h $\lambda\phi^4$ - String Field Theory as a Dynamics of Self Avoiding Random Surfaces

Luis C. L. Botelho
Departamento de Física, Universidade Federal do Pará
Campus Universitário do Guamá, 66075-900, Belém, PA, Brasil

Received February 2, 1992; revised manuscript received August 8, 1992

We introduce an analogous self avoiding random surface representation for a $\lambda\phi^4$ - Closed String Field Theory at critical dimension by following Symanzik's self avoiding contour representation for a $\lambda\phi^4$ - Point Like Field Theory.

One of the most interesting problems in String Field Theories is related to the implementation of non-perturbative analysis. We address in this paper the problem of implementing a non-perturbative self avoiding random surface representation for a $\lambda\phi^4$ - closed string field theory by the same procedure used years ago by Symanzik in his non-perturbative self avoiding contour representation for the $\lambda\phi^4$ (Point Like) Field Theory[1].

Let us start our analysis by considering the generating functional of the following mathematical $\lambda\phi^4$ Closed String Field Path Integral. On the critical dimension D = 26 (see Appendix A for the general covariant discussion).

$$Z[J(C)] = \int D^F[\Phi(C)] \times \exp\left\{ -\sum_{[C]} (\Phi(C)\hat{\Delta}_C\Phi(C) + J(C)\Phi(C)) + \lambda^2 \left(\sum_{[C_0,C_1]} \delta^{(D)}(C_0 - C_1)\Phi^2(C_0)\Phi^2(C_1) \right) \right\}. \tag{1}$$

The notation is as follows: i) the string field is given by a functional $\Phi(C)$ defined over the space of all closed string configurations $C = \{X_\mu(\sigma), -\pi \le \sigma \le \pi, X_\mu(-\pi) = X_\mu(\pi)\}$; ii) The sum over all closed string configurations is defined by the path integral

$$\sum_{[C]} = \int d^0x \left(\int_{X_\mu(-\pi)=X_\mu(\pi)=X_\mu} D^F[X_\mu(\sigma)] \exp\left(-\frac{1}{2}\int_{-\pi}^{\pi} (\dot{X}^\mu(\sigma))^2 d\sigma \right) \right); \tag{2}$$

iii) The critical $D = 26$ string free kinetic term is associated to the string D'Alembertian[2]

$$\hat{\Delta}_C = \frac{1}{2}\frac{\delta^2}{\delta^2 X_\mu(\sigma)} - \frac{1}{2\pi X'}|X'_\mu(\sigma)|^2; \tag{3}$$

iv) The string functional measure in equation (1) is given by the usual Feynman product measure

$$D^F[\Phi(C)] = \prod_{\{X_\mu(\sigma)\}} d\Phi(X_\mu(\sigma)); \tag{4}$$

and iv) The interaction action in equation (1) is given by the following vertex with D-dimensional delta functions supported on the string configurations and involving a positive λ^2 coupling constant in the extrinsic space

$$\lambda^2 \sum_{\{C_0,C_1\}} \delta^{(D)}(C_0 - c,). \tag{5}$$

The proposed interaction vertes was defined in such way that allows the replacement of the four string field interaction in equation (5) by an independent interaction of each string with a extrinsic Gaussian stochastic field $W(x)$ followed by an average over the fluctuating field $W(x)$. It is instructive to point out that similar procedure is well known in many-body path integral quantum field theory[3]. So, we can write equation (1) as

$$Z[J(C)] = \left\langle \int D^F[\Phi(C)] \exp\left\{ -\sum_{\{C\}} \Phi(C)(\hat{\Delta}_C \to i\lambda W(C))\Phi(C) + J(C)\Phi(C) \right\} \right\rangle_W . \tag{6}$$

