Influence of the Fifth-Order Nonlinearity on the Pulse Propagation in Optical Fibers

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We analyze tlie effects of tlie fifth-order optical nonlinearity, $n_4 \propto Re\chi^{(5)}$, on pulse propagation in fiber waveguides. It is sliown tliat tlie fifth-order term affects tlie pulse evolution a; appropriate intensities sucli tliat contributions to the effective nonlinearity arising from terms of order liiglier tlian n_4 can be neglected. Tlie phenomena of modulational instability aid solitoii propagation in tlie presence of n_4 are studied witli applications to semiconductor doped glass fibers.

I. Introduction

Pulse propagation in optical fibers lias been intensively studied over tlie past few years, witli important teclinological results as well as new insiglits in basic phenomena of nonlinear propagation of ultrasliort piilscs in waveguides. Tlie n-iajority of tliese results are wcll described by Agrawal[1] and Alfano[2] in reccnt books. Among the well studied nonlinear effects in optical fibers, self-phase modulation (SPM) due to tlie intensity dependent refractive index leads to a spectral broadening of tlie pump pulse. Upon propagation in a regime where group velocity dispcrsion (GVD) is important, tlie interplay between SPM aiid GVD can lead to the generation of optical solitons or a temporal instability (modulation instability) if tlie medium is self-focusing aiid has anomalous (negative) dispersion. In general, tlie third-order nonlinearity (in fact $Re\chi^{(3)}$) and the second order dispcrsion terin are sufficient to explain pulse propagation in optical fibers and can be well described by tlie nonlinear Scliroclingcr equation[1,2]. When a pulse lias its duratiori sliortcr tlian 100fs or wher the pulse central wavelength lies around tlie minimum dispersion wavelength value of tlie fiber, high-order nonlinear and dispersive terms can be included in the propagation equation (generalized nonlinear Schrodinger equation) to account for experimental rcsults.

Another high-order terin tliat lias bcen tlicorctically studied[3-7] is the fifth-order optical nonlinearity, $n_4 \propto Re\chi^{(5)}$. For standard silica bascd fibers, sucli effect is experimentally very difficult to observe because tlie value of $|\chi^{(5)}|$ is negligible in comparison witli $|\chi^{(3)}|$ [6]. However, in the recent years, nem materials witli liiglier optical nonlinearities tlian in SiO_2 have been studied. Tlie ternary compounds of $Cd(S,Se)$ nanocrystsllitcs embedded in glass matrix represent an important class of sucli materials now available as optical fibers[8] as well as waveguidcs structures[9]. Its nonlinear optical propcrties have bccn thoroughly studied since 1983[10,11]. Tlie main characteristics of tlie semicoiiductor doped glasscs (SDG) are a bandgap tliat varies according to tlie $Se(S)$ concentration, spanning tlie visible to near infrared region, a high third-order nonlinear susceptibility ($|\chi^{(3)}| \sim 5 \times 10^{-10} cm^2/W$ near bsndgap) and a response time of tlie nonlinearity, $T_1 \leq 10ps$, which is intensity dependent for above gap excitation. Tlie material dephasing time T_2 lias been measured to be $\sim 20fs$ [12]. High-order susceptibilities in commercially available SDG have also been measured, up to $\chi^{(7)}$ on rcsonance[13]. For instance, tlie modulus of $\chi^{(5)}$ is only four orders of magnitude smaller tlian $\chi^{(3)}$ (on resonance) and for $500cm^{-1}$ off-rcsonance, lve estimate tlie ratio $|\chi^{(5)}/\chi^{(3)}| \sim 10^{-6}$. It is also known that tlie sign of $n_2 \propto Re\chi^{(3)}$ is negative for detunings up to $\sim 3000cm^{-1}$ bclow tlie conduction band[14].

