Phase Diagram of a Spin-1 Ising Model with a Random Crystal Field

W. Figueiredo
Departamento de Física, Universidade Federal de Santa Catarina
88049, Florianópolis, SC, Brasil

Received May 20, 1992; revised manuscript received July 27, 1992

The mean field renormalization group is applied to determine the critical properties of a spin - 1 Ising model with a random crystal-field. The random crystal-field is given by a two-peaked distribution probability. As in mean-field calculations, three different types of phase diagrams are obtained as a function of the concentration of crystal-field. Meanwhile, a lower critical concentration is determined above which there is no more a stable ferromagnetic phase at low temperatures.

I. Introduction

In this work the simplest version of the mean field renormalization group by Indekeu et al[1] is applied to determine the phase diagram of a spin-1 Ising model with a random crystal-field. This model was treated recently in the mean field approximation by Benyoussef et al[2] and by Carneiro et al[3]. They considered the following Hamiltonian model

$$H = -J\sum_{i,j} S_iS_j + \sum_i \Delta_i S_i^2, \qquad (1)$$

where $S_i = -1, 0, +1$, and the first sum runs over all pairs of nearest neighbours. The random crystal-field is given by the probability distribution,

$$P(\Delta_i) = p\delta(\Delta_i - \Delta) + (1-p)\delta(\Delta_i). \qquad (2)$$

Benyoussef et al[2] have shown that depending on the values of p the phase diagram in the $(T-\Delta)$ plane can exhibit a single critical line, first and second order transition lines with a critical and a double critical end-point and first and second order transition lines with a tricritical point. Carneiro et al[3], determined the existence of two ferromagnetic phases at low temperatures, except at $p = 1$, which is the Blume-Capel model[4]. Also, one of the ferromagnetic phases was found to be stable at low temperatures, for arbitrarily large values of Δ and at any concentration $p < 1$. The case $p = 1$ was also studied by De Alcantara Bonfim[5] within a mean field renormalization group scheme and he determined a tricritical point in the phase diagram which improved the mean field results. Recently Carneiro et al[6] have performed a ϵ-expansion on this model and concluded that their Heisenberg fixed point is fully stable although it cannot be reached from any physical initial conditions. However, when they disregard replica fluctuations, the mean-field and renormalization group calculations can be reconciled. In the present work it is shown that mean field renormalization group calculations improve the earlier mean field results for the second order lines and give a lower critical concentration p_c above which the ferromagnetic phase is no more stable at low temperatures. This behavior is very similar to that found for the Ising model in a transverse random field[7].

In order to study the critical properties of the model given by eq. (1) we follow the mean field renormalization group approach by Indekeu et al[1] by considering the simplest choice for the clusters, namely the one- and two-spin clusters. This method has been applied to a great variety of systems and quite good results have been obtained even using small block sizes (see also Ref. 8 and references therein). The Hamiltonian for the single spin reads

$$H_1 = -zS_1J'b' + \Delta_1' S_1^2, \qquad (3)$$

where b', assumed to be very small, is the fixed magnetization of its z nearest neighbours. The magnetization m_1 of the single spin, after averaging over the probability distribution given in eq. (2) is

$$m_1 = \left[2(1-p)/3 + \frac{2p'/\exp(-d')}{1+2\exp(-d')}\right] K'zb', \qquad (4)$$

where $K' = \beta J'$, $d' = \beta\Delta'$, $\beta = (k_BT)^{-1}$ and primed quantities were used to indicate scaled parameters of the present Hamiltonian according to the mean field renormalization group.

The usual mean field approximation is obtained by setting $b' = m_1$ in the above equation. In this case, after dropping the primes the critical lines are given by

$$K_c^{-1} = z\left[2(1-p)/3 + \frac{2p\exp(-d)}{1+2\exp(-d)}\right]. \qquad (5)$$

In particular, for large values of Δ, the critical temperature k_BT_c/zJ tends asymptotically to $2(1-p)/3$ for all values of tlie concentration p, except at p = 1. At p = 1 (Blume-Capel model) eq. (5) gives a second order line for all values of tlie anisotropy **(A)** up to a tricritical point at $k_BT/zJ = 1/3$ and $\Delta/zJ = 2\ln(2)/3$. Beyond tliis poiiit, eq. (5) lias no more solutions and the critical line becomes a first order one.