Here, $W(C)$ means that the external stochastic field $W(x)$ is projected on the string configuration C

$$W(C) = \int_{-\pi}^{\pi} d\sigma W(X_\mu(\sigma)) \tag{7}$$

and satisfies tlie white noise stochastic correlation function with $x \leftarrow R^0$

$$< W(x)W(x') >_W = \delta^{(D)}(x-x'). \quad (8)$$

In tlie frce case, $\lambda = 0$, tlic String Path Integral Ficld Theory eqiiation (1) is exactly soluble with tlie following quantum string field generating functional

$$\frac{Z[J(C)]}{Z[J(C)\equiv 0]} = \exp\left\{+\frac{1}{2}\sum_{[C,\bar{C}]} J(C)\hat{\Delta}^{-1}(C,\bar{C})J(\bar{C})\right\}. \quad (9)$$

Here $\hat{\Delta}^{-1}(C,\bar{C})$ denotes tlic Green's Function for tlie string Laplacian and is given explicitly by tlic Random Surface Path Integral

$$\hat{\Delta}^{-1}(C,\bar{C}) = \int_0^\infty dA < c|e^{-A\hat{\Delta}_c}|\bar{C} >, \quad (10)$$

with

$$< C|e^{-A\hat{\Delta}_c}|\bar{C} >= = \int_{\substack{X^\mu(\sigma,O)=C^\mu(\sigma)\\ X^\mu(\sigma,A)=\bar{C}^\mu(\sigma)}} D^F[X^\mu(\sigma,\tau)] \times \exp\left(-\frac{1}{2} \times \times \int_0^A d\tau \int_{-\pi}^{\pi} d\sigma(\partial X^\mu)^2(\sigma,\tau)\right). \quad (11)$$

In order to reformulate the closed string field tlieory equation (1) as a dynamics of self-Avoiding Random Surfaces[4], we evaluate formally tlie Gaussian Ficld Path Integral in eqiiation (6)

$$Z[J(C)] = \left\langle [\det(\hat{\Delta}_c + i\lambda W(c)]^{-1/2} \times \exp\left\{+\frac{1}{2}\sum_{\{C,\bar{C}\}} J(C)(\hat{\Delta}_c + i\lambda W(c))^{-1}J(\bar{c})\right\}\right\rangle_W. \quad (12)$$

Lct us define tlie string functional determinant in equation (12) by tlie proper-time technique

$$\frac{1}{2}\log\det[\hat{\Delta}_c + i\lambda W(c)] = = -\int_0^\infty \frac{dA}{A}\sum_{(c,\bar{c})}\delta^{(F)}(c-\bar{c}) \times < c|\exp(-A(\hat{\Delta}_c + i\lambda W(c))|\bar{c} > \quad (13)$$

with

$$< C|\exp(-A(\hat{\Delta}_c + i\lambda W(c)))|\bar{C} >= = \int_{\substack{X^\mu(\sigma,O)=C^\mu(\sigma)\\ X^\mu(\sigma,A)=\bar{C}^\mu(\sigma)}} D^F[X^\mu(\sigma,\tau)] \exp\left\{-\frac{1}{2}\int_0^A dJ \int_{-\pi}^{\pi} d\sigma(\partial X^\mu)^2(\sigma,\tau) -i\lambda\int_0^A d\tau\int_{-\pi}^{\pi} d\sigma W(X^\mu(\sigma,\tau))\right\}. \quad (14)$$

By substituing equations (13) and (14) into equation (12) and making a power expansion in tlic coupling constant A, we obtain tlie String Ficld Tlicory eqiiation (1) as a Tlicory of Random Cylindrical Surfaccs (with boundarics being closed string configurations) iiiteractiiig with a external Gaussian Stochastic Ficld $W(x)$. The Gaussian average $< \ldots >_W$ may be straightforwardly evaluated at each order of the λ-power expansion and produccs self-avoiding interaction among tlie cglindrical random surfaccs similar to tlie usual self-avoiding Symanzik contour gas for the $\lambda\phi^4$ Ficld Theory[1]. For instance, by neglecting tlie functional determinant en Eq. (12), which physically means supressing surfaccs creation - annihilation (second-quantization) process, we have the following esprcssion for tlic theory's propagator