Tlie results presented liere were motivatcd by tlie rescarch on SDG, wliicli lcd to tlie results pointed out above. In particular, we have exploited the off-resonant region (below the gap) wliere $n_2 < 0$ and tlie time response is not clamped by resonant effects. Tliis regime may lead to tlie existence of solitons in tlie normal dispersion regime of optical fibers (considering tliat tlie inclusion of semiconductor crystallites in tlie glass imply little effect on its dispcrsive properties **as** it accounts for only $\sim 1\%$ in volume). Furthermore, bascd on tlie results of ref.[13] wliere for detunings of $\sim 500cm^{-1}$ bclow tlie gap a rclativcly liigli value of $|\chi^{(5)}|$ was measured, we extrapolated those results to larger ($\sim 2000cm^{-1}$) detunings to use approximate values of $|\chi^{(5)}|$ (or actually the ratio $|\chi^{(5)}/\chi^{(3)}|$) to verify tlie effects of $\chi^{(5)}$ on pulse propagation in SDG fibers.

In our calculatioii we take into account tliat for offrcsonance excitation tlie sign of the nonlinearity in the expansion of the optical susceptibilities alternate (see for instance ref.[15]). Wlien the intensity increases it is necessary to take into account more and more terms in tlie expansion of nonlinear polarization. For high enough intensities the expansion is not valid at all. Tlierefore, one needs to work at power levels low enough to avoid tliis problem. Even in tliis regime, there is some difference in the pulse propagation when the term due to n_4 is taken into account, as will be described bclow. On tlie other hand, if the pump frequency is close enough or above the band gap, saturation effects are much more pronounced and lead to novel results. Tliis subject has also been recently treated by Gatz and Herrmann[16] wlio considered the case of saturable noiilinearities. In what follows, we numerically analize (Section II) the effect of n_4 positive on the soliton propagation in tlie normal dispersion regime, where $n_2 < 0$ provides tlie required balance for soliton formation. In Section III, we analyze, analitically and numerically, tlie effect of n_4 to tlie plienomenon of modulation instability, also in tlie normal dispersion regime. In both cases realistic parameters for SDG fibers are used. In Section IV we summarize our main conclusions.

II. Soliton Propagation

Tlie equation which models the pulse propagation in optical fibers, the nonlinear Schrodinger equation (NLSE), has been examined in several occasions[1,2]. Here we follow the notation and conventions of ref.[1] and write tlie propagation equation disregarding fiber loss but including the fifth-order nonlinearity term. Tlie inclusion of losses does not affect tlie qualitative results to be obtained. Tlie equation to be solved has tlie form

$$i\frac{\partial A}{\partial Z} = \frac{1}{2}\beta^2\frac{\partial^2 A}{\partial T^2} - \gamma|A|^2 + \gamma'|A|^4 A, \qquad (1)$$

wliere β_2 is the GVD parameter, y $= 2\pi n_2/\lambda_0 A_{eff}$ and $\gamma' = 2\pi n_4/\lambda_0 A_{eff}^2$ are tlie nonlinearities parameters, λ_0 is tlie central wavelength, A_{eff} is the effective core area and $A(Z,T)$ is the amplitude of the pulse envelope.

An analytical solution of eq.(1) has been given by Gagnon[5], where a large set of solutions are presented. ln this Section, we describe numerical results of eq.(1), solved using the split-step method (see [1] and refs. tlierein). As mentioned before, eq.(1) has been analyzed for SiO_2 fibers but due to the extremely small value of $n_4 \sim 10^{-30}$ cm^4/W^2 its effect is negligible[4,6]. For SDG fibers we estimated, from the results of ref.[13], $n_4 \sim 10^{-20} cm^4/W^2$ for detuning Aw $\sim 2000 cm^{-1}$. Eq. (1) can be rewritten in the normalized form

$$i\frac{\partial U}{\partial \xi} = \frac{sgn(\beta_2)}{2}\frac{\partial^2 U}{\partial \tau^2} - N^2|U|^2U + r|U|^4U, \quad (2)$$