Analogously, the hamiltonian for the two-spin cluster can be written as

$$H_2 = -JS_1S_2 - J(z-1)(S_1+S_2)b + \Delta_1 S_1^2 + \Delta_2 S_2^2, \quad (6)$$

where b is the magnetization of tlie $(z-1)$ nearest-neighbors of each spin of tlie cluster. Again, after averaging over tlie crystal-field distribution the magnetization per spin for tliis cluster can be written as

$$m_2 = \left[p^2 \frac{4\exp(K-2d) + 2\exp(-d)}{1+4\exp(-d)+4\exp(-2d)\cosh(K)} + (1-p)^2 \frac{2+4\exp(K)}{5+4\cosh(K)} + 2p(1-p). \frac{1+\exp(-d)+4\exp(K-d)}{3+2\exp(-d)+4\exp(-d)\cosh(K)}\right] K(z-1)b. \quad (7)$$

According to tlie two-cell mean field renormalization group tlie magnetizations m_1 and m_2 are assumed to scale as tlie symmetry breaking fields b' and b, tliat is, $m_1 = \zeta m_2$ and $b' = \zeta b$. It is tlien obtained a renormalization recursion rclation ainongst K', d', p' and Ir', d, p which is independent of ζ. As it is not possible to determine tlie complete renormalization flow diagram from just tliis single equation its fixed point structure is studied for tlie case $K' = K = K_c$, $d' = d$ and $p' =$ p. From equations (4) and (7) we then obtain

$$p^2 \frac{4\exp(K_c-2d)+2\exp(-d)}{1+4\exp(-d)+4\exp(-2d)\cosh(K_c)} + (1-p)^2 \frac{2+4\exp(K_c)}{5+4\cosh(K_c)} + 2p(1-p)\frac{1+4\exp(K_c-d)+\exp(-d)}{3+2\exp(-d)+4\exp(-d)\cosh(K_c)} = \left[2(1-p)/3 + \frac{2p\exp(-d)}{1+2\exp(-d)}\right]\frac{z}{(z-1)}. \quad (8)$$

Equation (8) describcs the beliavior of the critical temperature as a function of anisotropy for different values of p. Three typical regions of p can be determined. Figure 1 shows the phase diagram in tlie $t = k_BT/zJ$ versus $D = \Delta/zJ$ plane with $z = 4$, for $0 \leq p < 0.8132$. In this case all transition lines are second order and it is observed that as D is very large the ferromagnetic phase is stable.

On the other hand for $0.8132 \leq p < 5/6$, eq. (8) presents two regions where tlie transition lincs are of second order. As shown in figure 2 there is a small region where eq. (8) does not have a solution. Although

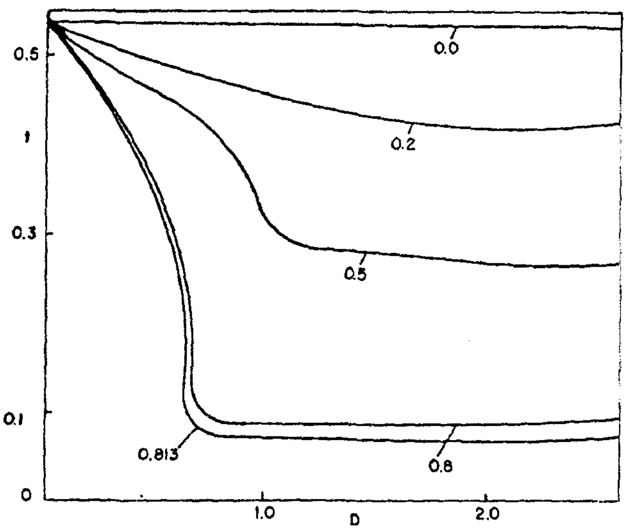

Figure 1: Phase diagram in tlie $t(= k_BT/zJ)$ versus $D(= \Delta/zJ)$ plane for different values of tlie concentration p; $z = 4$.

