Low Dimensional Semiconductor Structures

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Due to the development of new techniques it has become possible to control the growth of materials atomic layer by atomic layer. Potential profiles and impurity distributions could be made with a dimensional control close to interatomic spacing and with virtually defect free interfaces. The obtained structures, called *microstructures*, have small dimensions and remarkable physical properties. Instead of a complete overview we will discuss some of the most important structures with an emphasis on the concept of band structure engineering. Some of the new effects discovered in these structures are discussed like the integer and fractional quantum Hall effect and Wigner crystallisation.

I. Introduction

In recent years there has been an intense research effort on semiconductor heterojunctions. This field is an excellent example of how basic science and technology interact and influence one another.

The invention of molecular beam epitaxy (MBE) at Bell Laboratories by Arthur and Cho[1] in the sixties has been on the basis for progress in the area of heterojunction structures and quantum devices. Thanks to this epitaxial growth technique it has been possible to realize multilayered heterojunctions with atomically abrupt interfaces and precisely controlled compositional and doping profiles over distances as short as a few tens of Angstroms. Most of the fundamental research is concentrated on the 111-V alloys. Such structures include quantum wells, which are a key building block of band-gap engineering. The transport and optical properties of a serniconductor structure can be tailored to a specific device application.

Continuing advances in MBE and breakthroughs in nanolithography has made it possible to confine the electrons further in more dimensions which leads to *quasi-one* and quasi-zero dimensional semiconductor structures. Quasi-zero dimensional structures behave like artificial atoms. For example quasi-one-dimensional MOSFET in silicon have revealed the universal conductance fluctuations due to random quantum interference predicted by Altshuler et al.[2].

Essential in the physics of these low dimensional systems is their density of states. The latter is the number of available electron states per unit energy. Let us consider a gas of free electrons in different space dimensions. From basic solid state physics we can calculate the density of states (DOS) which is depicted in Table I and which is graphically represented in Figure 1. The DOS determines to a large degree the optical and transport properties of these systems and changes dramatically with the degree of confinement.

Table I: The density of states $D(E)$ per unit of volume, the Fermi energy E_F and the Fermi wavevector k_F of an electron in different dimensions. n_e is the electron density and m* is the electron effective mass.

	3 D	2 D	1 D
$D(E)$	$\frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^*}\right)^{3/2}\sqrt{E}$	$\frac{m^*}{\pi\hbar^2}$	$\frac{\sqrt{2m^*}}{\pi\hbar}\frac{1}{\sqrt{E}}$
E_F	$\frac{\hbar^2}{2m^*}\left(3\pi^2 n_e\right)^{2/3}$	$\frac{\pi\hbar^2}{m^*}n_e$	$\frac{\pi^2}{4m^*}n_e^2$
k_F	$\left(3\pi^2 n_e\right)^{1/3}$	$\sqrt{2\pi n_e}$	$\frac{1}{2}\pi n_e$

The concept of selective doping has been fruitful not only for devices but also for physical studies. Magneto-transport studies on Si/SiO_2 system and in selectively doped $AlGaAs/GaAs$ heterojunctions culminated in the discovery of two striking quantum phenomena in two-dimensional systerns in a high magnetic field: the

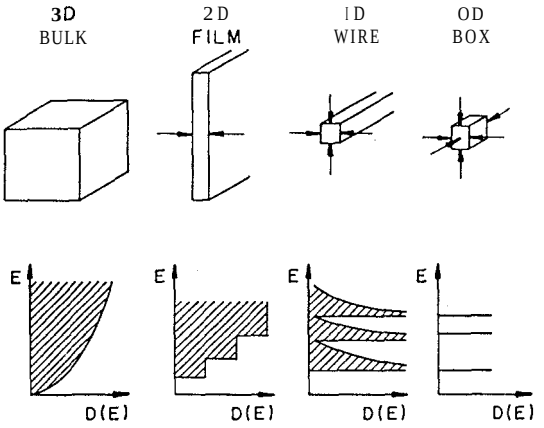

Figure 1.: The density of states D(E) for an electron gas in different dimensions. The arrows in the top part of the figure indicate the directions in which the electrons are confined.