where U is the normalized field envelope, $\xi = Z/L_D$, N^2 $= L_D/L_{n_2}$, $\tau = \frac{t-z/v_g}{T_0}$ and $r = L_D/L_{n_4}$, with $L_D = T_0^2/|\beta_2|$, L„ $= \lambda_0 A_{eff}/2\pi n_2 P_0$, $L_{n_4} = \lambda_0 A_{eff}^2/2\pi n_4 P_0^2$. T_0 is the half width at 1/e intensity point of the envelope ($T_0 = T_{FWHM}/1.763$ for tlie hyperbolic secant envelope used) and P_0 is tlie peak power. Tlie fiber parameters used were $n_2 = -10^{-12} cm^2/W$ aiid $n_4 = 10^{-20} cm^4/W^2$. In eq.(2), the peak power effects can be more conveniently analyzed tlirough tlie ratio L_{n_2}/L_{n_4}. For $L_{n_2}/L_{n_4} \leq 10^{-4}$, the quartic tcrm is negligible and tliercfore one lias tlie so-called soliton regime, where tlie negativc nonliiiearity is balaiiced by positive GVD. Figure 1 shows tlie soliton propagation for one soliton period $Z_0\left(\frac{\pi}{2}L_D\right)$ corresponding to N = 2 (N is tlie soliton order) for $L_{n_2}/L_{n_4} = 10^{-4}$. As an example of this situation, we consider a fiber with $\beta_2 \sim 0.06 ps^2/m$ and $A_{eff} \sim 10\mu m^2$. For a sech type pulse of central wavelengtli $\lambda_0 = 600nm$, $T_{FWHM} \sim 1ps$ and input peak power $P_0 \sim 0.08W$, tlie soliton period is $Z_0 = 6m$. The behavior of tlie fundamental soliton is analogous to tlie one at tlie anomalous dispersion regime[1]. We have verified tliat the pulse is actually a soliton by checking its reproducibility in many soliton periods and also that no frequency scanning is introduced in tlie pulse.

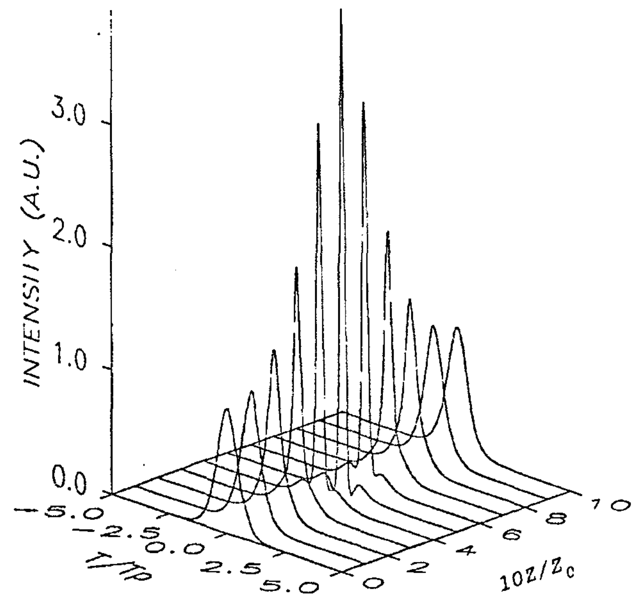

Figure 1: Soliton of order N = 2 calculated from eq.(2) with $L_{n_2}/L_{n_4} = 10^{-4}$.

As L_{n_2}/L_{n_4} increases to 10^{-3}, a quasi-soliton can be observed to evolve periodically as shown in fig.2. Such result could be experimentally obtained using $T_{FWHM} \sim 250fs$, $P_0 \sim 0.8W$ and $Z_0 = 0.6m$. Notice the slight asymmetry in the pulse evolution around $Z = Z_0/2$. We also observed departure from soliton behavior due to the pulse shortening througliout the pulse evolution, since the peak power increases and hence n_4 affects the effective chirp, which can no longer be properly compensated by dispersion. For $L_{n_2}/L_{n_4} = 1$, solitons, or even solitary waves, would no longer exist, but

