Meson Relativistic Spectroscopy*

H. S. Carvalho, H. F. de Carvalho
Universidade Federal do Rio de Janeiro, Instituto de Física
Caixa Postal 68529, Rio de Janeiro, 21944, RJ, Brasil
A. S. de Castro
Universidade Estadual Paulista, Departamento de Física e Química
Campus de Guaratinguetá, 12500, Guaratinguetá, SP, Brasil

Received February 2, 1992; revised manuscript received July 22, 1992

A two-body relativistic equation, derived from Dirac's constraint dynamics, is used to obtain the meson mass spectra. Spin-dependent effects are not considered. Comparison with recent experimental data and with the results given by a nonrelativistic approximation is made. The leptonic and hadronic decay widths and radiative transition rates are also calculated for some mesons.

I. Introduction

Hadrons are usually considered as being composed of quarks. The model describing the interaction among quarks is Quantum Chromodynamics (QCD) with the symmetry group given by colour SU(3). In this model mesons are interpreted as quark-antiquark bound states belonging to the color singlet representation. The discovery of the families $J/\psi(c\bar{c})$ e $\gamma(b\bar{b})$ represented a great contribution to the understanding of the quark-antiquark bound states. The ratios v/c for these systems are relatively small and consequently the nonrelativistic approximation to the Bethe-Salpeter equation can be used for the calculation of the spectra. Relativistic corrections are added through pertubative calculations. The static potential in the nonrelativistic approximation has one part related to asymptotic freedom and another part, purely phenomenological, related to confinement. Since confinement is not well understood, a particular confining potential is only justifield when the output is compared with the experimental data. But still the justification for using the Bethe-Salpeter equation is also partial. The problem can be dealt with in another way: we need a relativistic two-body equation which could be easily handled, and in general this is not the case with the Bethe-Salpeter equation. In this sense an equation that fits in this context is the Crater-Alstine equation[1], which is a relativistic equation obtained from Dirac's constraint dynamics for two spinless particles interacting through an interaction potential which can have a timelike vector part, a spacelike vector part and a scalar part. In the derivation of this equation mass-shell constraints and interactions among particles are introduced by the minimal substitutions $p_i^\mu \rightarrow p_i^\mu - A_i^\mu$ for the vector potential, and $m_i \rightarrow m_i + S_i$ for the scalar one. The important result is the attainment of a Klein-Gordon-like two-body equation. For weak potentials this equation reduces to the homogeneous quasipotential equation of Todorov[2] for stationary states of two spinless particles. The advantage of the Crater-Alstine equation is that while being relativistic it can be applied to two interacting bodies in a nonperturbative way. This does not happen with the Breit equation, which is obtained from the Bethe-Salpeter equation in the ladder approximation[3], where the relativistic corrections are made perturbatively. Besides, with the Crater-Alstine equation we have three kinds of potentials at our disposal: one of them corresponds to the electromagnetic potential and the other two correspond to phenomenological choices. This is the case of the confining potential used in the nonrelativistic approximation for calculation of quark-antiquark bound states. In the nonrelativistic approximation various potentials have been proposed and the calculated spectra are, in general, compatible[4] with them. The purpose of this paper is to calculate the meson spectra using the Crater-Alstine equation and to confront the results with those obtained through a nonrelativistic approach.

II. The Potential Model

The Breit-Fermi equation, obtained through the instantaneous approximation to the Bethe-Salpeter equation, is commonly used in the calculation of the meson mass spectroscopy. The Breit-Fermi equation provides a Schrödinger equation plus spin-dependent and spin-independent relativistic corrections

$$\Big(m_1 + m_2 + \frac{\vec{p}^2}{2\mu} + V(\vec{r}) + \text{relativistic corrections}\Big)\psi = E\psi. \qquad (1)$$

*Work partially supported by CNPq, FAPESP and FINEP.

Tlicse corrections depend on tlie behaviour of the potential under Lorentz transformations. Tlie Coulombian part of tlie potcntial, due to one gluon exchange, is vcctorial. Nevertlieless, tlie confining poteiitial does not have a Lorentz structure known from first principles, but oiily informations from a phenomenological poiiit of view. Tlie confining potciitial has been considered in previous work[5] as a misture of vcctorial and scalar coupling, i.e.,

$$V(r) = V_{\rm Coul}(r) + V_{\rm conf}(r) + C(q_a\bar{q}_b), \qquad (2a)$$

$$V(r) = V_v(r) + V_s(r) + C(q_a\bar{q}_b), \qquad (2b)$$

$$V_v(r) = (1-f)V_{\rm conf}(r) + V_{\rm Coul}(r), \qquad (2c)$$

$$V_s(r) = fV_{\rm conf}(r). \qquad (2d)$$

Tlie confining potciitial is purcly vcctorial for f = 0 aiid purcly scalar for f = I. Comparison witli experimental data[6] leads to a scalar contribution for tlie confining potcntial of tlie ordcr of 50% to 60%.

