Modern Spin-Wave Nonlinear Dynamics

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A review of our own experimental and tlieorctical work on high-power ferromagnetic resonance phenomena is given. Four topics are examined, namely, (1) tlie oiiset of spin-wave instabilities, **(2)** tlie oiiset of spin-wave auto-oscillatioiis, **(3)** liiglicr-ordcr bifurcations as transitions to chaos aiid (4) tlie control of spin-wave cliaos.

I. Introduction

Universality lias greatly increased tlie interest in dissipative dynamical systems during tlie last decade. Iii particular, spin waves driven by strong microwave ($1-10\ GHz$) fields in magiietic materials have proven very attract ve.

Tlie birth of high-power ferromagnetic resonance may be traced back to tlie pioneering experiments of Bloembergen, Damon and Wang in tlie early 50s [1]. They have observed a premature saturation of tlie main resonance absorption and tlie appearance of a subsidiary absorption wlien tlie microwavc power exceeded ccrtain thres iold levels. A tlieory by Suhl[2], introduced a few years later, is still tlie basis for explaining tlie phenomena. Sulil's tlieory rclicd upon tlie parainctric excitation of spin-wave pairs via two- and one-photon decay processes. Soon after, it was demonstrated[3] tliat spiii waves could also be parametrically pumped by a microwave field parallel to tlie dc magnetic field, $\vec{h}//\vec{H}_0$. This was called tlie parallel pumping proccss to distinguish from the standard magnctic rcsonancc configuration, $\vec{h} \perp \vec{H}_0$ Here, tlicse tliree processes are generically said to occur when tlie microwave field is iiicreased beyond the Suhl instability (SI) tliresliold, h_c.

Suhl was also tlie first to realize tliat tlie phenomena bore rcsemblance to fluid dynamics. Spin-wave experiments were then observed to exliibit a remarkable effect still in tlie early 60s. At a subscqucnt tliresliold $h'_c > h_c$, a low-frequency ($10\ kHz - 1\ MHz$) oscillation was observed as a iiiodulation iii tlie power absorbed by the sample[4]. This oscillatory instability, or auto-oscillation, was firstly attributed to tlie excitation of standing spin waves iii tlie sample[5], but tliis model was never completely understood[6]. On tlie otlier hand, several Sovict authors[7] had shown tliat tlie auto-oscillatioii could result from tlie nonlinear dynamics of spiii wavcs in an infinite medium but tlie quantitative results of their model were also unsatisfactory.

Spin-wave experimeiits have enjoyed a revival of interest in tlie 80s since it was predicted[8] and experimentally observed[9,10] tliat ferrornagnets exliibit tlie period-doubling route to cliaos at higher power levels ($h > h'_c$). A large number of theoretical and experimental results followed those findings aiid definitely placed tlie spin-wave among tlic most intercsting nonliiicar dissipative dynamical systems to be investigated[11].

In all three parametric processes, the SI tliresliold h_c is determined by tlie rate at which energy inust be pumped into tlie spin-wave mode to overcome tlie powcr dissipated to tlie lattice. Since both the coupling to tlie microwave field and tlie relaxation rate depend on tlie wave vector $\vec{k}$, there is a ccrtain mode for which h_c is a minimum. Increasing tlie microwave field beyond tliis value results in tlic excitation of many otlicr modes aiid complexity increases accordingly. As far as thcory is concerned, we sidestep tliis problein by considering tlie dynamics of only two niodes.

In tlie first part of tliis paper we review tlie experimental situation of nonlinear phenomena associated with spin-wave iiistabilities, including tlie intriguing step-likc behavior of trnnsients prior to auto-oscillation[12] aiid tlie control of spin-wave chaos[13] recently observed. Tlie oiigoing discussion regarding the nature of tlie excited modes is considered in tlie second part, where we point out tlie successes aiid failures of tlic two-mode inodel.

II. Spin-wave experiments

Tlie results presented liere were obtained with yttrium-iron-garnet (YIG) sphcres and films at room temperature, using tlie experimental set-up shomn schematically in Fig.1. Tlie sample is lield at tlie center of a critically coupled TE_{102} microwave cavity with $Q \simeq 2000$, placed between the poles of an electromagnet tliat provides tlie static field $\vec{H}_0$. Tlie cavity can be oriented so that tlie microwave magnetic field $\vec{h}$ is eitlier parallcl or perpendicular (subsidiary resonance) to $\vec{H}_0$. The microwave radiation is generated by a X-band backward-wave oscillator (BWO) and amplified to 2.0 Watt by a traveling-wave tube (TWT). The BWO frequency is stabilized by an external crystal oscillator and

manually adjusted to tlic center of tlie cavity resonance. Tlie radiation power is then controlled witli a variable precision attenuator aiid dirccted by a circulator to the resonant cavity, where it drives spiii waves in tlic sample. Tlie reflected microwave signal is detected witli a sensitive Schottky-barrier diode at tlic output port of tlic circulator aiid recorded in a digital storage oscilloscope or observed with a spectrum analyzer. In order to avoid sample heating tlic microwave racliatioii is pulsed (duratioii 100 μsec at 100 $pulses/sec$) with a PIN modulator placed before the TWT amplifier. This causes no difficulty for rccordiiig tlic stcady-statc rcgiiiic of tlie sample rcsponse because tlic transient tiiiic is normally much shorter than tlic pulse duration.

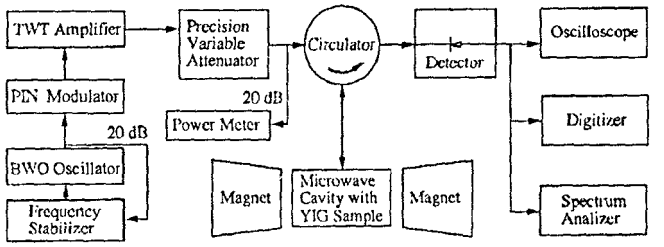

Figure 1: Schematic diagram of the microwave system uscd to excite spin-wave instabilities.