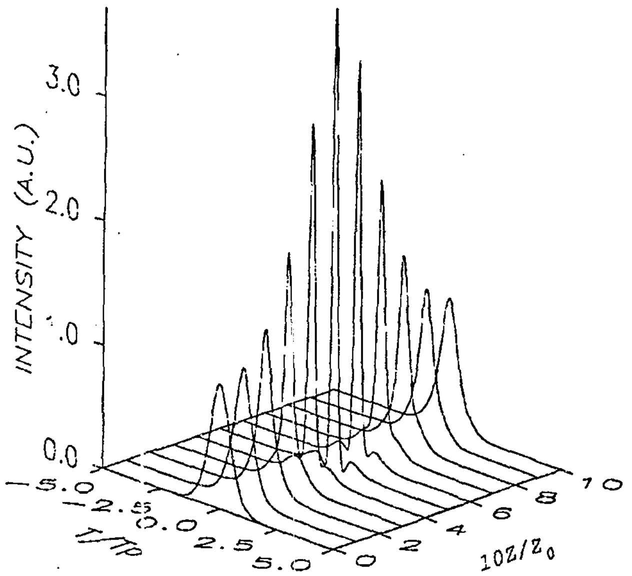

Figure 2: Soliton-like pulse propagation calculated using parameters corresponding to a soliton of order N = 2, from eq.(2) with $L_{n_2}/L_{n_4} = 10^{-3}$.

as this case is physically incorrect to treat in the present formalism, no further discussion is given.

From tlie viewpoint of ultrashort pulse generation, tlie effect of n_4 would have no dramatic consequence, but from tlie viewpoint of true solitary wave propagation, tlie inclusion of n_4 would lead to a disturbance of proper soliton wave propagation, although the solitary wave beliavicr would be kept.

III. Modulational Instability

Modulational Instability (MI) has been introduced in tlie context of optical fibers in 1980[17] and was first experimentally observed in 1986[18]. Since then, MI has been well studied with view to applications in the generatioii or pulsetrains with Terahertz repetition rates, both single pass or in lasers based upon MI[19,20]. Nakazawa et al.[21] have shown the connection between MI and high-order solitons. On the basis of the discussion presented in Section I, we analyze here tlie effect of n_4 on the MI where SDG fibers are again considered as the propagating medium, and therefore $n_2 < 0$ and $\beta_2 > 0$. MI in the normal dispersion regime of optical fibers has been studied, but exploiting other mechanisms such as optical wave breaking[22] or cross-phase modulation[23]. More recently MI has also been analyzed for a saturable nonlinearity (double-doped fiber) and anomalous (negative) dispersion[24]. By neglecting fiber losses and higher order terms, we analyze analitically the stability of the solution of eq.(1) following the usual procedure (see ref.[1]). The steady state solution of eq. (1) is given by

$$\bar{A} = \sqrt{P_0}\exp(i\phi_{NL}) \tag{3}$$

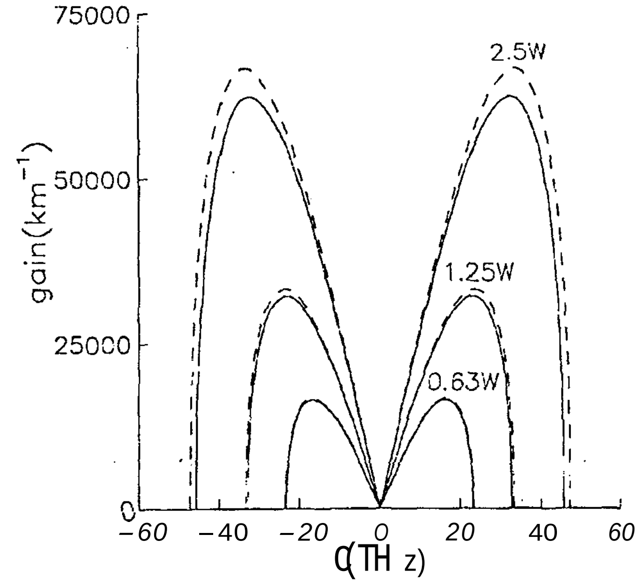

Figure 3: Gain spectra of modulation instability for $n_2 = -10^{-12}cm^2/W$, $n_4 = 10^{-20}cm^4/W^2$ and $\beta_2 = 0.06ps^2/m$ (solid line) and $n_2 = -10^{-12}cm^2/W$, $n_4 = 0$ and $\beta_2 = 0.06ps^2/m$ (dashed line).