For liglit quark-antiquark systems tlie nonrelativistic approach is safe only if tlie ratio v/c is small. As a matter of fact, only tlie $b\bar{b}$ system fits nicely this approach, although tlie $c\bar{c}$ system may also be considered as acceptable. With tlie relativistic equation

$$((\vec{p}^2 - (\epsilon_w - A)^2 + 2\epsilon_w \mathcal{V} - \mathcal{V}^2 + (m_w + S)^2$$
$$+\frac{1}{2}\nabla^2 \ln G + \frac{1}{4}(\nabla \ln G)^2)\phi = 0, \qquad (3)$$

Crater and Alstine[1] obtained tlie liglit aiid heavy meson mass spectra taking into account only time-like vector couplings (A = Q C = 1 and V # 0) and scalar coupling. The expression for tlic interaction V aiid S were obtaincd witli tlie Ricliardsoii potential[4]. This potential, in tlie nonrelativistic approximation, describes only tlie heavy meson spectroscopy, and is not adequate for liglit mcson spectroscopy. Cratcr and Alstine extended consistently tlie applicability of tlie Richardson potential lo liglit meson spcctra.

In previous work we have uscd another potential to dcscribe the liglit aiid lieavy mcson spectra in tlie nonrelativistic approximation[5]. Tlie same potcntial is uscd here in the relativistic equation for two scalar particlcs. In this scnse tlic obtained spectra is spin-averaged. Tlie potcntials ivliicli appear iii Eq. *(3)* are defined by

$$\mathcal{A} = V_{\rm Coul}(r) = -\frac{4\alpha_s}{3r}, \qquad (4a)$$

$$G = \left(1 - \frac{2\mathcal{A}}{W}\right)^{-1/2}, \qquad (4b)$$

$$\mathcal{V}(r) = (1-f)V_{\rm conf}(r) = (1-f)Kr^{1/2}, \qquad (4c)$$

$$S(r) = fV_{\rm conf}(r) + C(q_a\bar{q}_b) = fKr^{1/2} + C(q_a\bar{q}_b). \qquad (4d)$$

With tliese potcntial functions Eq. (3) takes tlic form

$$\Big[\vec{p}^2 - 2\alpha_s\frac{\epsilon_w}{\rm r} - \frac{\alpha_s^2}{\rm r^2} + \frac{5\alpha_s^2}{4r^2(Wr + 2\alpha_s)^2} \qquad (5)$$
$$+2K(\epsilon_w(1-{\rm f}) + fm_w + fC)r^{1/2}$$
$$+K^2(2f-1){\rm r} + 2m_wC + C^2\Big]\psi = b^2\psi.$$

The parametcrs of tlie potential and quark masscs are fitted in a way sliglitly different from that of tlie nonrclativistic case[5]. Instead of givcn as inputs, the quarks masscs are fitted together with tlie potential parametcrs. Again tlie parameter K is universal for all pairs of quark-antiquark. Tlie strong coupling is not constant but depends of tlic transferred momentum aiid the number of flavors of tlie quarks

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2N_f)\ln(Q^2/\Lambda^2)}. \qquad (6)$$

Tlie scale parameter Λ is fixed by tlic bottomonium spectrum aiid N_f assumes tlie valucs 5, 4, 3 and 2, for the systems $b\bar{b}$, $c\bar{c}$, $s\bar{s}$, $q\bar{q}$, respectively. Here q represents a liglit quark u or d. Using Eq. (5) we obtain f = 1, A = 0.118 GeV, K = 0.740 GeV$^{3/2}$, and b, c, s, and u quark masses of 4.5, 1.41, 0.337 and 0.16 GeV, respectively, whereas $\alpha_s(b)$ = 0.187, $\alpha_s(c)$ = 0.231, $\alpha_s(s)$ = 0.324 aiid $\alpha_s(u)$ = 0.346. $C(q_a\bar{q}_a)$ takes tlie values -0.275, -0.800, -1.129 and -1.207 GeV, for tlie $b\bar{b}$, $c\bar{c}$, $s\bar{s}$ and $u\bar{u}$, respectively. $C(q_a\bar{q}_b)$ is fitted as

$$C(q_a\bar{q}_b) = 0.004x^2 + 0.099x - 0.894, \qquad (7a)$$

where

$$x = \ln\left[m_{q_a}^2 m_{\bar{q}_b} + m_{q_a} m_{\bar{q}_b}^2\right]. \qquad (7b)$$

Tlie rcsulting spectra are shown in tables Ia and Ib. Tlic experimental results for tlie statcs $u\bar{b}$ aiid $s\bar{c}$ have a striicture $^1S_0$ aiid are oiily shown as a hint to tlie values of tlie $^3S_1$ states. Tlie masscs of the bound states $u\bar{b}$, $s\bar{c}$, $c\bar{b}$ are calculated using (7). In Table II we compare our results witli tliose obtained by Cratcr and Alstine[1], and also with those obtained using a nonrelativistic approach.

