi}_E|p\rangle = 0. \tag{20}$$

This is not a surprising result if we remember the superconductor nature of the ring. The current in the ring must be in a steady state. Therefore there can not be an electric field in the ring. However if one can form a mixed state (below we will discuss how it can be done) given by

$$|\alpha\rangle = a|p\rangle + b|p+1\rangle, \tag{21}$$

subjected to the normalization condition

$$a^*a + b^*b = 1, \tag{22}$$

we have

$$\langle\alpha|\hat{\phi}_E|\alpha\rangle = a.b^*\langle p+1|\hat{\phi}_E|p\rangle + a^*.b\langle p|\hat{\phi}_E|p+1\rangle.$$

But

$$\langle p+1|\hat{\phi}_E|p\rangle =$$
$$\frac{1}{e}\int_{-\frac{e}{2}}^{\frac{e}{2}} d\phi_E \phi_E e^{-i\frac{2\pi}{e}\phi_E} = -i\frac{e}{2\pi},$$

and therefore

$$\langle\alpha|\hat{\phi}_E|\alpha\rangle = \frac{e}{\pi} Im\{a.b^*\}. \tag{23}$$

It follows from equation (23) that the non nullity of the electric flux requires the mixture of, at least, two magnetic states. The maximum of $\langle\hat{\phi}_E\rangle$ happens when a = 6. Using (22) we have $\langle\hat{\phi}_E\rangle = \frac{e}{2\pi}$ at equal two level mixture.

In general, if we construct a state given by

$$|\psi\rangle = \sum_p a_p|p\rangle, \tag{24}$$

it is easy to show that

$$\begin{aligned}\langle\hat{\phi}_B\rangle &= \langle\psi|\hat{\phi}_B|\psi\rangle = \sum_p p|a_p|^2 \\ \langle\hat{\phi}_E\rangle &= \langle\psi|\hat{\phi}_E|\psi\rangle = \sum_{p>p'} Im\{a_p a^*_{p'}\}\frac{e}{\pi|\Delta p|}(-1)^{\Delta p}.\end{aligned} \tag{25}$$

with A = p − p'

These expressions give us the relations among the physical states of the operators $\hat{\phi}_B$ and $\hat{\phi}_E$ but we do not know how to describe the dynarnics of these operators. So given a physical situation with electromagnetic fluxes as observables, how are the quantities connected among thernselves and with the external conditions imposed to the ring? To answer this question we look for a Hamiltonian describing the interaction between the electric and magnetic fluxes. If we can write it, we will understand and calculate how the states can be mixed, like the ones described by equation (24). It was postulated[1] that the Hamiltonian

$$\hat{H} = \frac{\epsilon^2}{2C}\hat{\phi}_E^2 - f_e.\hat{\phi}_E + \frac{1}{2L}\hat{\phi}_B^2 - \frac{b.S_1}{L}.\hat{\phi}_B \tag{26}$$

describes the interaction of an external magnetic b and electrical f_e fields with the system described by equation (7), and S_1 is the area of the device exposed to the external magnetic field b.

Let us calculate the magnetic flux tied to the ring as function of the magnetic field imposed to the system. Take again the limit $\epsilon \to 0$ as above, and consider the Hamiltonian of a superconductor ring in a magnetic field as

$$\hat{H}' = \frac{1}{2L}\hat{\phi}_B^2 - \frac{b.S_1}{L}.\hat{\phi}_B \tag{27}$$

The first point to be noted is that this Harniltonian commutes with the $\hat{\phi}_B$ operator. So, their eigenstates are also eigenstates of $\hat{\phi}_B$. Using (13) and (18) we see that the energy of (27) is

$$E = \frac{1}{2L}(\frac{2\pi}{e}p)^2 - \frac{b.S_1}{L}\frac{2\pi}{e}.p. \tag{28}$$

and the ground state of this system is characterized by the integer p given by

$$p=\{\frac{e}{2\pi}.b.S_1\}, \qquad (29)$$

where $\{x\}$ stands for the nearest integer contained in x.