$$< \Phi(C^{\rm in})\Phi(C^{\rm our}) >^{(0)}= = \int_0^\infty \int_{\substack{X^\mu(\sigma,O)=C^{\rm in}\\ X^\mu(\sigma,A)=C^{\rm out}}} D^F[X^\mu(\sigma,\tau)] \exp\left\{-\frac{1}{2}\int_0^A d\tau\int_{-\pi}^{\pi} d\sigma(\partial X^\mu)^2(\sigma,\tau) -\frac{\lambda^2}{2}\int_0^A d\tau - i\lambda\int_0^A d\tau\int_{-\pi}^{\pi} d\sigma W(X^\mu(\sigma,\tau))\right\}. \quad (15)$$

Note that self-intersecting lines are invariant under reparametrizations of tlic full string world sheet.

Tlie next string quantum field correction for eq. (15) in our proposed framework will be given by

$$< \Phi(C^{\rm in})\Phi(C^{\rm our}) >^{(1)}= = \int_0^\infty d\bar{A}\int_0^\infty \frac{dA}{A}\sum_{\{C,\bar{C}\}}\delta^{(F)}(C-\bar{C}) \times << C|\exp[-A(\hat{\Delta}_c + i\lambda W(c))]|\bar{C} > \times < C_{\rm in}|\exp[-A'(\hat{\Delta}_c + i\lambda W(c))]|C_{\rm out} >>_W, \quad (16)$$

where

$$\delta^{(F)}(C-\bar{C}) = \prod_{\pi\leq\sigma\leq\pi}\delta^{(D)}(C_\mu(\sigma)-\bar{C}_\mu(\sigma)). \quad (17)$$

We may write eq. (16) in tlie form of a two-body random surface path integral witli self-avoiding interactions[1,4].

$$\int < \Phi(C^{\rm in})\Phi(C^{\rm our}) >^{(1)}= = \int d\bar{A}\int_0^\infty \frac{dA}{A}\sum_{\{C,\bar{C}\}}\delta^{(F)}(C-\bar{C}) \times \int_{\substack{X^\mu_{(1)}(\sigma,O)=C_\mu(\sigma)\\ X^\mu_{(1)}(\sigma,A)=\bar{C}_\mu(\sigma)}} D^F[X^\mu_{(1)}(\sigma,\tau)]$$

$$\int_{\substack{X^{\mu}_{(2)}(\sigma,O)=C^{in}_{\mu}(\sigma)\\ X^{\mu}_{(2)}(\sigma,A)=\bar{C}^{out}_{\mu}(\sigma)}} D^F[X^{\mu}_{(2)}(\sigma,\tau)]$$
$$\exp\left(-\frac{1}{2}\int_0^A d\tau\int_{-\pi}^{\pi} d\sigma(\partial X^{\mu}_{(1)})^2(\sigma,\tau)\right)\times$$
$$\exp\left(-\frac{1}{2}\int_0^{\bar{A}} d\tau\int_{-\pi}^{\pi} d\sigma(\partial X^{\mu}_{(2)})^2(\sigma,\tau)\right)\times$$
$$\exp\left(-\frac{\lambda^2}{2}\int_0^A d\tau\int_0^{\bar{A}} d\tau'\int_{-\pi}^{\pi}\right.$$
$$\left. d\sigma\int_{-\pi}^{\pi} d\sigma'\delta^{(D)}X^{\mu}_{(1)}(\sigma,\mathrm{r})-X^{\mu}_{(2)}(\sigma',\mathrm{r'})\right)\times$$
$$\exp\left(-\frac{\lambda^2}{2}\int_0^A d\tau\int_0^A d\tau'\int_{-\pi}^{\pi}\right.$$
$$\left. d\sigma\int_{-\pi}^{\pi} d\sigma'\delta^{(D)}X^{\mu}_{(1)}(\sigma,\tau)-X^{\mu}_{(1)}(\sigma',\tau')\right)\times$$
$$\exp\left(-\frac{\lambda^2}{2}\int_0^{\bar{A}} d\tau\int_0^{\bar{A}} d\tau'\int_{-\pi}^{\pi}\right.$$
$$\left. d\sigma\int_{-\pi}^{\pi} d\sigma'\delta^{(D)}X^{\mu}_{(2)}(\sigma,\tau)-X^{\mu}_{(2)}(\sigma',\tau')\right). \quad (18)$$