integer[3] and the fractiona1[4] quantum Hall effects. The quantization of the Hall resistance with a precision of one part in 10^7 has lead to a new resistance standard and to a more accurate determination of the fine structure constant which is the coupling strength parameter in electromagnetism. The fractional quantization of the Hall resistance instead, has revealed the existence of a new quantum fluid of correlated spin polarized electrons. These discoveries demonstrate that semiconductor devices can be exceilent tools for the study of fundamental physical phenomena.

From a more practical point of view the resonant tunnelling transistor is an important new device in this field of low dimensional semiconductor structures. The resonant tunnelling bipolar transistor was proposed by Capasso and Kieh1[5] in 1984. This type of transistor allows the implementation of a large class of circuits with greatly reduced complexity. Up to now the progress of integrated circuits has so far been marked by increased levels of miniaturization. Due to interconnection limitations this scaling strategy will probably approach practical limits at a minimum lateral dimension of $0.25\mu m$. When these limits are reached electronics will have to find new paths for its evolution in order to survive. New devices and circuit architectures will be needed.

The best studied system is the two-dimensional electron gas and we will devote a large part of this review on this subject and the physics of this intriguing system. A new development is the creation of even smaller structures where the electron motion is further confined such that the electron has only one free dimension to move in: *quantum wires.* Although already an appriciable body of work has been done on semiconductor quantum wires there is still a lot to be done. From the experimental point of view it has been extremely difficult to make the wires sufficiently uniform and most of the wires are rather broad. Total confinement of the electrons into *quantum boxes* is in its infancy. One expects that when the experimentalists have mastered the technology to make high quality quantum dots (boxes) a new range of interesting physics will appear. For example they are similar to atoms, and are also called *artificial atoms,* but with excitation energies which are several orders of magnitude smaller than in the traditional atoms. Furthermore with these artificial atoms a new table of Mendelev can be constructed in which there will be no upper bound on the number of electrons which can be bound (or rather confined).

The present paper is organized as follows. First we introduce the concept of band structure engineering in Section II which is the basis for electron or hole confinement into low dimensional semiconductor structures. As examples we discuss quantum wells, superlattices, the field effect transistor and heterostructures. As an application of these concepts we discuss resonant tunnelling in Section III. In Section IV special emphasis is paid to the properties of the two-dimensional electron gas (2DEG) which is very important from the fundamental physics point of view. The system of electrons on liquid helium is the purest and most simple example of 2DEG which is discussed in Section V. The important discoveries of the integer and fractional quantum Hall effect in the early eighties is reviewed in Section VI. Finally in Section VII we give a short overview of the next steps in confinement which leads to quantum wires and quantum boxes.

11.1. Band structure engineering

For illustrative purposes we confine ourselves to the most popular, and successful system which is based on the materials *GaAs* and the alloy $Al_xGa_{1-x}As$. The band gap of the latter[6] is $E_g = 1.520 + 1.155x + 0.37x^2$ expressed in eV. With MBE it is possible to grow layers of the alloy *AlGaAs*. After a while one can stop this growth process and start to grow *GaAs*. After growing several layers of *GaAs* one can continue to grow *AlGaAs*. The resulting potential profile for the bottom of the valence and conduction band is shown in Figure 2. Because of the small difference in lattice constant between the two materials (see Figure 3) and the similar chernical composition the interface is very sharp and well defined. The band gap shows a discontinuity at the interface of magnitude $\Delta E_g = (1.155x+0.37x^2)$eV. Experimentally one has found that this discontinuity is as follows: 60% drop in the conduction band and 40% in the valence band. For example for $x = 0.3$ the electrons will be confined in a *quantum* well with a barrier height of $V_o = 228meV$.