From ecuation (29) we see that as we change b we do not necesarily change p. As p is an integer number, it is not a continuous function of b. It is easy to see that it changes forming electromagnetic flux plateaus. One can note that the positions of the plateaus are exactly in the positions that they were experimentally observed. Like in the rxperiment, the plateau transitions happen just when $\frac{e}{2\pi}.b.S_1$ assumes a semi-integer value. But in the expeiiment, as was noted by Doll et al.[4] , there are no discontinuity between a plateau and another one. It was observed a very steeped, but continuous, curve joining a p ateau to the next one. Most exactaly it was observed points in the region between the plateaus. Taking the words of Deaver et al.[3], "Near the transition ... the fluctuations in the data are greater, and in addition points lie between the steps."

To understant how that can happens we remember that the limit $\epsilon\rightarrow 0$ taken above does not mean that the term $f_e.\hat{\phi}_E$ of equation (26) must be zero. If we assume that this term is present in the Harniltonian, with a very small f_e, we see that

$$\hat{H}''=\frac{1}{2L}\hat{\phi_B}^2-\frac{b.S_1}{L}.\hat{\phi}_B-f_e.\hat{\phi}_E, \qquad (30)$$

does not commute with the operator $\hat{\phi}_B$. The states $|p\rangle$, in equation (19), are no more eigenstates of (30). So, the high definite plateaus of figure 1 are not exact; they are smoothed by the term $f_e\hat{\phi}_E$ of (30). To see that let

$$|\alpha\rangle=\sum_p\alpha_p|p\rangle, \qquad (31)$$

and determine α_p in such a way that $|\alpha\rangle$ is an eigenstate of H''

$$\hat{H}''|\alpha\rangle=\mathcal{E}|\alpha\rangle. \qquad (32)$$

Using (25) we obtain an equation for α_p, given by

$$\sum_p\{H'_p.\delta_{p,p'}-i.f_e.\frac{e}{2.\pi.\Delta p}$$
$$.(-1)^{\Delta p}.(1-\delta_{p,p'})\}\alpha_p=\mathcal{E}.\alpha_{p'} \qquad (33)$$

with $H'_p=\langle p|H'|p\rangle$.

To solve this equation we will truncate the series (31), and use only the terrns that are close to the ground state of (28), because we admit $f_e\hat{\phi}_E$ acting as a perturbative term. Furthermore, we will take just two'terms to diagonalize (33); the ground state $|p\rangle$ of (28), and the nearest state $|p+j\rangle$, where $j=1$ or $j=-1$ according the nearest state is $|p-1\rangle$ or $|p+1\rangle$. Calling

$$R=f_e.\langle p|\hat{\phi}_E|p+j\rangle=-i.\frac{f_e.e}{2.\pi}.j, \qquad (34)$$

we arrive at

$$\langle\hat{\phi}_B\rangle=p+j.\frac{|R|^2}{|R|^2+(H_{p+j}-\mathcal{E})^2} \qquad (35a)$$

$$\langle\hat{\phi}_E\rangle=-\frac{|R|.(H_{p+j}-\mathcal{E})}{(H_{p+j}-\mathcal{E})^2+|R|^2}, \qquad (35b)$$

where E is given by the ground state solution of the determinantal equation

$$\begin{vmatrix} H'_p-\mathcal{E} & R^* \\ R & H'_{p+j}-\mathcal{E}\end{vmatrix}=0. \qquad (36)$$

The mixture of states is obvious from equations (35a-b), but it can be much more appraised from Fig. 1, where we see that in the plateaus transition region we have not a definite flux in the ring. It is a mixture of flux eigenstates. When that happens, we see by equation (25), that we have a non zero electric flux $\langle\hat{\phi}_E\rangle$ in the superconductor. Odd as it is, this result can be easily understood by a semi-classical argument. When we have a magnetic fiux plateau we have a constant current in the ring, therefore there are no difference of electric potential between any two of its points. When the magnetic flux raises from a plateau to another, it must occur an increasing in the current moving around the ring. During this transition the electric flux must be different of zero.