Let us pɔint out tliat the perturbative renormalizability of tl e interacting string propagator eq. (15), may be given by the renormalization group of tlie self-avoiding random surfaces theories[4]. An alternative regularization ɛtudy for eq. (15) may be implemented in a pure geornetrical framework as proposed in ref.[5] for tlie loop space formulation of point particle field theories[1]. Ir order to implement this study for random surface, we start by extracting the trivial self-intersect points $X_\mu(\sigma,\mathrm{r}) = X_\mu(\sigma',\tau')$ with a = a', r = r' from tlie λ^2 interaction term of eq. (15). Thus, let us introduce a D-dimensional regularization parameter A on tlie self avoiding D-dimensional interaction in order to extract the (geometrical) infinitics associated to tlie trivial self-intersect surface points

$$I[X_\mu(\sigma,\mathrm{r})] = \frac{\lambda^2}{2}\int_0^A d\tau\int_0^A d\tau'\int_{-\pi}^{\pi} d\sigma\int_{-\pi}^{\pi} d\sigma'$$
$$\left(\int_{|k|<\Lambda} d^0K\exp[iK_\mu(X_\mu(\sigma,\zeta)-X_\mu(\sigma',\tau'))]\right). \quad (19)$$

The above equation may be written in the more suitable form after introducing the extrinsic λ coupling constant as a scale of the $X_\mu(\sigma,\mathrm{r})$ - field, i.e.:

$$I[X_\mu(\sigma,\tau)] = \frac{1}{2}C(D)\int_0^A d\zeta\int_0^A d\zeta'\int_{-\pi}^{\pi} d\sigma\int_{-\pi}^{\pi} d\sigma'$$
$$\left(\int_0^{\Lambda'} d|K|\cdot|K|^{D/2}\frac{1}{\lambda^{2/D}}|X_m u(\sigma,\zeta)\right.$$
$$-X_\mu(\sigma',\zeta')|^{1-D/2}$$
$$\left.\mathcal{J}_{\frac{D}{2}-1}\left(\frac{|K|}{\lambda^{2/D}}|X_\mu(\sigma,\zeta)-X_\mu(\sigma',\zeta')\right)\right), \quad (20)$$

where $C(D)$ is a constant depending only on tlie space time dimension and $\mathcal{J}_\nu(x)$ denotes the usual Bessel Function of order ν. Let us remark now that the new regularization parameter A' is dimensionlcss.

By power expanding the Bessel Function we reduce equation (17) to a sum of the form

$$J[X_\mu(\sigma,\tau)] = \frac{1}{2}\, C(D)\;\cdot$$
$$\sum_{K=0}^{\infty}\frac{(-1)^k(\lambda^{2/D})^k}{k!\partial^{2k}\Gamma\left(\frac{D}{2}-k\right)}I^{(k)}(X_\mu(\sigma,\tau),\Lambda'), \quad (21)$$

where the partial contributions in equation (18) are of tlie form

$$I^{(K)}[X_\mu(\sigma,\tau),\Lambda'] =$$
$$\int_0^{\Lambda'} d|K|\cdot|K|^{D+2K-1}\int_{-\pi}^{\pi} d\sigma\int_{-\pi}^{\pi} d\sigma'$$
$$\int_0^A d\tau\int_0^A d\tau'|X_\mu(\sigma,\tau)-X_\mu(\sigma',\tau')|^{2K}. \quad (22)$$