Tlie expressions for tlie leptonic and hadronic decays of tlie n^3S_1 states, including perturbative corrections of QCD are given by[7]

$$\Gamma_{q\bar{q}\to e^+e^-} = 16\pi\alpha^2 e_q|\psi(0)/M_{q\bar{q}}^2|^2\left(1 - \frac{16\alpha_s}{3\pi}\right), \qquad (8a)$$

$$\Gamma_{\psi\to\ {\rm hadrons}} = \Gamma^{(0)}\left(1 + (4.9 \pm 0.5)\frac{\alpha_s}{\pi}\right), \qquad (8b)$$

$$\Gamma_{\Upsilon\to\ {\rm hadrons}} = \Gamma^{(0)}\left(1 + (3.8 \pm 0.5)\frac{\alpha_s}{\pi}\right), \qquad (8c)$$

where

$$\Gamma^{(0)} = \frac{160}{81}(\pi^2 - 9)\alpha_s^3|\psi(0)/M_{q\bar{q}}^2|^2. \qquad (8d)$$

Table Ia - Mass spectra of tlie light aiid heavy mesons, in GeV

	$b\bar{b}$	$c\bar{c}$	$s\bar{s}$	$u\bar{u}$
$C(q_a\bar{q}_a)$	-0.275	-0.800	-1.129	-1.207
1S theory	9.463	3.097	1.020	0.768
exp	$\gamma(9.460)$	$\psi(3.097)$	$\phi(1.020)$	$\rho(0.768)$
2S theory	10.010	3.686	1.679	1.451
exp	$\gamma(10.023)$	$\psi(3.686)$	$\phi(1.680)$	$\rho(1.450)$
3S theory	10.353	4.092	2.171	1.953
exp	$\gamma(10.355)$	$\psi(4.040)$		$\rho(1.712)$
4S tlieory	10.617	4.418	2.567	2.360
exp	$\gamma(10.580)$	$\psi(4.415)$		
5S tlieory	10.838	4.696	2.906	2.705
exp	$\gamma(10.865)$			
GS tlieory	11.030	4.941	3.201	3.006
exp	$\gamma(11.019)$			
1P tlieory	9.838	3.510	1.431	1.176
exp	(9.900)	(3.525)	(1.476)	(1.262)
2P theory	10.220	3.956	1.996	1.774
exp				
1D tlieory	10.138	3.795	1.761	1.516
exp		$\psi(3.769)$		$\rho(1.691)$
2D tlieory	10.437	4.171	2.249	2.030
exp		$\psi(4.159)$		

Table Ib - Mass spectra of tlie light and heavy mesons, in GeV

	$u\bar{b}$	$s\bar{c}$	$u\bar{c}$	$u\bar{s}$
$C(q_a\bar{q}_a)$	-0.511	-0.926	-0.928	-1.224
1S theory	5.156	2.117	2.010	0.892
exp	B(5.271)	$F^*(2.140)$	$D^*(2.010)$	$K^*(0.892)$
2S theory	5.661	2.712	2.600	1.565
exp				$K^*(1.370)$
3S tlieory	6.011	3.143	3.034	2.066
exp				$K^*(1.678)$
1P tlieory	5.493	2.509	2.392	1.309
exp			$D_2^*(2.459)$	$K_2^*(1.430)$
2P tlieory	5.887	2.990	2.879	1.893
exp				
1D tlieory	5.861	2.811	2.684	1.648
exp				$K^*(1.780)$

Table II - Mass spectra of the light and heavy mesons, in GeV. Values with a asterisk are $^1S_0$ states

