Diffusion Limited Aggregation with Competitive Particles

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A rule for aggregation growth from lattice gas, which includes competition between particles is proposed. The new aggregate present varying fractal dimension (from DLA value $d_f \sim 1.7$ to $d_f = 1$), depending on the (increasing) number of particles launched simultaneously.

The DLA (diffusion limited aggregation), as proposed by Witten and Sander[1], is a model that simulates the process by which particles combine to form fractal objects presenting multi-branched forms that can be encountered on aerosol, dielectric breakdown, colloidal and polymer science (see, for instance, references [2,3,4]). The rules of this original model are simple: the aggregate starts with a particle, hereafter called seed, that is put at the origin of the lattice. Another partirle is then lauchend from the limit of the lattice, supposed far away from the seed, and walk randomly unti! reaching some site on the neighbourhood of the seed. When this occurs, the particle is incorporated to the aggregate at this site and a new one is launched, randomly walks until reaching some site in the periineter of the aggregate, and is also incorporated, and so forth. After a large number of repetitions of these steps a very interesting multi-branched pattern is formed because the tips of the cluster have a greater probability of growth than the inner part. Thus the particles are captured mainly on the tips, forming the branches. The DLA is a fractal that, for the lattice spatial dimension $d = 2$, is characterized by the fractal dimension $d_f \approx 1.71$. Meakin and Havlin[5] have suggested that the DLA is not a simple fractal but it has a multifractal structure. In ref.[8], a multifractal analysis of this aggregate is performed and the momenta D,, up to the order q = 32, present the same value. Accordingly, this aggregate would be a simple fractal. Despite of this discussion we will consider, in this paper, only the global fractal dimension $d_f \sim 1.71$ of DLAs grown by the above defined rules and compare it with our model defined below.

A modiication in DLA rule can be introduced if not only one particle is launched but if an aggregate is grown in a lattice gas of density n_g [6,7,8]. In ref.[7] it is shown an interesting transition from the DLA to the Eden mode [9], when n_g is increased. If we consider the aggregation process inside a lattice gas, several particles are diffusing at the same time. When a particle is aggregated, another one is launched but all the others remain diffusing. In the limit of low gas density, this new aggregate is similar to the DLA.

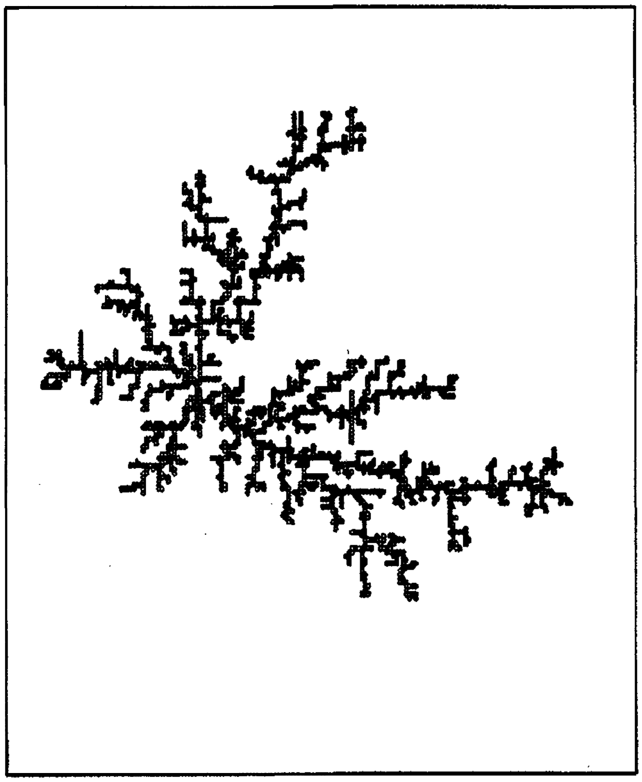

Figure 1: Aggregate grown with $\lambda = 0.1$ for AR = 10. It is evident the crossover between DLA for few particles aggregated and the tendency to grow in a preferential direction.

In this work, we propose a new modification, by introducing competition between the particles in the lattice gas. The difference concerning the model quoted above is the following: when a particle reaches the aggregate, all the others are discarded, and a new set of particles is launched from points, choosen at random, in a circurnference centered on the seed which radius is AR lattice parameters larger than the current aggregate radius. We lept the linear density λ constant during the growth process and thus as soon as the aggregate radius increases, the number of walkers also increases in order to keep constant the density of walkers around the outer circunference quoted above. We used a parallel dynamics, i.e., each particle is allowed to walk one lattice parameter by time (we used square lattice in our simulations). It is also allowed for two particles to occupy the same site at the same time, on the lattice. According to this rule the particle that will be aggregated will be the one that reaches the aggregate with the lowest number of steps. In this sense we can say that "competition" between the particles is introduced. As in the Uhawa and Saito[7] work, a transition in fractal dimension is observed, but now in the opposite direction. Our aggregates change to a linear form when more particles are launched simultaneously, as seen in fig. 1 and fig. 2, whereas in the ref.[7] the aggregate changes to a more compact form.