tlie mean field renormalization group can not give information about first order transitions, it is expected that in tliis region the transition is a first order one. The points A and B, wliere the second order lines end are possibly critical end-points. This type of phase diagram is equivalent to tlie figure 2 of Benyoussef et al[2], although they do not consider the long tail of tlie ferromagnetic phase as was done by Carneiro et al[3]. Figure **3** exhibits the region of the phase diagram wliere $p \geq 5/6$. In this region the second order line ends at a tricritical point and there is no more a long tail in the ferromagnetic phase for large values of D. After a straightforward calculation it is easy to show that tlie critical temperature, for large values of D, is given by

$$t_c = \left[\ln\left(\frac{z/(z-1)-p}{p_c - p}\right)\right]^{-1}, \quad (9)$$

where

$$p_c = \frac{2z-3}{2(z-1)}. \quad (10)$$

If $z = 4$, $p_c = 5/6$ and the figures 1 and 2 show that eq. (9) describes the predicted long tail of the ferromagnetic phase in the mean renormalization group. On the other hand, if $p > 5/6$, figure **3** clearly shows that there is no more a stable ferromagnetic phase at low temperatures, as predicted by Carneiro et **al**[3] in their mean field calculations. Finally, figure 4 shows the behavior of the critical temperature of the ferromagnetic phase as a function of the concentration p. As long as $p > 5/6$, only the paramagnetic phase is stable for large values of D.

Before concluding it would be interesting to point out some features of this simple mean field renormalization group calculation. The exact solution, based

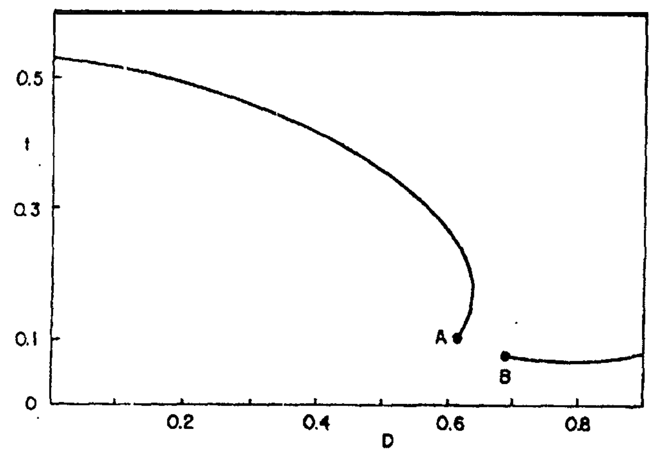

Figure 2: Typical (t,D) phase diagram for $0.8132 \leq p < 5/6$. Here $p = 0.82$, $z = 4$; A and B are critical end-points.

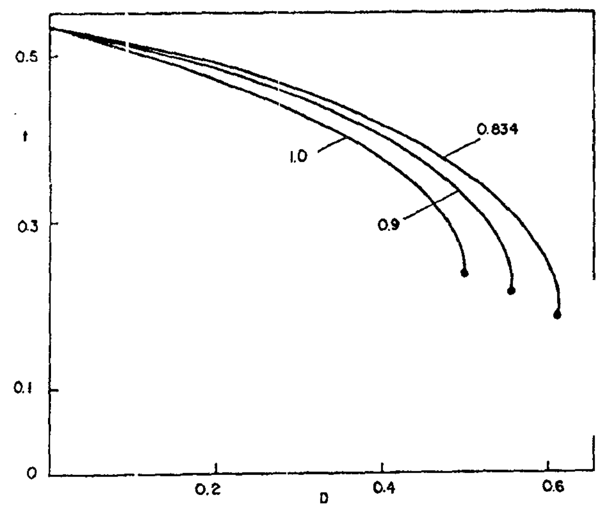

Figure 3: Phase diagram in tlie (t, D) plane for values of $p > 5/6$. Here $z = 4$ and tlie sccond order lines end at tricritical points.

on tlie Curie-Weiss version of tlie mean field[3], gives an expression for the free energy tliat permits to determine all tlie lines of first and second order transitions. With this renormalization group calculation only continuous transitions can be determined. In this way we are not able to determine tlie two coexisting ferromagnetic phases at low temperatures in the figures 1 and 2, as was dore by Carneiro et al[3]. Unfortunately, with only a single order parameter, it is not possible to derive a complete flow diagram, because in this problem it would be necessary to use tliree independent order parameters. Nevertlieless, it is possible to obtain a flow diagram in tlie context of the mean field renormalization group. For instance, for the two-parameter Ashkin-Teller model, where two order parameters can be defined, a complete flow diagram was determined[9] using this mesn field renormalization group. Contrary