		Nonrelativ.[5]	Relativ.	Relativ.[1]	Exp.[6]
	$b\bar{b}$	9.467	9.463	9.460	9.460
	$c\bar{c}$	3.094	3.097	3.097	3.097
	$u\bar{c}$	2.008	2.010	1.990	2.010
	$s\bar{s}$	1.020	1.020	1.020	1.020
	$u\bar{s}$	0.892	0.892	0.892	0.892
(1S)	$u\bar{u}$	0.770	0.768	0.759	0.768
	$u\bar{b}$	5.271	5.156	5.311	5.271*
	$c\bar{b}$	6.329	6.330	6.337	
	$s\bar{b}$	5.383	5.318	5.414	
	$s\bar{c}$	2.140	2.117	2.140	2.140*
	$b\bar{b}$	10.012	10.010	10.021	10.023
	$c\bar{c}$	3.696	3.686	3.661	3.686
	$u\bar{c}$	2.694	2.600	2.575	
	$s\bar{s}$	1.727	1.679	1.706	1.680
	$u\bar{s}$	1.616	1.565	1.606	1.370
(2S)	$u\bar{u}$	1.511	1.451	1.509	1.450
	$u\bar{b}$	5.941	5.661	5.830	
	$c\bar{b}$	6.904	6.887	6.879	
	$s\bar{b}$	6.028	5.845	5.939	
	$s\bar{c}$	2.804	2.712	2.685	
(3S)	$b\bar{b}$	10.352	10.353	10.349	10.353
	$c\bar{c}$	4.093	4.092	4.055	4.040
	$s\bar{s}$	2.208	2.171		
	$u\bar{u}$	2.015	1.953		1.712
(4S)	$b\bar{b}$	10.614	10.617	10.604	10.580
	$c\bar{c}$	4.406	4.418	4.383	4.415
	$b\bar{b}$	9.875	9.883	9.935	9.900
	$c\bar{c}$	3.516	3.510	3.556	3.525
	$u\bar{c}$	2.475	2.392	2.457	2.459
	$s\bar{s}$	1.499	1.431	1.564	1.476
	$u\bar{s}$	1.381	1.309	1.456	1.434
(1P)	$u\bar{u}$	1.269	1.176	1.353	1.262
	$u\bar{b}$	5.729	5.493	5.573	
	$c\bar{b}$	6.740	6.478	6.786	
	$s\bar{b}$	5.827	5.681	5.840	
	$s\bar{c}$	2.594	2.509	2.569	

Table III - Leptonic and hadronic decay widtlis for the bottomonium and charmonium S-states, in KeV.

	Nonrelativ.[5]		Relativ.		Exp.[6]	
	$b\bar{b}$	$c\bar{c}$	$b\bar{b}$	$c\bar{c}$	$b\bar{b}$	$c\bar{c}$
$\Gamma(1S \rightarrow e^+e^-)$	0.88	5.26	0.99	3.70	1.34	4.72
$\Gamma(2S \rightarrow e^+e^-)$	0.44	2.37	0.54	2.57	0.59	2.14
$\Gamma(3S \rightarrow e^+e^-)$	0.31	1.56	0.41	2.10	0.44	0.75
$\Gamma(4S \rightarrow e^+e^-)$	0.25	1.16	0.35	1.85	0.24	0.47
$\Gamma(1S \rightarrow \text{hadrons})$	59.51	93.60	68.00	150.00	32.00	58.00

Table IV - Radiative transitions in bottomonium and charmonium, in KeV.

	Nonrelativ.[5]		Relativ.		Exp.[6]	
	$b\bar{b}$	$c\bar{c}$	$b\bar{b}$	$c\bar{c}$	$b\bar{b}$	$c\bar{c}$
$\Gamma_{E_1}(2^3S_1 \rightarrow \gamma 1^3P_2)$	2.0	40.0	1.872	25.83	0.7 ± 0.9	17 ± 5
$\Gamma_{E_1}(2^3S_1 \rightarrow \gamma 1^3P_1)$	2.0	57.8	1.898	38.46	1.6 ± 0.8	19 ± 5
$\Gamma_{E_1}(2^3S_1 \rightarrow \gamma 1^3P_0)$	1.1	64.7	1.218	46.81	1.0 ± 0.7	21 ± 6
$\Gamma_{E_1}(1^3P_2 \rightarrow \gamma 1^3S_1)$	49.5	601.7	40.763	493.83		330 ± 170
$\Gamma_{E_1}(1^3P_1 \rightarrow \gamma 1^3S_1)$	43.2	436.7	35.353	359.00		< 700
$\Gamma_{E_1}(1^3P_0 \rightarrow \gamma 1^3S_1)$	36.9	206.7	28.064	162.69		97 ± 38

Our rcsults are shown in Tablc III. For liglit mesons, tlie hadronic decay width is calculatcd from QCD without perturbativc corrcctions.