To regularize the infinities in equation (22), we propose to introduce the already used dimensionless parameter A' in eq. (20) on tlie two dimensional string space-time {(a, r);−?r $\le$ a $\le \pi$;0 $\le \tau \le A$) by using tlie following unity decomposition into tlie integrand of equation (22)

$$1 = \delta^{(2)}_{(\Lambda')}((\sigma,\tau)-(\sigma',\tau'))$$
$$+[1-\delta^{(2)}_{(\Lambda')}((\sigma,\tau)-(\sigma',\tau'))], \quad (23)$$

where the regularized two dimensional delta function is given explicitly by

$$\delta^{(2)}_{(\Lambda')}((\sigma,\tau)-(\sigma',\tau')) = \begin{cases}\Lambda' & \sigma-\frac{1}{\Lambda'}\le\sigma'\le\sigma+\frac{1}{\Lambda'} \\ & \tau-\frac{1}{\Lambda'}\le\tau'\le\tau+\frac{1}{\Lambda'} \\ 0 & \text{otherwise}\end{cases}. \quad (24)$$

By Taylor expanding the integrand of eq. (22) around the point $\xi = (\nu',\tau')$, where $\xi = (\mathrm{a},\mathrm{r})$,

$$|X_\mu(\xi)-X_\mu(\xi')|^{2k} = \left\{\sum_{\ell=2}^{\infty}\left[\sum_{r_1\ge1,r_2\ge1}^{r_1+r_2=\ell}\right.\right.$$
$$\left.\left.D_\xi^{r_1}X^\mu(\xi)D_\xi^{r_2}X_\mu(\xi)|\xi-\xi'|^{r_1+r_2}\right]\right\}^k, \quad (25)$$

inserting the identity eq. (24) and eq. (25) into eq. (22) and making use of the result

$$\int_{-\pi}^{\pi} d\sigma \int_{0}^{A} d\tau \delta_{\Lambda'}^{(2)}((\sigma,\tau)-(\sigma',\tau'))$$

$$(\sigma-\sigma')^n(\tau-\tau')^m f(\sigma',\tau')$$

$$= \begin{cases} \Lambda'^{(n+m)/2} f(\sigma,\tau) & n,m = \text{ even} \\ 0 & \text{otherwise }, \end{cases} \tag{26}$$

we are able to show that Ilic most general extrinsic counter term arising from tlic trivial self-intersect limit $h \to \infty$ is an exponential of a four variable quadratic polinomial with a renormalized extrinsic Λ^R coupling constant,

$$\int_{-pi}^{\pi} d\sigma \int_{0}^{A} \exp\{\mathcal{P}[\partial_\sigma X^\mu, \partial_\tau X^\mu, \partial_\sigma^2 X^\mu, \partial_\tau^2 X^\mu]\}. \tag{27}$$

All other contributions oii tlic derivative order greater than tlic *second derivative* on the random surface vector position vanish in tlic trivial self intersect limit of $\Lambda' \to \infty$.

The contribution of tlic non trivial self intersect points associated to tlic term $(1-\delta_{\Lambda'}((\sigma,\tau)-(o',\tau')))$ at $\Lambda' \to \infty$ leads to a kind of surface self-avoiding topological index[4]

$$\int_{0}^{A} d\tau d\tau' \int_{-\pi}^{\pi} d\sigma d\sigma' \delta^{(D)}(X_\mu(\sigma,r)-X_\mu(\sigma',\tau')). \tag{28}$$

Tlic slash in tlic integration symbols $\int$ in eq. (28) means that tlic trivial self iiitersect points a = a', r = r' are excluded from tlic integrand.