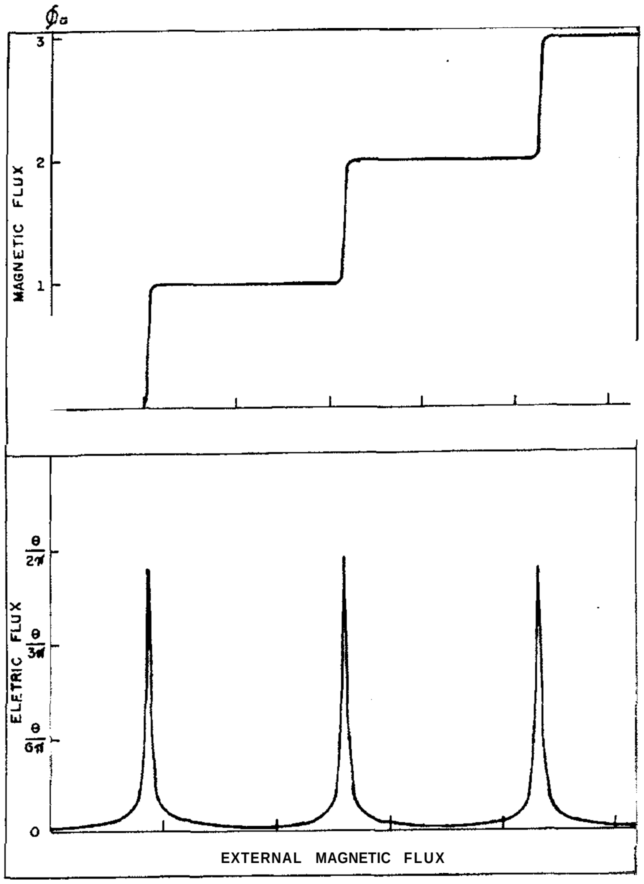

Figure 1: Magnetic flux and Electric flux in the ring described by equation (30), as we change the external field, we have used $f_e = 0.01$.

C - Magnetoresistivity

Of course, from the first experiments in which the magnetic flux quantization was measured we had no data connected to the electric flux; the purpose of these experiments was just to show the magnetic flux quantization. We show in the following $\langle\hat{\phi}_E\rangle$ can be measured in a physical experiment. Indeed we will show that it is related to the magnetoresistivity.[9,10,11]

Consider the following circuit. Passing by the point p_1 an electric current is splited into two identical paths C_1 and C_2, with lenght L, which are joined again in the point p_2. These two paths lace a hole, where we have a magnetic flux $\phi_B = \int \vec{B}.d\vec{S}$, with $\vec{B}$ being an external magnetic field, and the integral cover the hole's surface.

Our aim is to show how the quantization of the electromagnetic flux can give us information on the electric properties of the circuit between the points p_1 and p_2.

If there was no magnetic flux ϕ_B through the hole, the electric identity of C_1 and C_2 would warranty the equality of i_1 and i_2. The magnetic flux causes the difference between them, because

$$i_1 = \frac{1}{L}\frac{e}{m}\int \vec{p}\cdot d\vec{l}_1 \quad and \quad i_2 = \frac{1}{L}\frac{e}{m}\int \vec{p}\cdot d\vec{l}_2, \tag{37}$$

and therefore

$$i_1 - i_2 = \frac{1}{L}\frac{e}{m}\oint \vec{p}\cdot d\vec{l}. \tag{38}$$

The canonical momentum $\vec{P}$ of a particle with charge e in an electromagnetic field is given by

$$\vec{P} = \vec{p} + e\vec{A}, \tag{39}$$

when $\vec{p}$ is the kinetic momentum and $\vec{A}$ is the vector potential.