11.1 Tlic oiisct of spin-wave instabilities and auto-oscillations

At low-powcr levels the pulse reflected from tlie cavity lias essentially tlic same shape as tlie incoming microwave pulse (Fig.2(a)). As the power is iricrcascd, an abrupt change occurs at tlic end of tlie rcílcctcd pulse, corresponding to tlic buid-up of a spin-wave mode after a delay time r. Actually, tlic build-up time is infinite at $h = h$, aiid dccrcascs rapidly with increasing field, according to a scaling law $\mathrm{r} \sim (h/h_c - 1)^{-\gamma}$. Fig.2(b) shows tliis stage of parametric excitation in a 1 mm diameter YIG sphere with subsidiary-resonance pumping for $h/h_c \simeq 1.05$. At larger excitation, anotlicr spin-wave mode builds up, as shown in Fig.2(c) for $h/h_c \simeq 1.12$. As h increases further, no otlicr mode appears bcfore tlie onset of auto-oscillation, shown in Fig.2(d) for $h'_c/h_c \simeq 1.7$. Oiily recently[12] we have obscrvcd tlic step-like bchavior shown iii Fig.2, which sheds light on tlie unsolved problem regarding tlic number of modes excited above tlic SI tlircsliold, as we will discuss latcr. However, we stress that tlie overall situation is rather complex. Iii some cases only one step appcars before tlic onset of auto-oscillatioii and in otlicr situations up to three steps have been obscrvcd.

Tlie gross fcaturcs of tlic spiii-wavc instabilities driven in subsidiary resonance are similar to those in parallel pumping. Fig.3(b) shows tlic phase boundaries for spin wave excitation and auto-oscillatioii measured in the sarne 1 mm YIG sphere of Fig.3(a) with $\vec{H}_0$ along tlie [110] axis aiid microwave frequency 8.87 GHz. Instead of a sharp minimum, tlic "butterfly curve" for subsidiary resonance has a broad minimum aiid three characteristic rcgioiis in field H_0. For $H_0 < H_c \simeq$ 1580 Oe tlic modes with lowest tlircsliolcl are plane waves witli $k > 0$ propagating at an angle $\theta_k = \pi/4$ witli $\vec{H}_0$ (or $\theta_k \simeq 35°$ depending oii tlic effect of the relaxation rate). For $H_c < H_0 < H'_c = 2190$ Oe the modes with minimum tlircsliold are magnetostatic waves witli $k \approx 0$. For $H_0 > H'_c$ tlic tlircsliold riscs sharply with H_0 as $\theta_k \to 0$. Siiicc in tlic various field regions tlic modes excited have different nonlinear coupling parameters, a variety of bifurcations leading to self-oscillations can be observed at different field values, as indicated by tlic bouiidary lines in Fig.3(b). In tlie parallel-pumping case, modes with $\theta_k = \pi/2$ aiid wave number $k = [(H_c - H_0)/D]^{1/2}$ are excited at tlie minimum SI tlircsliold li, $= 2\omega_p\gamma_k/\gamma\omega_M$. Here, D is tlie exchange stiffness (D = 5.4×10^{-9} $Oe.cm^2$ for YIG), $\omega_p = 2\pi f_p$ is tlic pumping frequency, $\gamma = g\mu_B/h$ is tlie gyromagnetic ratio, $\omega_M = \gamma 4\pi M$, M is the saturation magnetization ($4\pi M$ = 1750 Oe for YIG at 300 K) and γ_k is tlic phenomenological relaxation rate. Notice tliat this is valid for $H_0 < H_c$ (Fig.3(a)) aiid in tliis field range tlic parallel pumpiiig process enables one to measure γ_k as a function of k.

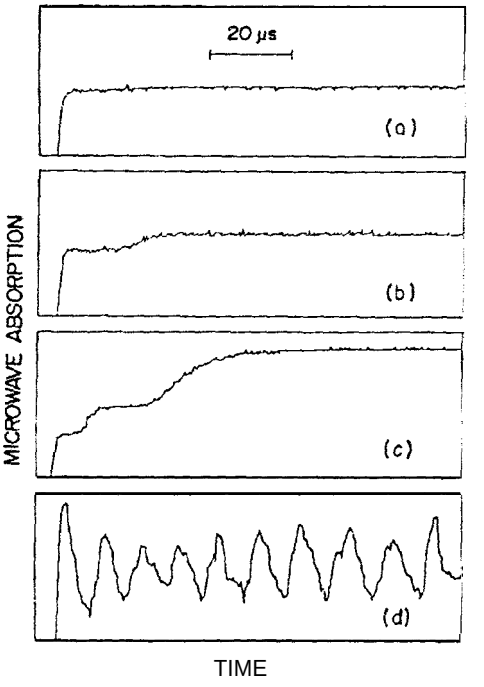

Figure 2: Storage oscilloscope recordings of tlie reflected microwave pulse obtained with subsidiary resonance in a $1mm$ YIG sphere, with $H_0 = 2.0$ kOe along [111] and $f_p = 8.2$ GHz. Tlie various traces corrcspoiid to microwave fields (a) R = $h/h_c \simeq 0.05$, (b) 1.05, (c) 1.12 and (d) 1.7.

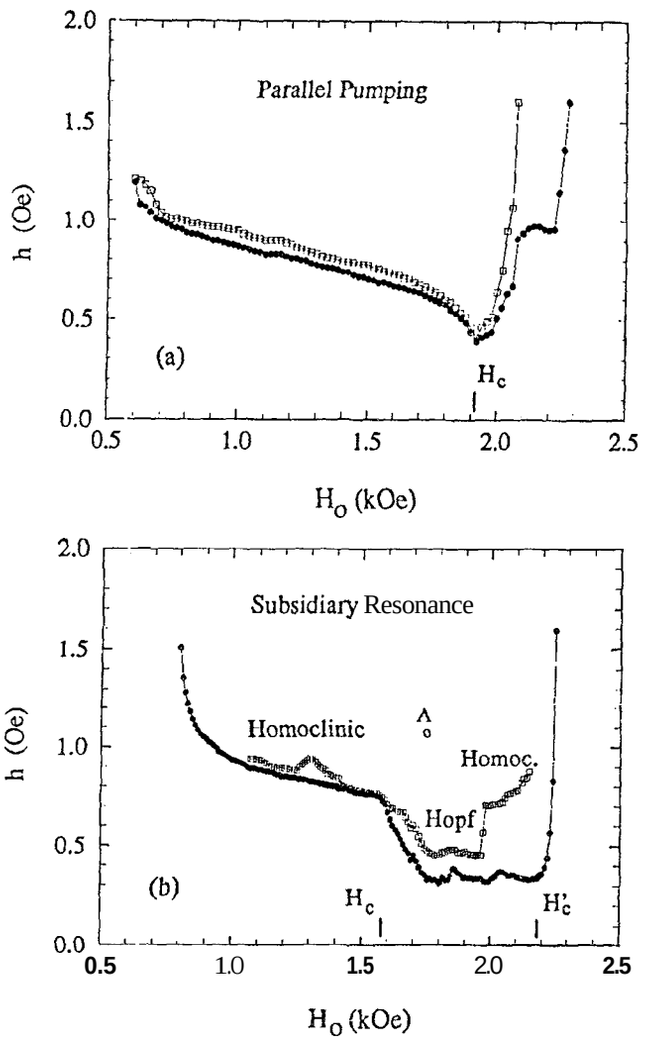

Figure 3: TI reshold microwave fields h, (•) and h'_c (□) versus dc ficld H_0 in a $1mm$ YIG sphere for **(a)** parallcl pumping at f_p = 12.0 GIIz, $\vec{H}_0$ aloiig [111] and (b) subsidiary rcsonance at $f_p = 8.87GIIz$, H_0 aloiig [110].