where P_0 is the incident power at $Z = 0$ and

$$\phi_{NL} = \left(\gamma P_0 - \gamma' P_0^2\right) Z. \tag{4}$$

The stability of the steady-state solution against small perturbations is studied by slightly perturbing the steady state such as:

$$A = \left(\sqrt{P_0} + a\right)\exp(i\phi_{NL}). \tag{5}$$

Tlien, we find the linearized form of eq.(1) as:

$$i\frac{\partial a}{\partial Z} = \frac{1}{2}\beta_2\frac{\partial^2 a}{\partial T^2} - \gamma P_0\left(1 - \frac{2\gamma'}{\gamma}P_0\right)(a + a^*). \tag{6}$$

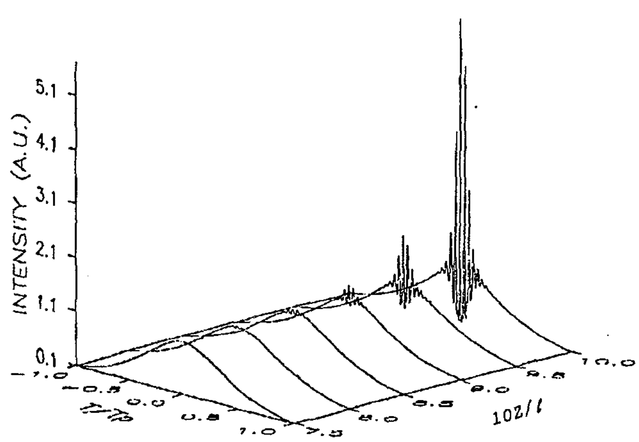

Figure 4: Evolution of soliton pulse with N = 60 and $n_2 = -10^{-12}cm^2/W$, $n_4 = 0$ $\beta_2 = 0.06ps^2/m$ between $Z = 0.75\ell$ and $Z = 1.0\ell$ ($\ell = 0.68m$).

The nontrivial solution for eq.(6) has the same form as for the case where $\gamma' = 0$ and is given by

$$K = \pm\frac{1}{2}|\beta_2|\Omega(\Omega^2 + sgn(\beta_2)\Omega_c^2)^{1/2}. \qquad (7)$$

Therefore, even with the inclusion of n_4, the MI effect occurs but in this case the critical frequency Ω_c is given by

$$\Omega_c^2 = \frac{4\gamma P_0\left(1 - \frac{2\gamma'}{\gamma}P_0\right)}{|\beta_2|}, \qquad (8)$$