We remark tliat eq. (15), after being renormalized as described above, describes a two dimensional super-renormalizable field tlicory oii tlic striiig spacetime $\{(a,\tau), -\pi \le \sigma \le \pi, 0 \le r \le A\}$ since tlie couiiter term, eq. (27), generates n term to be added to tlie "ficc kiiietic action" with tlic form $\approx C(\lambda^R)[(\partial_\sigma^2 X^\mu)^2 + (\partial_\tau^2 X^\mu)^2]$ where $C(\lambda^R)$ is a function of tlie extrinsic renormalized self-supressing coiipliiig constant.

Finally we comment that oiir proposed striiig quantum field theory is, iii principle, different from those already proposed by other authors[6] siiice oiir interaction vertes, eq. (5), is a combination of D-dimensional delta functions aiid iiot functional delta functions as in ref. [G].

A complete study of tlie 2D field tlicory, eq. (15), will be presented elsewhere.

hcknowledgement

This work was supported by CNPq - Brasil.

Appendix A - A covariant version of the proposed $\lambda\phi^4$ String Field Theory

In this appendix we will make comments on tlic covariance of tlic tlicory under tlie action of the striiig diffeomorphism groiip.

In order to have from tlic beginning a covariant string field theory we must consider oiir tlicory for sul>-critical strings $D \le 2G$ ([7]). Tlic main change in oiir study is that we have to take iiito account 2D induced pure quantum gravity which is needed by the dynamical status acquired by tlie intrinsic metric field $g_{ab}(\sigma,\tau)$. This step may be easily implemented oii the random surface path integrals, eqs. (15)-(18). For instance, tlic theory's propagator, eq. (15), will take the *reparametrization invariant form*[7].

$$\begin{aligned} &< \Phi(C^{\text{in}})\Phi(C^{\text{out}}) >^{(0)} = \\ &\int_0^\infty dA \int_{\substack{X^\mu(\sigma,0)=C^\mu_{\text{in}}(\sigma) \\ X^\mu(\sigma,A)=C^\mu_{\text{out}}(\sigma)}} D^C[X^\mu(\sigma,\tau)] \\ &\int D^C[g_{ab}(\sigma,\tau)] \times \\ &\exp\left[-\frac{1}{2}\int_0^A d\tau \int_{-\pi}^{\pi} d\sigma(\sqrt{g}g^{ab}\partial_a X^\mu \partial_b X_\mu)(\sigma,\tau)\right] \\ &\times \exp\left[-\frac{\lambda^2}{2}\int_0^A d\tau \int_0^A d\tau' \int_{-\pi}^{\pi} d\sigma \int_{-\pi}^{\pi}\right. \\ &d\sigma' \sqrt{g(\sigma,\tau)}\delta^{(D)}(X_\mu(a,r)-X_\mu(\sigma',\tau')) \\ &\left.\times\sqrt{g(\sigma',\tau')}\right]. \end{aligned} \tag{A.1}$$

Unfortunately, tlic tlicory of sub-critical strings was not exactly solved yet. However, at D = 26 we can show tliat tlic $g_{ab}(\sigma,r)$ field decouples from tlic full striiig propagator, eq. (A-I), at least for tlic weak perturbative coiipliiig phase for tlic λ-constant (the result for $\lambda = O$ was proved by Polyakov[6]). This result afford us to clioose and, thus, to fix tlic decoupling gauge $g_{ab}(\sigma,r) - \delta_{ab}$ in our proposed tlicory.

It is worth to point oiit tliat in a rigorous mathematical procedure one should consider, as iii usual gauge tlicories, first Ward - Takahashi identities associated to tlic diffeomorphism (non-conformal) group at D $\le$ 26. Thus, take tlic limit D = 26 oii these identities. Anyway, tlic physical objects in striiig tlicories are not tlic striiig propagators but the scattering amplitudes which are physical observables aiid may be calculated directly from tlic eq. (A-I) aiid tested to have tlic necessary invariances as shown by a perturbative analysis in λ coiipliiig constant.