The auto-oscillations tliat appear at h > h'_c have shape and frequency which depend on tlic pumping lcvcl, dc field value, crystal axis orientation witli rcspect to $\vec{H}_0$ and a so on sample dimensions, surface polishing and temperature. Fig.4(a) shows tlie oscilloscope traces of tlie self-oscillations with increasing pumping iii the parallel pumping configuration at H_0 = 1570 Oe, f_p = 9.5 $GIIz$. Tiie oscillations appear at a field h'_c just above the SI threshold with a finite frequency $f_0 \simeq 220$ $kIIr$ and vanishing amplitude A. As h increases the amplitude iiicreascs smoothly while llie frequency stays constant. Tlie detailed data reveal tliat A varies witl tlie reduced driviiig field r = $h/h'_c - 1$ in tlie critica region as A $\sim r^\beta$, where $\beta = 0.5$. This is typical of a Hopf bifurcation and is analogous to a second-order phase transition[14–17].

Quite different behavior is shown iti Fig.4(b) by tlie oscilloscope traces obtained in subsidiary resonance pumping with $\vec{H}_0$ aloiig tlie [110] axis of tlie YIG sphere, as in Fig.3(b), with H_0 = 1950 Oe, f_p = 8.87 $GIIz$. In this case the auto-oscillation sets in with finite amplitude aiid vanishing frequency, in analogy with a first-order phase transition and in marked contrast witli tlie scenario of Fig.4(a). This has been identified as a homoclinic bifurcatioii [16,17].

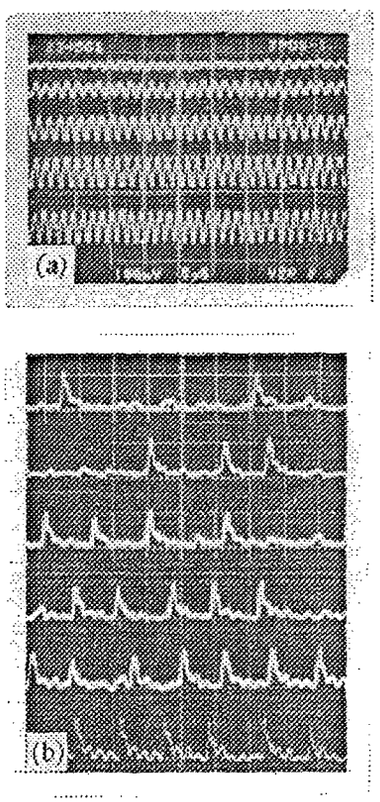

Figure 4: Oscilloscope traces showing tlie behavior of tlie auto-oscillatioii with increasing microwave ficld. (a) Hopf bifurcation observed witli parallel pumping in a YIG sphere, H_0 = 1570 Oe along [111], $f_p = 9.5$ $GIIz$, aiid R = 1.224, 1.255, 1.293, 1.321 aiid 1.342 from top to bottom. (b) Homoclinic bifurcation observed in subsidiar resonance witli tlic same sphere, H_0 = 1950 Oe along [110] and R = 1.862, 1.872, 1.883, 1.894, 1.905 aiid 1.916 from top to bottom [Ref.17].

In Fig.5 we experimentally characterize the onset of auto-oscillation by measuring tlie rntio of tlie fundamental harmonic to the relaxation rate, f_0/γ_k, for YIG spheres (Fig.5(a)) aiid films (Fig.5(b)) with different dimensions. Tlic results unequivocally slioiv tliat tlie sample dimension plays a role in determining tlie auto-oscillation frequency for tlie k values accessible in parallel-pumping experiments. The higher the dimension, tlic lower tlic ratio f_0/γ_k.

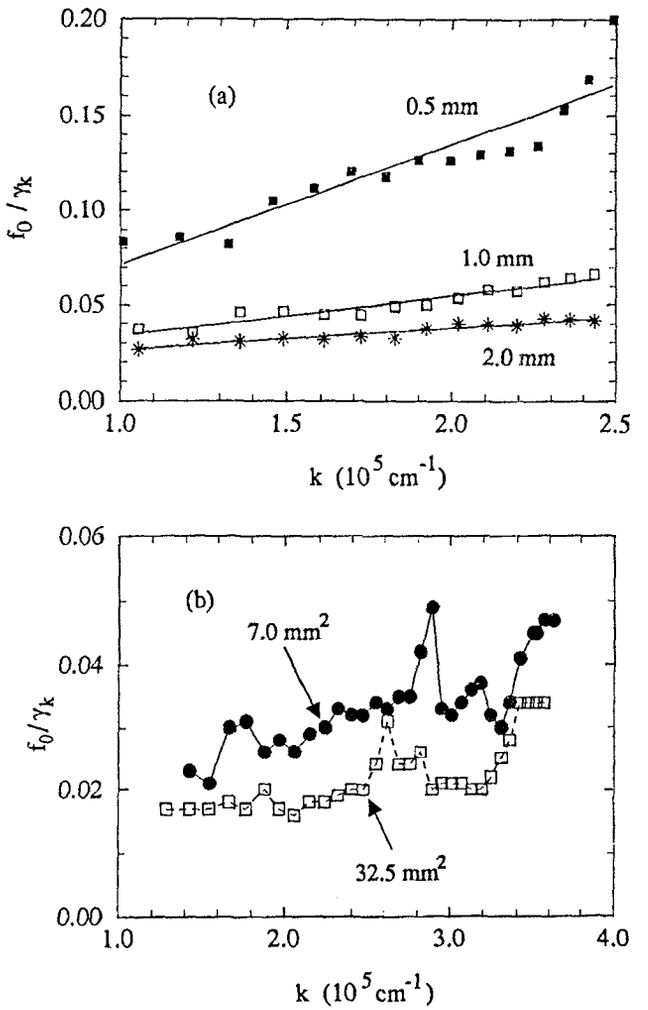

Figure 5: Ratio of tlie auto-oscillation frequency f_0 to tlie relaxation rate γ_k at threshold, as a function of k for YIG spheres (a) and films (11) with different dimensions, obscrved in parallel pumping experiments. Tlie solid lines in (a) are linear fits and in (b) are guide to tlie eycs.