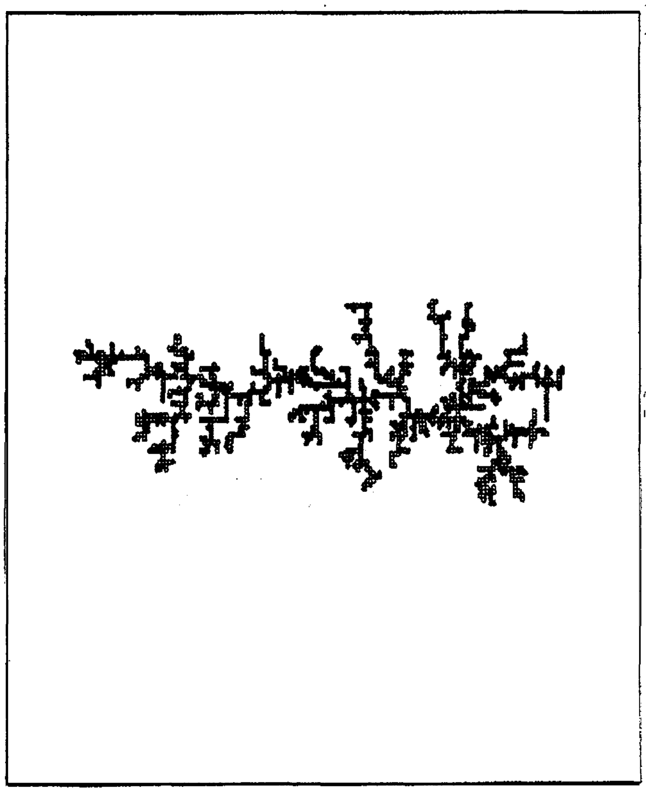

Figure 2: Aggregate grown with $\lambda = 1$ for AR = 10. This aggregate has a fractal dimension lower than the one shown in figure 1.

It is easy to understand this tendency to linear behavior. As soon as a particle is aggregated, a preferential direction of growth appears because, as the particles are launched from the same distance from the seed, those launched from points,closer to the tips can reach the aggregate with a lower number of steps than the farthest ones, reducing the shot noise. The point of maximum probability is more sampled than in traditional DLA rule, therefore fluctuations have been suppressed. The larger the number of particles launched simultaneously, the greater the probability of the particle that would be aggregated have been launched from a region in the circumference that is closer to the preferential direction.

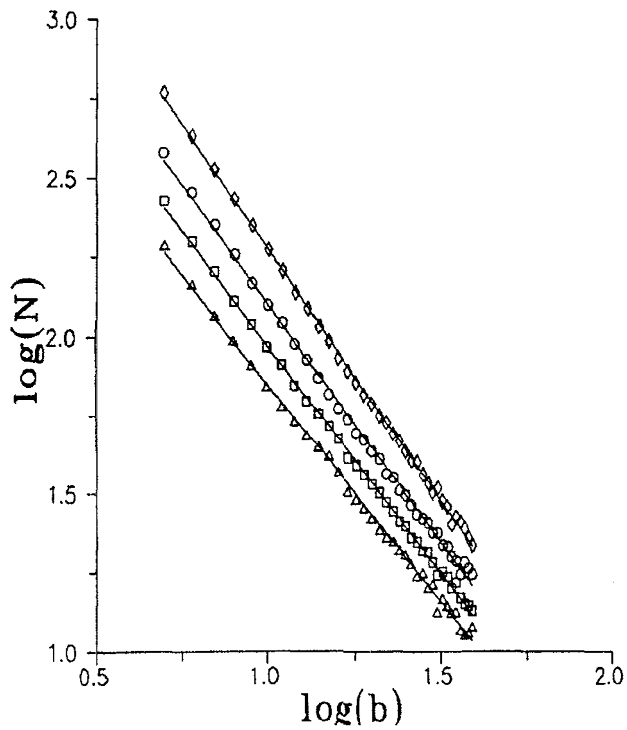

Figure 3: Logarithm of number of boxes of size b containing one particle at least versus logarithm of size b (box counting method) for some values of λ (λ = 0.0625(◇), 0.125 (○), 0.25(□) and $\lambda = 0.5$(△)). Here AR = 10 for all plots.

An example of "diffusion with competition" is the spermatozoa's race in a fecundation process. In this case, only the fasted spermatozoon reaches the ovule and all others are discarded. This is not a growth process but clearly a situation where competition is important. The competition leads to an anisotropy in the aggregates, if compared to the original DLAs. The anisotropy can occur in other processes where preferential directions appear either during the growth process or are defined a *priori*. In this new aggregate the largest probability of growth in the tips with relation to the inner part is still enhanced. We can compare our results with other methods of noise enhancement, like introduction of counters or η-model in DBM (see for instance T. Vicsek in ref.[4] and references therein). In our model no underlying lattice is needed. This is the most important result presented in this paper.