The electron is now confined into a square well where the electron will reside mainly in the *GaAs*. Because of the finite width of the well (W) the electron

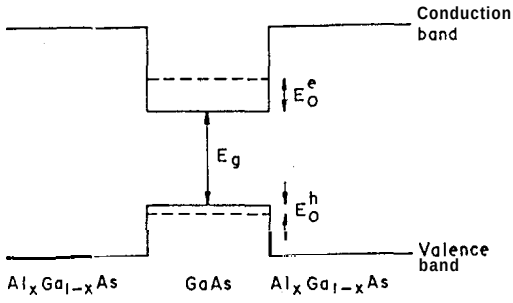

Figure 2.: Bottom of the conduction band and top of the valence band in a $GaAs/AlGaAs$ quantum well. E_g is the band gap in GaAs, and E_o^e and E_o^h are the zero point energies for the electron and hole in the quantum well, respectively.

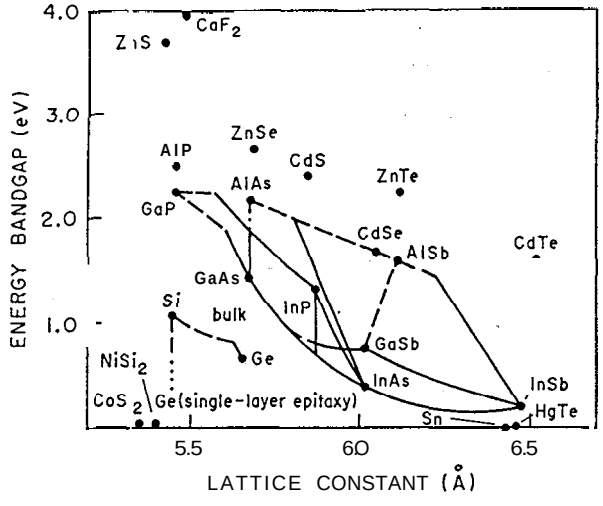

Figure 3.: The energy band gap and lattice constant for different semiconductors.

will have a zero-point energy, which approximately is given by $E_o = \pi^2\hbar^2/2m^*W^2$ (this is the result in the limit of a square well with infinite high barriers). In order to excite electrons into the well we have to create an electron-hole pair by shining light on the quantum well. An electron is created in the quantum well and a hole in the valence band. The energy needed is the band gap eriergy corresponding to GaAs plus the zero point energies of the electron and the hole. Thus the energy at wliich the quantum well starts to absorb light will depend on the properties of the quantum well, i.e. the width of the quantum well. Electrons are created and electrical conduction becomes possible which can be measured electrically. In this way an optical signal can be transformed into an electrical one.

By applying an electric field perpendicular to the well, the position of the energy levels can be varied and, as a consequence, the energy at which aborption starts will be changed. This is the so-called Stark effect and can be used in electro-optical devices to convert electrical signals into optical and visa versa.

The reverse process of emission of light is also possible. By some external means it is possible to put electrons in the quantum well and holes in the corresponding valence quantum well. They will recombine and emit photons. This is the basic principle of the quantum well laser. Such a laser is for example used in the compact disk player. For optical communications, in which optical signals have to traverse optical fibers which have a rninimum loss for wavelengths in the range $1.3 - 1.55, \mu m$ a quantum well laser with this wavelength is needed. Lasers based on the material $InGaAsP$ satisfy this requirement because this material has an energy gap in the correct energy range.

If many quantum wells are grown on top of each other and the barriers are made so thin (typically $< 50\mathring{A}$) that tunnelling between the coupled wells becomes important, a superlattice is formed (see Figure 4). This concept was first proposed in 1969 by Esaki and Tsu[7] at IBM. Superlattices are new materials with novel optical and transport properties introduced by the artificial periodicity.