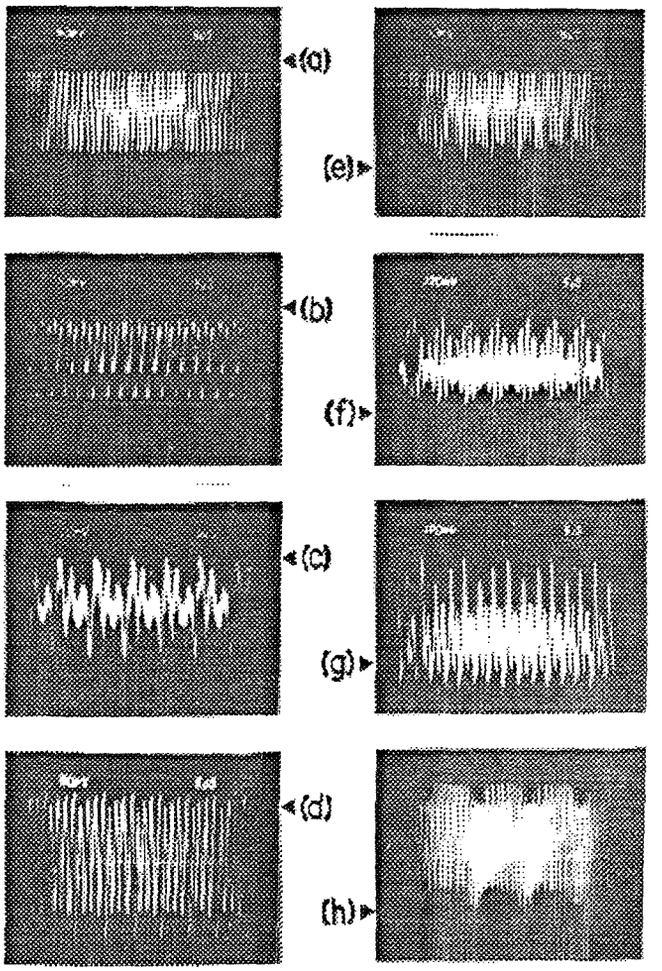

Figure 6: Oscilloscope traces showing (a) period-1, (b) period-2, (c) period-3, (d) period-4, (e) period-5, (f) period-5, but with different pattern from (e), (g) period-6 and (h) quasiperiodic oscillations observed in subsidiary-resonance experiments in a YIG sphere for different combinations of crystal orientation, H_0 and R [Ref.18].

II.2 Higher-order bifurcations

The nonlinear signature of the spin-wave auto-oscillation is clearly demonstrated at higher power levels ($h > h$:). Universal beliavior sucli as relaxation oscillations, period multiplication, quasiperiodicity, intermittency, cham and chaotic transieiits lias been reported by many authors[11], in all three paramctric processes described above. Ilere we givc a fcw exainples of this rich dynamical beliavior of spin waves, observed in the perpendicular pumping of the subsidiary resonance in the YIG sphere for whicli tlie tlireshold fields $h_c(H_0)$ and $h_c'(H_0)$ are given in Fig.3(b).

Let us define $R \equiv h/h_c$ as the control parameter. In general, the auto-oscillation amplitude and frequency increase with R. For ccrtain values of H_0 and crystal directions we may observe tlie appearance of even or odd subharmonics or unlocked (quasiperiodic) oscillatory regimes, as depicted in Fig.6 [18]. Ordcred transitions to châos may also be observed either witli fixed H_0 and varying h or with fixed h and varying H_0. The period-doubling route to chaos is nicely seen in Fig.7 for $H_0 = 1950$ Oe and $2.12 \leq R \leq 2.54$ [19]. Chaotic behavior in spin-wave experiments has been characterized as low-dimensional chaos[19a,19b], indicating that few modes participate in tlie spin-wave nonlinear dynainics, evcn in tlie regime of hyperchaos[19c].

Another iiitercsting scenario is sliown in Fig.8 for h = 0.46 Oe and the sample oriented with tlie [111] direction along $\vec{H}_0$. In this case, an intermittent transition to chaos is seen with decreasing H_0, with val-

ues 1900, 1868, 1860, 1850 and 1840 Oe, from bottom to top, rrspectively. This scenario has been investigated in the light of the theory of crisis in dissipative systems[20,21,22] and quantitative measurements of the distributions of the laminar periods duration and tlie dependence of the mean time on tlie control parameter are underway.