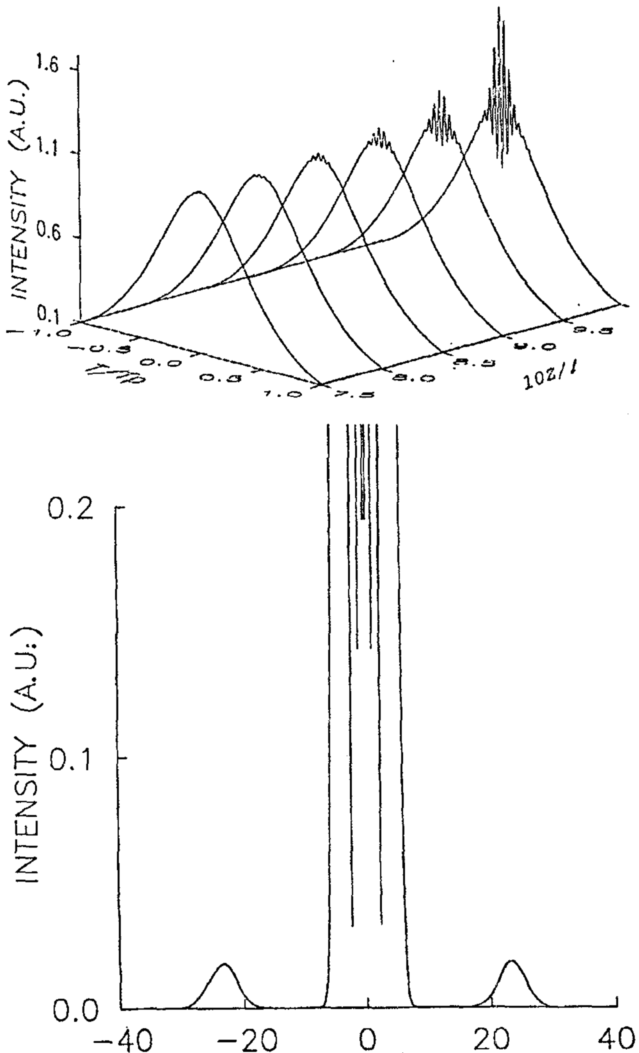

Figure 5: (a) Evolution of soliton pulse between $Z = 0.75\ell$ and $Z = 1.0\ell$ ($\ell = 0.68m$). (b) Magnified spectra corresponding to $Z = 0.8\ell$ showing the caracteristics sidebands of MI ($n_2 = -10^{-12}cm^2/W$, $n_4 = 10^{-20}cm^4/W^2$ and $\beta_2 = 0.06ps^2/m$).

which reproduces the usual result when $\gamma' = 0$, as expected. Eq.(7) leads to an instability (K imaginary) when $sgn(\beta_2) < 0$ and $\Omega < \Omega_c$, because the nonlinearity is positive in the case of silica based fiber. For the case of interest to us, where n_2 is negative, the result still holds for $sgn(\beta_2) > 0$, as can be verified after some algebra. In this case, the gain given by $g(\Omega) = 2I_m(K)$ can be written as

$$g(\Omega) = |\beta_2|[\Omega_2^2 - \Omega^2]^{1/2}. \qquad (9)$$

Fig.3 shows a plot of gain versus the frequency shift, from eq. (9), where $\beta_2 = 0.06ps^2/m$, $n_2 = -10^{-12}cm^2/W$ and $n_4 = 10^{-20}cm^4/W^2$ have been used. The heavy line shows the results for $n_4 \neq 0$, while the dashed line shows the results for $n_4 = 0$, all at the same input power levels as indicated in the figure. As can be observed, if n_4 is taken into account, a small reduction in the gain and a frequency shift is observed. This can be understood if one notices at eq. (8) that the effect of n_4 is to reduce the net nonlinearity, because of the opposite signs of n_4 and n_2.

In order to verify the effect of n_4 on the pulse propagation, we employed the same approach as in ref.[21], where the MI can be observed through the evolution of a very high order soliton. For an input pulsewidth $T_0 = 2.6ps$ ($sech$ shape assumed) the dispersive length is $L_D = 108m$. Using the fiber core radius $W_0 = 5\mu m$ and $P_0 = 2.5W$, we simulated the pulse evolution for a soliton of order $N = 60$. In this case $L_{n_2} = 0.03m$. Fig.4 shows the pulseshape evolution corresponding to propagation of the $N = 60$ soliton pulse between 0.6ℓ and 0.8ℓ for the case where $n_4 = 0$, and for a fiber length $\ell = 0.68m$, such that $Z/L_D \leq 10^{-2}$. The frequency ripple corresponds to the MI frequency. In fig.5(a) we show numerical simulations using the same parameters as for fig.4 but with the inclusion of the n_4 term, such that $L_{n_2}/L_{n_4} \sim 0.03$. The spectrum corresponding to the pulse at fig.5(a) is shown fig.5(b), where the sidelobes characteristics of MI are clearly shown. Inspection of the phase and frequency chirp show results similar to that of ref.[21]. For the same input parameters and fiber length, it can be observed, as predicted by the analytical results, that the MI would be delayed by the n_4 term, and longer fiber length (or higher power) would be required to reproduce the same gain and MI frequency as in fig. 4.

IV. Conclusion

We have analyzed the influence of the fifth-order nonlinearity on the pulse propagation in highly nonlinear optical fibers. The results show that for appropriate light intensities the n_4 term would affect soliton propagation and modulation instability generation. One interesting aspect of the present study is the feasibility of experimental verification of the studied phenomena in the normal dispersion regime of semiconductor doped fibers because of their negative third-order nonlinearity. Our study does not apply, however, for very high optical intensities or saturable nonlinearities.

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