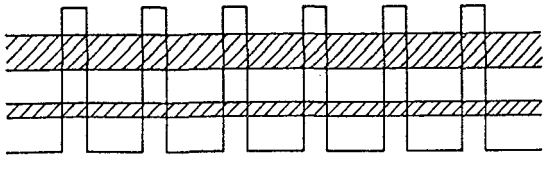

Figure 4.: Superlattice with the corresponding minibands (hatched area).

The rich variety of available combinations of band gaps (see Table I), semiconductor alloys and lattice constants is the main feature of band-gap engineering. As discussed above, we need two materials with a different band gap energy and almost the same lattice constant. The latter is required in order to make sure that the interface does not contain defects, dislocations, vacancies,... If the lattice constants are different then the chemical bonding at the interface will try to impose the same lattice constant for both materials (or at least for the one which is grown on top of the other) which leads to strain. Such systems are called strained-layers. Examples of such systems are: $In_xGa_{1-x}As/GaAs$, $CdTe/MnTe$, $GaP/GaAs_xP_{1-x}$, $Si/SiGe$,...

11.1. Field effect transistor

Dimensional confinement of charge carriers near the interface between two appropriate materials and their usage for device operation has been known for a long time[8]. The best known example being the Si/SiO_2 bases heterostructure transistor. One exploits the discontinuity between the conduction and valence band

energies of Si and SiO_2, combined with the bending of the Si band achieved via appropriate doping and biasing, to create a 'triangular' potential at the interface. Such a potential well gives rise to confinement of the carriers in the direction normal to the interface. When its size becomes comparable to the carrier de Broglie wavelength, new quantized energy states for motion in the normal direction arise. A gas of electrons which is restrained to move in two dimensions, i.e. along the interface, is formed.

Most of the early work on two-dimensional systems was done on insulator-semiconductor heterojunctions. Examples of such structures are MIS (metal-insulator-semiconductor) and MOS (metal-oxide-serniconductor) structures. The most popular and wide spread technological device is the Si-MOSFET (see Figure 5) which was developed in the 1960s and 1970s and used as amplifying and switching devices in integrated circuits. It is today one of the major electronic components in memory and logic circuits used in modern computers and chips.

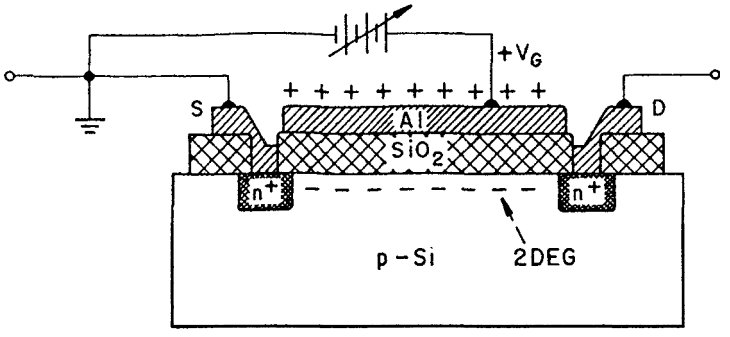

Figure 5.: Schematic view of a Si-MOSFET. The two-dimensional electron gas is located on the Si-side of the Si/SiO_2 interface. Current can flow through the 2DEG by using the source (S) and drain (D) which are electrically connected with the 2DEG through the heavily doped n^+ regions.

11.2. Heterostructures

In more recent times, the same notion has been realized in lattice matched and mismatched combinations of epitaxial heterostructures of compound semiconductors belonging to the 111-V, 11-VI and group IV columns of the periodic table. A particular notion introduced is the controlled placement of dopants in the larger band gap semiconductor (hereafter called the barrier layer) so as to achieve spatial separation between the ionized dopants and the charge carriers in the well created at the interface[g]. This leads to reduction of the ionized impurity scattering of the carriers in the well. In high quality MBE grown $GaAs/Al_{0.3}Ga_{0.7}As$ heterojunctions low temperature electron mobilities near $5.10^6 cm^2/Vs$ have been achieved under suitable conditions using this notion called modulation doping (see Figure 6). Such a heterojunction has been exploited for transistor action and is known under high electron mobility transistor (HEMT)[10].