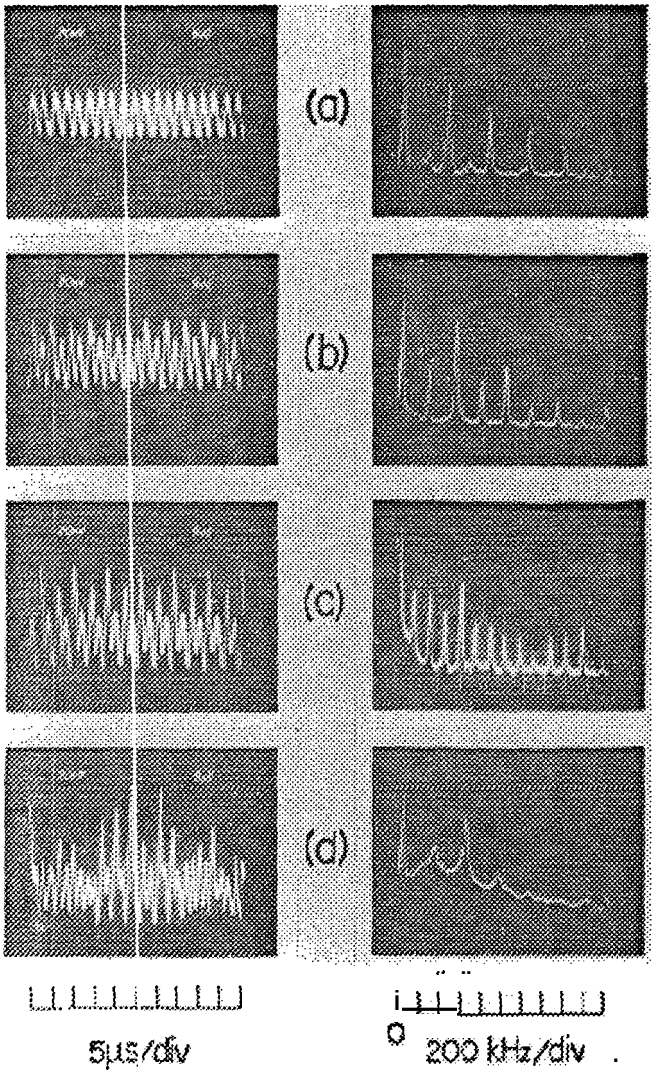

Figure 7: Oscilloscope traces showing (a) period-1, (b) pcriod-2, (c) period-4 and (d) cliaotic oscillations aiid corresponding power spectra observed in subsidiary-resonance experiments in a YIG spliere at R = 2.12, 2.17, 2.29 aiid 2.54, respectively [Ref.19].

11.3 Controlling spin-wave chaos

Tlie feas bility of controlling cliaos by a small time-dependent perturbation of tlie biasing magnetic field applied to ,he sample was recently demonstrated[13]. Tlie method is based on the fact that a chaotic attractor usually lias embedded within it an infinite number of unstable periodic orbits. By applying a small time-dcpendent niodulation to a conveniently chosen paramcter one car stabilize some of these orbits to achieve control of the chaotic state, as predicted by Ott, Grebogi aiid Yorke[23].

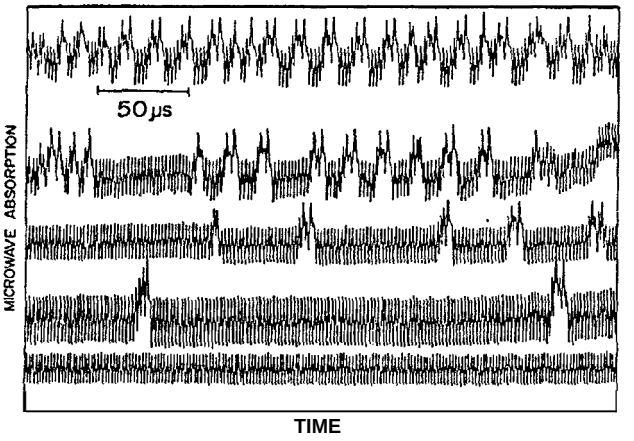

Figure 8: Intermittent transition to cliaos observed in subsidiary-resonance experiments in a YIG spliere with h = 0.46 Oe and $\vec{H}_0$ along [111]. H_0 = 1900, 1868, 1860 and 1810 Oe from bottom to top, rcspectively [Refs.20,21].

Tlie control of spin-wave cliaos was acliieved in tlie subsidiary resonance configuration witli the 1 mm YIG spliere witli tlie [110] direction aligned witli $\vec{H}_0$. We have added inside the microwave cavity a loop of diameter 1.5 cm made witli a 0.5 mm copper wire to allow the modulation of the static magnetic field, $H = H_0 + \delta H\ cos(2\pi f_m t)$, over a broad frequency range ($0 < f_m < 10\ MHz$), typically witli $\delta H/H_0 \sim 10^{-4}$. Tlie results presented here were observcd after tlie system lias been driven to a fully cliaotic regime with h = 1.1 Oe and H_0 = 1750 Oe, represented by tlie point A in Fig.3(b). Fig.9(a) shows tlie power spectrum of a chaotic auto-oscillation for $\delta H = 0$, displaying a cliaractcristic broadband feature. By increasing δH, tlie spectrum becomcs progressively cleaner, with sliarp lines characteristic of a periodic signal. Fig.9(b) shows tlie rcsult for $\delta H = 0.435\ Oe$ and $f_m = 1480\ kHz$. Tlie spectrum in this case corresponds to a quasiperiodic signal with fundamental frequencies $f_0 = 740\ kHz$ aiid $f_1 = 1975\ kHz$, and a subharmonic component at $f_0/2$. In Fig.10 we show the variation of the critical amplitude δH^* necessary to control cliaos witli tlie modulation frequency f_m measured at point A in Fig.3(b). Notice that δH^* lias minima at values commensurate with the fundamental frequency f_0, i.e., $f_0/f_m = p/q$, where p and q are integers. In addition, for the f_m range shown in Fig.10, the minima are strikingly ordcred according to the devil's staircase, i.e., the minimum detected between p/q and p'/q' is characterized by the ratio $(p + p')/(q + q')$.

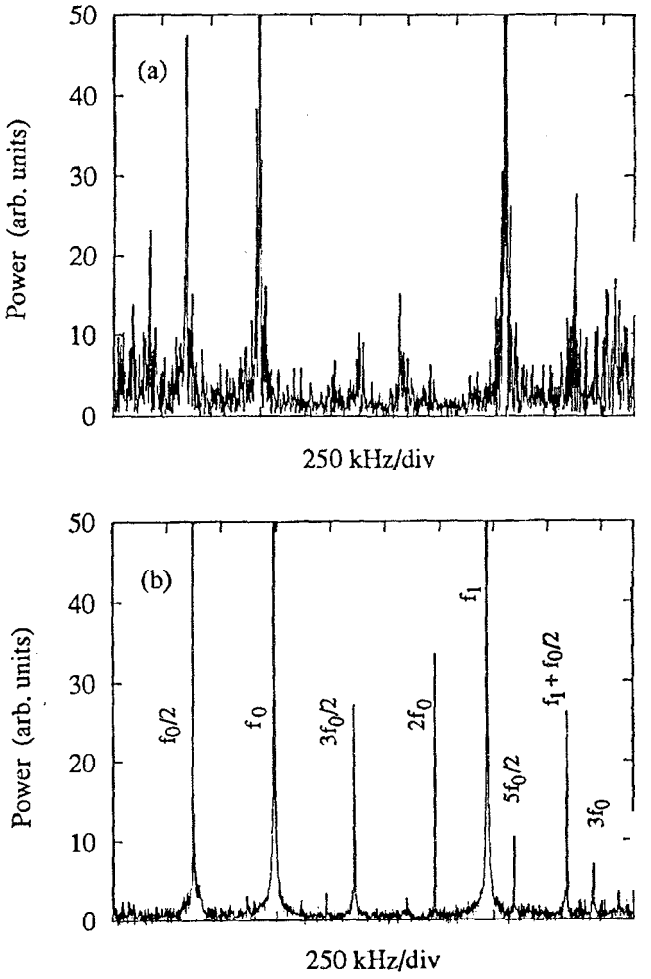

Figure 9: Power spcctra of auto-oscillations observed at point A of Fig.3b for different valucs of tlie amplitude δH of tlie field modulation. (a) Cliaos for $\delta H = 0$. (b) Cliaos under control with $611 = 0.435\ Oe$ and $f_m = 1480\ kHz$. [Ref.13].