A straightforward calculation shows that

$$i_1 - i_2 = \frac{2e^2}{Lm}\phi_B \equiv \varphi_B. \tag{40}$$

If we now use the fact that the electric current arriving at p_2 is the sum of the currents travelling along C_1 and C_2 paths

$$i = i_1 + i_2, \tag{41}$$

it follows that

$$i_1 = \frac{1}{2}(i + \varphi_B) \qquad i_2 = \frac{1}{2}(i - \varphi_B), \tag{42}$$

and hence the net electrical resistance at each branch are different. By imposition, the potential gap between the points p_1 and p_2 independs on the path. This defines the electric field between p_1 and p_2 as:

$$V = e\int_{p_1}^{p_2} \vec{E}.d\vec{l}. \tag{43}$$

We can decompose this electric field in two terrns. The first comes from the variation of the magnetic field in the hole, which induces an electric field, and the other one comes from the electric field $\vec{\mathcal{E}}$ imposed to the system by the battery. So,

$$\vec{E} = \vec{\mathcal{E}} + \frac{\phi_E}{S}\vec{n}, \tag{44}$$

where $\vec{n}$ is an unitary vector in the direction of the electric field in the circuit, and S is the transversal area of the ring's wire. Therefore, along C_1 we have

$$V = e\int_{C_1} \vec{E}.d\vec{l}_1 = V_1 - \frac{\phi_E}{S}L \equiv V_1 - \varphi_E. \tag{45}$$

In the same way we write

$$V = V_2 + \varphi_E. \tag{46}$$

Putting (42), (45) and (46) together we arrive at

$$R_1 = \frac{V + \varphi_E}{\frac{1}{2}(i + \varphi_B)} \qquad R_2 = \frac{V - \varphi_E}{\frac{1}{2}(i - \varphi_B)}, \tag{47}$$

which are the electric resistance along C_1 and C_2. The total resistance satisfies

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}, \quad (48)$$

and

$$R=r\{\frac{1-(\frac{\varphi_E}{V})^2}{1-(\frac{\varphi_E}{V})(\frac{\varphi_B}{i})}\}, \quad (49)$$

where r $=\frac{V}{i}$ is the electric resistance of the circuit when we have no magnetic flux in the hole.

Some observations must be made: a) when we have a magnetic plateau we know that $\langle\hat{\phi}_E\rangle\approx 0$ so R = r; the resistance in the circuit is the normal resistance. b) in the points where are the plateau transitions we have no more $\langle\hat{\phi}_B\rangle\approx 0$, and in the first order in $\hat{\phi}_E$ we see that

$$R=r\{1+(\frac{\varphi_E}{V})(\frac{\varphi_B}{i})\}, \quad (50)$$

which gives a peak in R once we pass through a plateau transition point. These peaks are measured, for example, in reference (8).

III. The Electromagnetic Flux Quantization

We will show here how equation (2) can be deduced from the principles of quantum eletrodynamics. Let us take the usual electromagnetic Lagrangian

$$\mathcal{L}=-\frac{1}{4}F^{u\nu}F_{u\nu} \quad (51)$$

with $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$, from which we immediately get

$$\pi_o=\frac{\partial\mathcal{L}}{\partial A_o}=0 \quad and \quad \pi_k=\frac{\partial\mathcal{L}}{\partial A_k}=F_{ok}=E_k \quad (52)$$

The quantization of the temporal term will not concern us here. The spacial quantization rule is given by

$$[\hat{E}_i(\vec{x}\ t),\hat{A}_j(\vec{y},t)]=i\delta_{i,j}\delta^3(\vec{x}-\vec{y}). \quad (53)$$

We will take the ring shown in figure 2, and define the electric flux $\hat{\phi}_E$ and magnetic flux $\hat{\phi}_B$ operators by

$$\hat{\phi}_E=\int\vec{\hat{E}}.d\vec{S}_2, \quad (54a)$$

and

$$\hat{\phi}_B=\int\vec{\hat{B}}.d\vec{S}_1=\oint\vec{\hat{A}}.d\vec{l}_1, \quad (54b)$$

where l_1, S_1 and S_2 are respectively the path and the surfaces shown in figure 2.