Tlic cxprcssions for tlie electromagnetic transitions in tlic clcctric dipole approximations are given by[8]

$$\Gamma_{E_1}(2^3S_1 \rightarrow \gamma + 1^3P_J) = \frac{4}{3}\frac{(2J+1)}{9}\alpha e_q^2\omega^3 \quad (9)$$
$$\left| \int_0^{+\infty} dr R_{1P}(r) r^3 R_{2S}(r) \right|^2,$$

and

$$\Gamma_{E_1}(1^3P_J \rightarrow \gamma + 1^3S_1) = \quad (10)$$
$$= \frac{4}{9}\alpha e_q^2\omega^3 \left| \int_0^{+\infty} dr R_{1S}(r) r^3 R_{1P}(r) \right|^2,$$

wlicre *w* is tlie energy of tlic pliotoii emitted and $R(r)$ is tlic normalized radial wave functioii. The rcsults for charmonium and bottomonium familics are shown in Tablc IV.

III. Conclusion

Comparison of tlic results obtained in this work with more rccciit experimental rcsults[6] sliows tliat this model dcscribcs well tlic meson inass spectra. The greatest discrepnncy takes place in the $u\bar{u}$ aiid $u\bar{s}$ systems. For tlie system uü, tlie resonances $\rho(1450)$ and $\rho(1700)$ are interpreted *as* 2S aiid 3S statcs, respectivcly, whereas for tlie $u\bar{s}$ system tlie resoiiances $K^*(1370)$ and $K^*(1680)$ are interpreted *as* 2S and 3S, rcspectively. Until *1988* tlie experimental results had furnished evidence tliat tlic rcsonance $\rho(1600)$ was a 2S state of tlie uü systcm. Since then, it is believcd tliat tliis resonance is a supcrposition of two others, E(1450) and E(1700). Table II sliows tliat our results are compatible witli otlier approaches.

For tlie bottomonium, the leptonic decay widths present a better agreement with the experimental results than in tlie nonrelativistic case. Tlie same docs not happen witli the results for cliarmonium. As a matter of fact, this is expected since a nonrelativistic a p proximation was done in tlie calculations of tlie leptonic decay widths and consequently the rcsults for bottomonium are favored. With tlie approach uscd here we do not obtain compatible rcsults between experimental aiid tlieorctical hadronic dccays widths. Iii fact, our rcsults are worse than tliosc obtaincd with tlie nonrelativistic approach[5]. Tliis is probably duc to the way by which thc liadronic dccay widths were calculated Iiere. The crucial differencc is tlic hadronization was made taking as modcl similar calculations in QED for tlie case of tlie three-photon dccay of tlie ortliopositroniuin. A bettcr understanding of tlic hadronization proccss may contribute to improve tlic results.

The radiative transitioii ratcs, in tlic clcctric dipole approximation, furnish practically tlic samc rcsults for tlie bottomonium when compared with tlic nonrelativistic case. Ncvertheless, for tlie charmonium tlic rcsults obtained witli tlie relativistic description are improved. This fact sliows tliat tlic relativistic effects are more important for tlie cliarmonium.

References

1. II. W. Cratcr aiid P.Van Alstine, Phys. Lctt **B100**, 166 *(1981);* II. W. Cratcr and P. Van Alstine, Pliys. Rcv. D36, *10* (1987).
2. I. T. Todorov, Pliys. Rcv. D3, 2351 *(1071).*
3. M. *A.* Stroscio, Pliys. Rep. C**22**, *215* (1975).
4. *E.* Eichten, K. Gottfricd, T. Kinoshita, J. Kogut, K. D. Lane and T. RI. Yan, Phys. Rcv. Lett. 34, *369 (1975);* II. F. de Carvallio, R. Chanda and **A.** B. d'Oliveira, Lett. Nuovo Cim. 22, 679 *(1978);* II. F. de Carvallio and *h.* B. d'Oliveira, Lctt. Nuovo Cim. 33, *572* (1982); J. L. Richardson, Pliys. Lctt. B**82**, *272 (1979);* C. Quigg and J. L. Rosner, Pliys. Rep. 56C, 167 *(1979);* A. Martin, Pliys. Lett. **100**B, *511* (1981).
5. **A.** S. de Castro, II. F. de Carvallio and A. B. d'Oliveira, Lett. Nuovo Cim. 43, 161 (1985).
6. Particle Data Group, Pliys. Lctt. B**239**, *1 (1990).*
7. R. Barbieri, *R.* Gatto, R. Kogerten and Z. Komszt, Pliys. Lett. B**57**, *455 (1975);* P. B. Mackenzie and G. P. Lepage, Phys. Rcv. Lctt. **47,** *1244 (1981).*
8. J. D. Jackson, SLAC report No. 198 (1976).