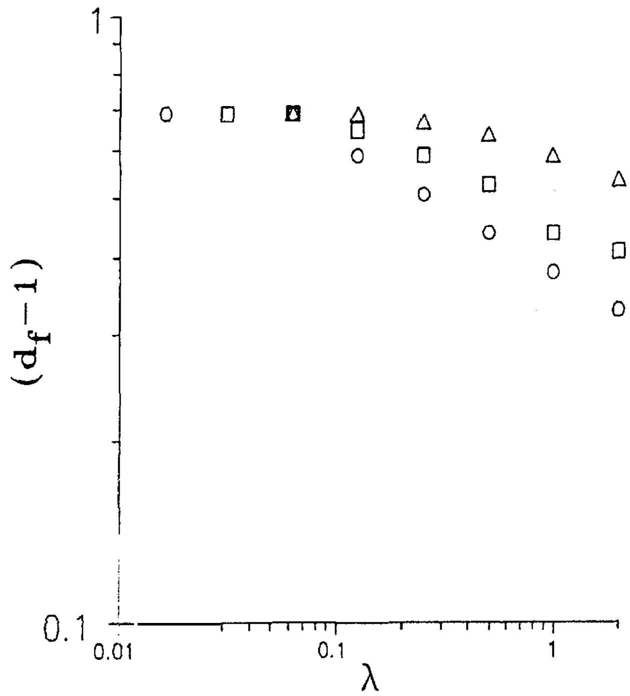

Figure 4: Log-log plots of fractal dimension versus $\lambda(\Delta R = 10(\bigcirc), 20(\square)$ and $80(\triangle))$.

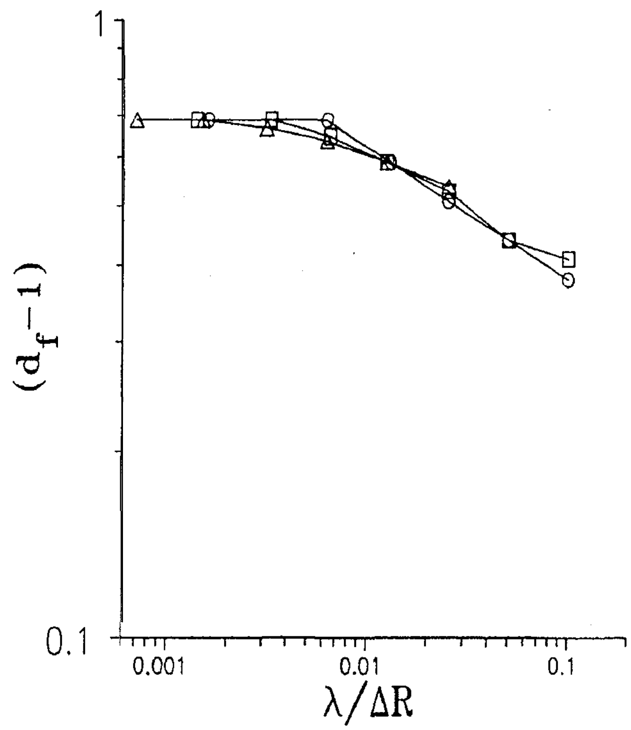

Figure 5: Data colapse of fractal dimension. The quantity to be preserved in order to reproduce the fractal dimension for different distances is $\lambda/\Delta R$.

We have tried a modification in this rule taking into account the change in the aggregate center-of-mass position during the process. In this second rule, the circurnferenre from which the particles are launched is centered on the current center-of-mass position (no more on the seed), which is updated when each new particle is aggregated. This new procedure should minimize the efècts of the preferential growth direction to be closed to the starting ring. Neverthless, for both versions we found the same results. We performed only on-lattice simulations, thus the center-of-mass changes only in discrete lattice parameters. If a new particle is aggregate in the preferential growth direction this will change to tlie opposite, but along the same line. However, this will not cause any change in the aggregate form or dimensions, and only the position of the seed will be close to the center-of-mass.

In order to extract more conclusive results, we grew several aggregates for different numbers of particles lauched simultaneously and performed the fractal analysis, using both *"sand box"* and *"box counting"* method[11]. Similar results were found. Fig. 3 presents the log-log plots showing the fractal dimension for different values of A, using the box counting routine. The value $d_f \approx 1.70$ found in our simulations for low density limit is in good aggreement with the previous one and it serves as a test for our routines.

Fig. 4 shows how the fractal dimension varies with respect to the linear density A. The crossover between the DLA (low density) regime and a region where the fractal dimension decreases becomes evident. In the high density limit we always found $d_f \approx 1$. These curves point out that for large values of AR, the DLA regime will stand for large values of A. This behavior was expected because the particles perform Brownian motion. Hence, after some steps the density around the agreggate will be lower than the initial density in the starting ring. Finally in Fig. 5 we plot the fractal dimension versus the density divided by AR. In this case we found the data collapse and thus we conclude that when the starting radius increases we must increase linearly the density and not the number of particles in order to obtain the same global fractal dimension.

Acknowledgements

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