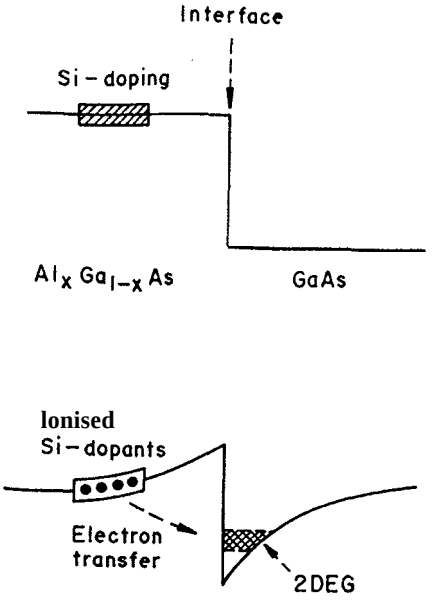

Figure 6.: $GaAs/Al_xGa_{1-x}As$ heterostructure with remote doping. Top figure shows the conduction band before transfer of electrons. The bottom figure is the thermodynamic stable case in which the Si-donors are ionized and a two-dimensional electron gas (2DEG) is formed at the interface.

III. Resonant tunnelling

With modern growth techniques (MBE and MOCVD) it has been possible to fabricate tunnel barriers by band-gap engineering. One can separate two GaAs regions by a thin barrier of $Al_xGa_{1-x}As$, and the tunnel barrier is formed by the conduction band discontinuity. In a similar way two or more barriers can be combined. Now the small gap GaAs layers are actually quantum wells which are weakly coupled one to another. Resonant tunnelling is the quantum mechanical concept that the tunnelling probability is quite low except for energies which equals the energy of the quantum levels in the individual wells (see Figure 7). This opens up the possibility of *negative differential conductivity* in the device characteristics. It is possible to engineer an entire range of tunnelling and quantum structures'' within more normal semiconductor devices in each case producing novel or enhanced performance characteristics.

The concept of resonant tunnelling is illustrated in Figure 7. Consider an undoped double barrier sandwiched between two heavily doped contact layers which provide the electrons. Electrons originate near the

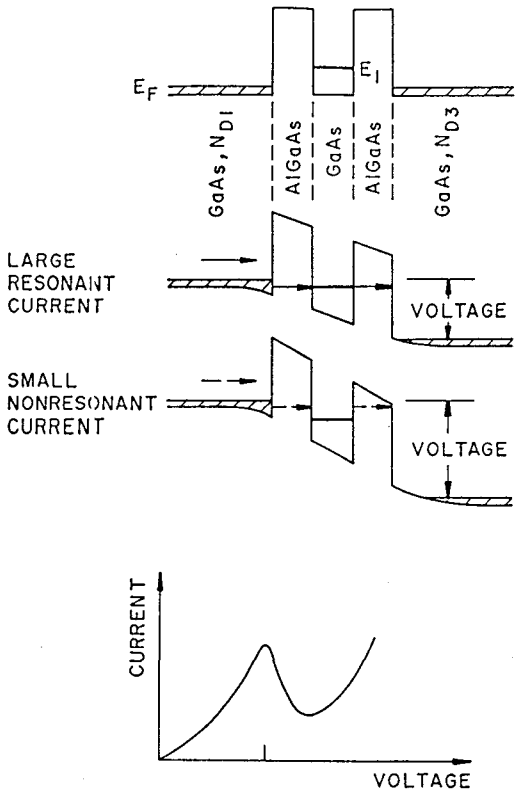

Figure 7.: The bottom of the conduction band in a double-barrier resonant tunnelling structure is shown for increasing bias voltage. The position of the resonant level in the quantum well is indicated by E_1. The bottom figure gives the tunnelling current as function of the bias voltage.