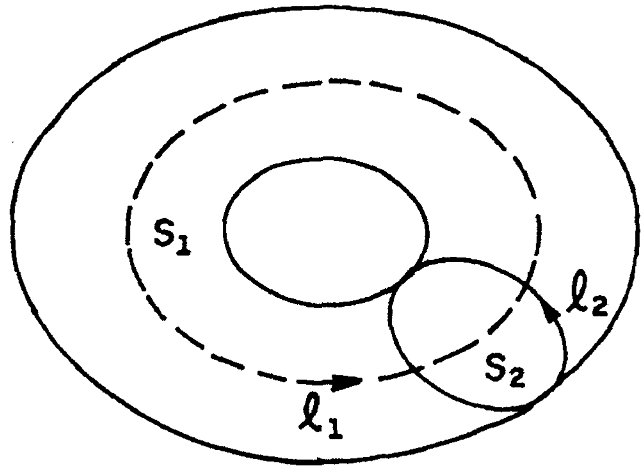

Figure 2: Ring where can be measured the eletromagnetic flux quantization.

Using equation (53) and the geometry of figure 1, we obtain

$$[\hat{\phi}_E,\hat{\phi}_B]= \\ =\int\oint[E_i(\vec{x},t),A_j(\vec{y},t)]dS_2^i(\vec{x})dl_1^j(\vec{y})= \\ =\int\oint i\delta_{i,j}\delta^3(\vec{x}-\vec{y})dS_2^i(\vec{x})dl_1^j(\vec{y})=i, \quad (55)$$

where we adopted the repeated index sum rule convention. A comment must be made about this deduction.

When we try to quantize any system using equation (53) we must be aware that it is incompatible with the vacuum condition

$$\vec{\nabla}.\vec{E}=0. \quad (56)$$

Usually the incompatibility between (53) and (56) is not a serious problem. In the electromagnetic quantization we impose commutation relations upon a non physical field A_μ. Their derivatives are physical measurable quantities, so the vector potential posseses more degrees of freedom than necessary and we have a gauge theory. Equation (56) is just a condition - restriction - imposed upon the non physical with excess of degrees of freedom, states constructed with the rule (53). But the same cannot be said about $\hat{\phi}_E$ and $\hat{\phi}_B$. One can easily see that they are gauge independent. Their states have no additional degrees of freedom to be eliminated with a condition like (56). So, our main purpose now is to show whether the equation (55) makes no sense because of incompatibility with Gauss law or whether, by a subtle way that is still not clear, it incorporates the condition (56).

The commutation relation (53) is not compatible with the Gauss' law in a region free of charges. To solve this problem we change the commutation relation

(53) in such a way that we have cornpatibility with (56). We take as usual[17]

$$[\hat{E}_i(\vec{x},t),\hat{A}_j(\vec{y},t)] = i\delta^{tr}_{i,j}(\vec{x}-\vec{y}), \qquad (57)$$

where $\sum_{i=1}^{3}\frac{\partial}{\partial x_i}\delta^{tr}_{i,j}(\vec{x}-\vec{y}) = 0$, which makes the commutation rule (57) compatible to the Gauss' law. Note that when we substitute (53) by (57) we are making the change

$$i\delta_{i,j}\delta^3(\vec{x}-\vec{y}) \rightarrow i\delta^{tr}_{i,j}(\vec{x}-\vec{y}) = \qquad (58)$$
$$= i\int\frac{d^3k}{(2\pi)^3}\left(\delta_{i,j}-\frac{k_ik_j}{k^2}\right)e^{\vec{k}.(\vec{x}-\vec{y})}$$

which makes Eq. (57) compatible with Eq. (56)