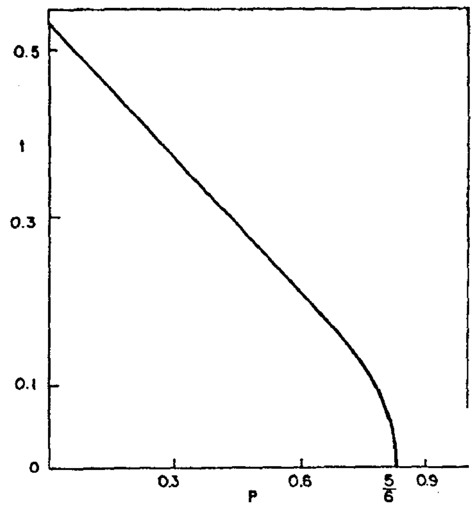

Figure 4: Critical temperature of the ferromagnetic pliase as a function of p for large values of D. If $p > 5/6$, tlie only stable pliase at low temperatures is tlie paramagnetic one.

to tlie mean field calculatioiis[2], where tlie slope of second order line at tlie tricritical point clianges from negative to positive values within a small range of values of p, here tlie slope goes to infinity at tlie tricritical region. In fact, tlie condition tliat $dt/dD \to -\infty$ was used by Benyouseff et al[2] to separate this sniall range of values of p. In tliis work a reentrant plienomenon is observed only for tlie values of p tliat correspond to figure 2. This is the main motivation to identify tlie poiiits A and B in figure 2 as being critical end-points as in the mean field approach.

In this work tlie mean field renormalization group is applied to the Blume-Capel model wlien tlie crystal-field is diluted. Depending on tlie valucs of dilution tliree different typcs of phase diagrams can be obtained. For $p < 0.8132$ tlie transition lines are all of second order with a stable ferromagnetic phase for very large values of anisotropy. Meanwhile, for $0.8132 \leq p < 5/6$, the transition lines are second order, except at a very narrow region of values of anisotropy, where first order transition must be present. For $p > 5/6$, tlie transition line ends at a tricritical point and tlie ferromagnetic phase is absent for large values of the crystal-field. As expected, this behavior is due to the fact tliat the mcan field renormalization group takes into account, to some extent, tlie spin fluctuations whicli are neglected in tlie mean field approximation. Use of bigger clusters of spins improve only a litlle the results of this work as was shown by De Alcantara Bonfim[5] for the pure Blume-Capel model.

Acknowledgements

This work has been partially supported by the

Brazilian Agency Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq). I also thanks Prof. V. B. Henriques for pointing me this problem and to Prof. N. S. Branco for a critical reading of the manuscript.

References

1. J. O. Indekeu, *h.* Maritan and *h.* L. Stella, 3. Phys. A15, 1201 (1082).
2. A. Benyoussef, T. Diaz, RI. Saber and M. Touzani, J. Pliys. C: Solid State Phys. **20**, 5349 (1987).
3. C. E. 1. Carneiro, V. B. Henriques and S. R. Salinas, J. Pliys. **1**, 3687 (1989).
4. M. Blume, V. J. Emery aiid R. B. Griffiths, Phys. Rev. A **4**, 1071 (1971).
5. O. F. de Alcantara Bonfim, Physica A 130, 367 (1985).
6. C. E. I. Carneiro, V. B. Henriques, M. N. Tamashiro and S. R. Salinas, in *Nonlinear Phenomena in Fluids, Solids and Other Complex Systems*, ed. by P. Cordeiro and B. Nachtergaele (Elsevier Science Publ, Amsterdam, 1991) p.351.
7. T. F. Cassol, W. Figueiredo and J. h. Plascak, Phys. Letters A **160**, 518 (1091).
8. J. O. Indekeu, A. Maritan and A. L. Stella, Phys. Rev. B 35, 305 (1987).
9. P. M. C. de Oliveira aiid F. C. Sá Barreto, J. Stat. Phys. 57, 53 (1989).