III. Theoretical model

Tlie basic pliysical idcas developed by Suhl in tlie lute 50s [2] regarding tlic theory of high-power ferromagnctic resonance retain tlicir validity nowadays. However, despite the recent progress in the understanding of spin-wave nonlinear dynamics, a definitive correspondence between experiment and tlieory remains elusive, even in the pre-diaotic regime[12]. In this section we give a brief review of the two-mode-model (TMM) approacli and point out tlie hitherto unexplained features of the experiments. A detailed dcscription of the TMRI might be found in refs. [11] and [17].

The Hamiltonian for a spin-wave system driven by a microwave field is given by

$$\mathcal{H} = \mathcal{H}^{(2)} + \mathcal{H}^{(4)} + \mathcal{H}'(t), \qquad (1)$$

where

$$\mathcal{H}^{(2)} = \sum_k \hbar\omega_k c_k^\dagger c_k, \qquad (2)$$

is tlie Hamiltonian for a system of independent harmonic oscillators (magnons) with frequency ω_k, $\mathcal{H}^{(4)}$ represents tlie four-magnon iiiteraction and $\mathcal{H}'(t)$ describes tlie interaction with tlie microwave field. Tlie expectation value of the magnon operator $< c_k^\dagger >$ is proportional to tlie transverse-precessing magnetization $m^+ =$ m, $\circledS\, im_y$. Since m^+ gives tlie transverse (circularly polarized) microwave susceptibility of the sample, it is related to tlie amplitude-modulation signal in tlie subsidiary resonance experiment. In tlic parallel-pumping configuration, tlie signal is related to tlie z-component of the magnetization and thus to tlie magnon occupation number $n_k =< c_k^\dagger c_k >$.

The nonlinear coupling of parametric magnons is provided by the four-magnon iiiteraction, which has relevant terms of tlic form [7]

$$\mathcal{H}^{(4)} = \hbar \sum_{k,k'} \left(\frac{1}{2} S_{kk'} c_k^\dagger c_{-k}^\dagger c_{k'} c_{-k'} + T_{kk'} c_k^\dagger c_{k'}^\dagger c_k c_{k'} \right). \qquad (3)$$

In general, exchange, dipolur and aiiisotropy interactions contributc to $\mathcal{H}^{(4)}$. For small values of k ($\sim 10^5 cm^{-1}$), such as YIG pumped by microwave fields, tlie contribution from tlie exchange energy is negligible[24].

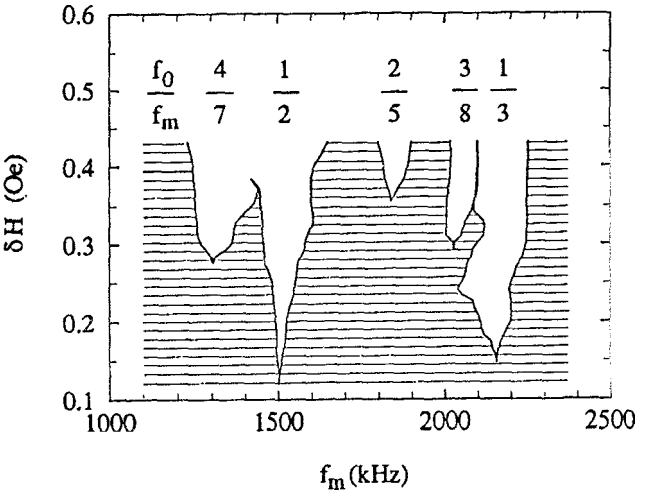

Figure 10: Critical modulation amplitude δH^* vs. inodulation frequency f_m. The boundarics between tlie chaotic and controlled regions have minima at values of f_m commensurate witli tlie fundamental frequency f_0 at ratios indicated at tlie top [Ref.13].

We usually treat the interacting spin-wave system as if it were driven by a uniform microwave field in an infinite mcdium, so that $\mathcal{H}'(t)$ preserves momentum. However, in a finite medium, $\mathcal{H}'(t)$ can be nonzero for pairs $\vec{k}$, $\vec{k'}$ for which k and k' differ by an amount of tlie order of tlie reciproca1 of tlie sample size L. In tliis case, it can be shown that tlie driving term of tlie Hamiltonian becomes, eitlier in parallel or subsidiary-

resonance pumpings,[17]

$$\mathcal{H}'(t) \simeq \frac{1}{2}h\sum_{k,k'}\hbar\rho_k e^{-i\omega_p t}c_k^\dagger c_{-k}^\dagger \cdot (\delta_{kk'} + \alpha_{\Delta k}\delta_{k,k'\pm\Delta k} + \cdots) + H.c., \quad (4)$$

where ρ_k expresses tlie coupling between tlie field $h(t) = he^{-i\omega_p t}$ and the spin wave. $a_{\Delta k}$ is a factor which depends on tlie wave-vector mismatch $\Delta\vec{k} = \vec{k} - \vec{k}'$. For instance, for two neigbouring standing waves along x with $\Delta k_x = \pi/L_x$, $\alpha_{\Delta k}$ attains its maximum value $2/\pi \simeq 0.64$. In an infinite medium, $\alpha_{\Delta k} = 0$.