Using the expression (57) in the commutator $[\hat{\phi}_E,\hat{\phi}_B]$ we arrive at

$$[\hat{\phi}_E,\hat{\phi}_B] = i\int\oint\delta^{tr}_{i,j}(\vec{x}-\vec{y})dS^i_2(\vec{x})dl^j_1(\vec{y}), \quad (59)$$

where the integrals are over the surface S_2 and the curve l_1 shown in figure 2.

To solve this integral we observe that

$$\delta^{tr}_{i,j}(\vec{x}-\vec{y}) = \delta_{i,j}\delta^3(\vec{x}-\vec{y}) - \int\frac{d^3k}{(2\pi)^3k^2}k_ik_je^{i\vec{k}.(\vec{x}-\vec{y})}$$
$$\equiv \delta_{i,j}\delta^3(\vec{x}-\vec{y}) - \tilde{\delta}_{i,j}(\vec{x}-\vec{y}). \qquad (60)$$

Thus, if one could show that

$$\oint\int\tilde{\delta}_{i,j}(\vec{x}-\vec{y})dS^i_2(x)dl^j_1(y) = 0, \qquad (61)$$

then equations (59) and (55) are equivalent. In this way note that

$$\oint\int\tilde{\delta}_{i,j}(\vec{x}-\vec{y})dS^i_2(x)dl^j_1(y)$$
$$= \oint\int\int\frac{d^3k}{(2\pi)^3k^2}\partial_{x_i}\partial_{y_j}\{e^{i\vec{k}.(\vec{x}-\vec{y})}\}dS^i_2(x)dl^j_1(y) =$$
$$= \oint\partial_{y_j}\{\int\int\frac{d^3k}{(2\pi)^3k^2}\partial_{x_j}\{e^{\vec{k}.(\vec{x}-\vec{y})}\}dS^i_2(x)\}dl^j_1(y) = 0 \qquad (62)$$

because

$\oint\{\partial_{y_j}f(y)\}dl^j(y) = \int\{\epsilon_{lkj}\partial_{y_k}\partial_{y_j}f(y)\}dS^l(y) = 0$, for any $f(y)$.

Therefore

$$[\hat{\phi}_E,\hat{\phi}_B] = i, \qquad (63)$$

which is exactly the equation that we have set up in (10), but now compatible with the Gauss' law, eq. (56) and furthermore, it comes straightly from the principles of quantum electrodynamics. We can now understand the contrast between the change in the local commutation rule, after imposition of Gauss's law, and the non changing in the flux's commutation relation, eq. (63). It is the flux non local character that turn into zero eq. (62). Note that this result comes from the integration $d\vec{l}_1(y)$ around the ring. So we can say that the compatibility of (53) with Gauss' law stresses the non locality of the electromagnetic flux quantization.

IV. Conclusion

We have studied the electromagnetic flux quantization, and shown that it comes directly from quantum electrodynamics, but with an elementary reasoning we can understand it also from the grounds of quantum mechanics. The magnetic and electric fluxes operators are our main objects of interest. Using them we have constructed a quantum version of Faraday's law. We have shown how it can afford the magnetic quantization and magnetoresistance. We emphasize the non local nature of these phenomena and remember that this is the character of the presented formulation. So, we claim that the electromagnetic flux quantization is suitable to study non local aspects of electrodynamics.

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15. Many ideas of this section have already appeared in ref [1] quoted above. We reintroduce them here in order to clarify our presentation.
16. This hypothesis presents no serious difficulties. In fact it can be show that when there are more than one free electron in the ring a similar result holds. See M. Simões, Doctoral Thesis, Universidade de São Paulo (1989).
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