We remark tliat difficulties in considering *non gauge fixed* tlicories is shared by others striiig field tlicorics considered in tlic literature as Llic B.R.S.T aiid light cone striiig field tlicorics[6].

As a final comment we notice that tlic important problem of invariances iii striiig field theory is waiting tlic solution of the tlicory of sub-critical strings.

Appendix B - Our proposed $\lambda\phi^4$ String Field Theory as an infinite component field theory of string excitations

Let us consider a harmonic oscillator expansion for the closed string configuration with $X_\mu(\sigma) = x_\mu$; i.e.:

$$X_\mu(\sigma) = x_\mu + \sum_{\substack{n\neq 0 \\ n=-\infty}}^{+\infty} \mathcal{O}_n^\mu e^{in\sigma}. \quad (B.1)$$

In this base, the second quantized string field will be decomposed in all possible string excitations[6]

$$\Phi[X_\mu(\sigma)] =$$

$$\mathcal{O}(x) + A_\mu(x)\mathcal{A}^\mu_{(1)} + \ldots\zeta_{\mu_1\ldots\mu_N}(x)\mathcal{A}^{\mu_1}_{(N)}\ldots\mathcal{A}^{\mu_N}_{(N)} + \ldots. \quad (B.2)$$

The sum over all closed string configurations are weighted by (see eq.(2))

$$\int_{-\infty}^{+\infty} d^0x \int \prod_{(N,\mu)} d\mathcal{A}^\mu_{(N)} e^{-|\mathcal{A}^\mu_{(N)}|^2}. \quad (B.3)$$

The Feynman product measure, eq. (4), is factorized in the product of all Feynman measures associated to the point-like field string excitations, eq. (B-2), and thus

$$D^F[\Phi(C)] = \prod_{N=1}^{\infty} D^F[\zeta_{\mu_1\ldots\mu_N}(x)]D^F(\varphi(x)], \quad (B.4)$$

with

$$\hat{\Delta}_c = -\frac{\partial^2}{\partial x_\mu^2} + \sum_{\substack{(N\neq) \\ N=-\infty}}^{\infty} \frac{\partial^2}{\partial \mathcal{A}^\mu_N \partial \mathcal{A}^\mu_{-N}}. \quad (B.5)$$

Finally our proposed vertex takes the form

$$\delta^{(D)}(C_0 - C_1) =$$
$$\int d^0k \exp iK^\mu \left[\sum_{N=-\infty}^{+\infty} \mathcal{A}_N^{\mu,(0)} e^{iN\sigma} - \sum_{N=-\infty}^{+\infty} \mathcal{A}_N^{\mu,(1)} e^{iN\sigma} \right]. \quad (B.6)$$

After substituting the above writen equations in our proposed action, eq.(1), we obtain an interacting infinite-component field theory associated to the string excitations.

Fortunately, our random surface representation is a compact and conceptualy simple way to understand and study this cumbersome infinite component point like field theory associated to the string field theory describe by eq. (1).

References

1. a) K. Symanzik, in *Local Quantum Theory*, Ed. R. Jost (Academic, London, 1969); b) L.C.L. Botelho and J.C. de Mello, J. Phys. A. Math. Gen. 22, 1915 (1989); G. Broda, Phys. Rev. Lett. **63**, 2709 (1989).
2. L.C.L. Botellio, Phys. Rev. **40D**, 660 (1989).
3. L.C.L. Botellio, Mod. Phys. Let. B 391 (1991).
4. a) M. Kardar and D.R. Nelson, Phys. Rev. Lett. 1964 (1989); b) B. Duplantier, Commun. of Math. Phys. **85**, 221 (1982).
5. A.M. Polyakov, Nucl. Phys. B **164**, 171 (1980).
6. M. Kaku, *Introduction to Superstrings.* Graduate Text in Contemporary Physics. (Springer, Berlin, 1988).
7. L.C.L. Botellio, J. Math. Phys. **30**, 2160 (1989); b) Rev. Bras. Fis. 21, 447 (1991).