The Hamiltonian (1) leads to 2N coupled nonlinear cquations describing tlie N nearly degenerate modes excited by tlie pumping field. For only two modes, tlie cquations of motion for tlie slowly varying spin-wave variables b:come[17]

$$\dot{c}_1 = (\gamma_1 + i\Delta\omega_1)c_1 - ih\rho_1(c_1^* + \alpha e^{i\beta/2}c_2^*) - i2(S_1c_1^2c_1^* + S_{12}c_1^*c_2^2 + 2T_{12}c_2c_2^*c_1), \quad (5)$$

$$\dot{c}_2 = (\gamma_2 + i\Delta\omega_2)c_2 - ih\rho_2(c_2^* + \alpha e^{-i\beta/2}c_1^*) - i2(S_2c_2^2c_2^* + S_{12}c_2^*c_1^2 + 2T_{12}c_1c_1^*c_2), \quad (6)$$

where $\Delta\omega_k = \omega_k - \omega_p/2$, $S_k \equiv S_{kk} + 2T_{kk}$, $a \equiv \alpha_{\Delta k}$ and $\beta \equiv \beta_{12}$ is tlie phase difference between modes 1 aiid 2 [17].

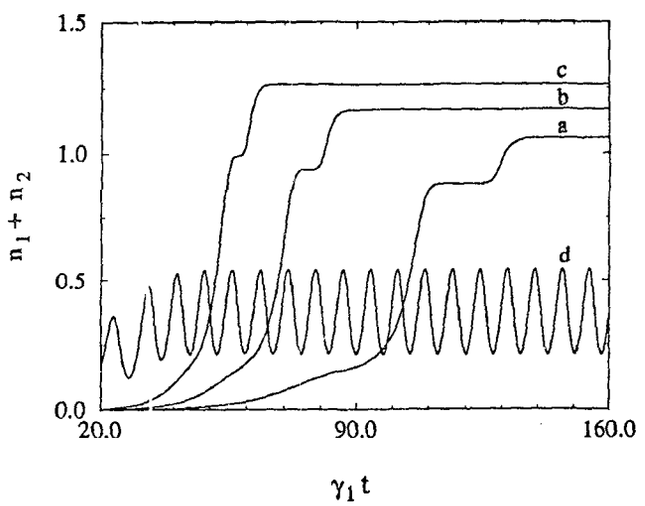

Figure 11: Numerical results of tlie magnon population sum $n_1 + n_2$ obtained frorn eqs.(5) and (6) witli $S_1 = S_2 = -0.5$, $S_{12} = 2.0$, $T_{12} = -1.0$, $\Delta\omega_1 = -0.2$, $\Delta\omega_2$ and $(\alpha, R) = (0, 1.07)$ **(a),** (0,1.09) (b), (0,1.11) (c) and (0.5, 1.20) (d). [Ref.12].

III.1 Numerical solutions of tlic spin-wave equations

Equations (5) and (6) with their 12 independent parameters still represent a very complex mathematical problem. In tlie numerical studies with an arbitrary set of parameters, one usunlly finds that tlic solutions are attracted to stable fixed points. In ccrtain rcgions of tlic parameter space, some fixed points become unstable and tlie solutions may exhibit a variety of dynamic behavior. Due to tlic unclear nature of tlie modes involved in tlie dynamics, it is difficult to relate tlic parameters with tlie microscopic ones[10,24].

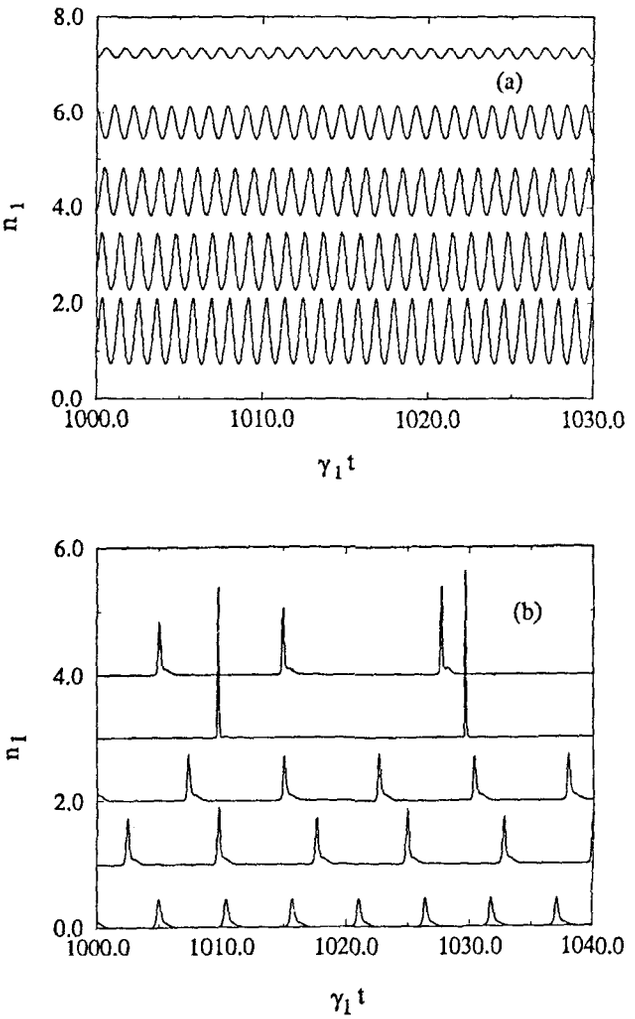

Figure 12: Numerical results for n_1 vs. $\gamma_1 t$ for two different scts of parameters in eqs.(5) and (6) (see text). (a) Hopf bifurcation and (b) homoclinic bifurcation.

Figure 11 shows the transient beliavior of tlie normalized magnon population $n_1 + n_2$ for several values of tlie pumping field $R = h/h_c$, for two modes described by (5) and (6) with normalized parameters $S_1 = S_2 = -0.5$, $S_{12} = 2.0$, $T_{12} = -1.0$, $\rho_1 = \rho_2$, $\Delta\omega_1 = -0.2$ and $\Delta\omega_2 = 0.5$. The tliree step-like curves were obtained with $\alpha = 0$ and $R = 1.07$, 1.09 and 1.11. The auto-oscillation was obtained with R = 1.2 but witli a = 0.5. The curves showing tlie build-up of tlie first mode and subsequently the second witli increasing R, have stricking similarity with the experimental data of Fig.2.

Quite different behavior may occur at the onset of auto-oscillation as we can see in the experiments of Fig.4. Here we show that this can be understood as a result of particular combinations of the parameters in the TMM. Fig.12(a) shows numerical results for n_1 *vs.* $\gamma_1 t$ with $\Delta\omega_1 = \Delta\omega_2 = 0$, $\rho_2 = 0.7\rho_1$, $\gamma_2 = 5.0\gamma_1$, $S_1 = -1.0$, $S_2 = 0.5$, $S_{12} = 2.5$, $T_{12} = 1.125$ and $\alpha = 0$. In this case, at $R = R'_c \simeq 1.37$ a *Hopf bifurcation* into a limit cycle occurs, corresponding to the onset of an auto-oscillation with a finite frequency $f_0 = 0.87\gamma_1$ and a vanishingly small amplitude. As R increases beyond R'_c, the amplitude obeys the scaling law $A \sim (R - R'_c)^{1/2}$, as in the experiments of Fig.4(a). Another remarkable result is shown in Fig.12(b) for $\Delta\omega_1 = (-2\pi)0.3$, $\Delta\omega_2 = (2\pi)0.2$, $S_{11} = S_{22} = S_{12} = 4.078$, $T_{11} = T_{22} = -1.896$, $T_{12} = 0$, $\rho_1 = \rho_2$, $\gamma_1 = \gamma_2$ and $\alpha = 0$ [25]. In this case, as the pumping field increases beyond $R'_c \simeq 1.64$, very-low-frequency aperiodic spiking behavior develops in the amplitude n_1. The (average) frequency increases rapidly with R, in good qualitative agreement with the experiments of Fig.4(b). This phenomenon has been characterized as a *homoclinic bifurcation*[16,17].

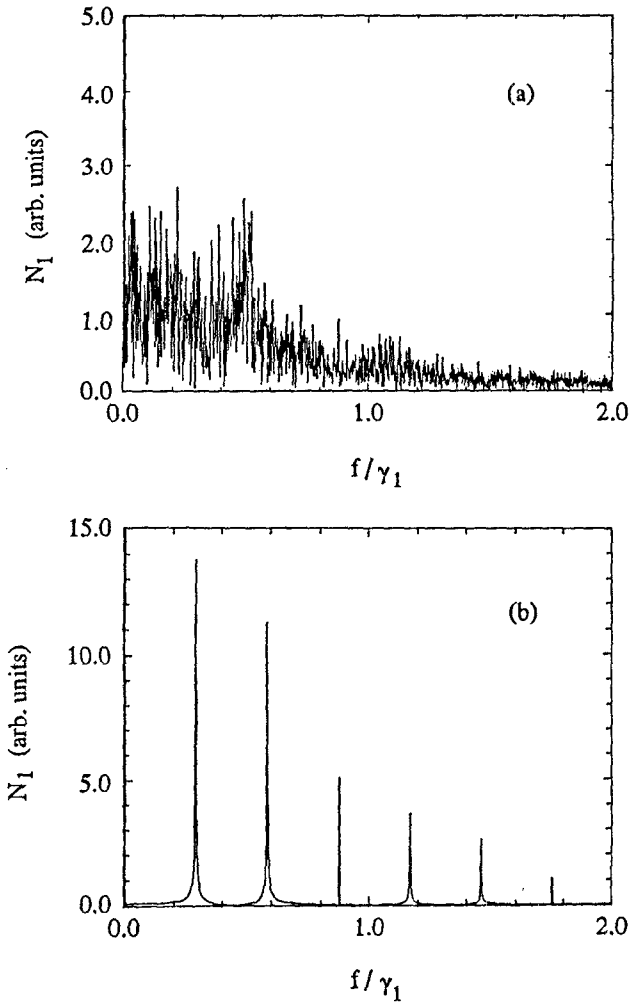

Figure 13: Power spectra of n_1 for two different values of the amplitude Ω of the modulation frequency. Parameters are given in the text. (a) Chaos for $\Omega = 0.6$. (b) Chaos under control with $\Omega = 0.9$ and $f_m/\gamma_1 = 0.875$.

Subharmonic routes to chaos were extensively studied since the introduction of the TMM by Nakamura and co-workers ten years ago[8,11]. Here we present an additional interesting feature of this model, namely, the control of chaos. A periodic perturbation in the static magnetic field $\delta H cos(2\pi f_m t)$, as the one described in the last section, corresponds to a time-dependent parameter $\Delta\omega_k$ in the TMM, $\Delta\omega_k \rightarrow \Delta\omega_k + \Omega cos(2\pi f_m t)$ in Eqs.(5) and (6). In Fig.13 we show preliminary results for the power spectrum of n_1 with $R = 1.81$, $\rho_1 = \rho_2$, $S_1 = S_2 = 1.0$, $S_{12} = -0.4$, $T_{12} = -0.75$, $\beta = 0$, $\alpha = 0.64$, $\Delta\omega_1 = \Delta\omega_2 = -1.0$, and $f_m/\gamma_1 = 0.875$. Fig.13(b) shows that the chaotic state shown in Fig.13(a) for $\Omega = 0.6$, is under control with $\Omega = 0.9$, again in good qualitative agreement with the experiments (Fig.9).

III.2 Discussion

The restriction to only two modes out of an *entire* degenerate manifold paved the way to a better qualitative understanding of the spin-wave nonlinear dynamics. However, the number and the nature of the excited modes still lack a definitive explanation. Considerable efforts have been focused at the onset of spin-wave auto-oscillation in the past few years. Unprecedent quantitative agreement between experiment and the microscopic theory was achieved by McMichael and Wigen[26]. They have used a very thin magnetic garnet film where only a few modes are excited and can be selectively tuned in the experiments. On the other hand, Suhl and Zhang[27] have used powerful tools of nonlinear dynamics to show that even if the entire degenerate spin-wave manifold is excited above thereshold, it organizes itself into new eigenmodes of which only a few are active. Therefore, the dynamics of the system in the vicinity of the oscillation threshold is effectively governed by two modes. We regard the step transients in Fig.2 as a strong support of this idea. However, the quantitative results of Suhl and Zhang presented similar problems to those of the two-mode numerical calculations, namely, too high auto-oscillation frequency and threshold field.

Improvements in the microscopic model have recently been proposed[17,28], through the inclusion of the boundary conditions in finite bulk samples. Although a better quantitative agreement for the auto-oscillation frequency and threshold field has been achieved, our model equations with $\alpha_{\Delta k} \neq 0$ do not seem to account for all details of the experimental results, probably because they do not describe correctly the collective modes[12]. On the other hand, it seems to us that the build-up of discrete modes, as presented in

Fig.2, cannot be described by tlic analytical approach of ref. [28]. Combined microwave and light-scattering experiments[29] should give important additional informations o ı the spatial and temporal patterns in spin-wave experiments. These examples indicate that research in spin-wave nonlinear dynamics is likely to attract attention for years to come.

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