Fermi level to the left of the first barrier and tunnel through the well. Resonant tunnelling occurs when the energy of the injected carriers becomes equal to the energy of the level in the well (E_1 in Figure 7). Maxima occur in the overall transmission through the double barrier and in the current-voltage curves. This negative differential resistance may be useful in ultrahigh-frequency device applications. The resonant tunnelling effect is essentially equivalent to the resonant enhancement of the transmission in an optical Febry-Perot interferometer, provided that scattering in the double barrier is negligible (that is, transport must be coherent).

IV. Two-dimensional electron gas

As indicated above there exist different ways to create a two-dimensional electron gas (2DEG) in a semiconductor[12]. For a quasi-two-dimensional system the electrons (or holes) are confined in one direction (e.g. the z-direction) by a potential which quantizes the energy levels for one spatial dimension but are free to move in the other two spatial dimensions. In the effective mass approximation the electron energy can be then written as

$$E_n(\vec{k}) = E_n + \frac{\hbar^2}{2m^*}(k_x^2 + k_y^2). \qquad (1)$$

The wave number is a good quantum number for two dimensions but not for the third dimension. The carrier motion in the direction perpendicular to the interface cannot be treated classically. Quantum mechanics tells us that the motion in this direction is quantized into discrete levels (E_n). These systems are not two-dimensional in a strict sense, because: 1) the electron wavefunction has a finite extent in the z-direction, and 2) because electromagnetic fields are not confined to a plane but spill out into the third dimension.

The best known examples of 2DEG are carriers confined to the vicinity of junctions between insulators and semiconductors, between layers of different semiconductors, and between vacuum and liquid helium (see next section). One common element is that they have at least one well-defined interface which is usually sharp on a nanometer scale or even less.

There are other (natural) systems in which the carriers also have two-dimensional character. Examples of such systems are layer compounds, intercalated graphite, and thin films. These systems, although interesting in its own right, are not discussed in the present paper.

A two-dimensional conduction band without spin or valley degeneracy has a constant density of states equal to $D_o = m^*/2\pi\hbar^2$, where m^* is the effective mass which we assume to be isotropic. For GaAs the effective mass is $m^*/m_e = 0.067$ and hence the zero field DOS is $2D_0 = 2.8 \times 10^{10}/meV^{-1}cm^{-2}$ including spin-degeneracy. Typically the first excited state of the confining potential is $20meV$ above the ground state and therefore about 5×10^{11} carriers per cm^2 can fill the first electric subband before the second one starts to be occupied.

Applying a magnetic field perpendicular to the 2D plane drastically alters the DOS. The electron motion is completely quantized and leads to discrete energy levels

$$E_{n,\pm} = (n + \frac{1}{2})\hbar\omega_c \pm \frac{1}{2}g\mu_B B, \qquad (2)$$

where $\omega_c = eB/m^*c$ is the cyclotron frequency, ω_B the Bohr magneton, and g the g-factor. From now on we will neglect the small spin splitting of the energy levels which actually happen for example for GaAs-type of structures in the magneto-resistivity for magnetic fields $B < 1-2T$. Each level is highly degenerate because the energy will not depend on the position of the cyclotron orbit center. Solving the Schrödinger equation with the Hamiltonian $H = (\vec{p} + \frac{e}{c}\vec{A})^2/2m^*$ and the vector potential $\vec{A} = (0, -Bx, 0)$ in the Landau gauge gives us the eigenfunctions

$$\Phi_{n,k_y} = e^{iyk_y}\phi_n(x - x_o), \qquad (3)$$

with the harmonic oscillator wavefunction

$$\phi_n(x) = Ne^{-x^2/2l_B^2} H_n(x/l_B), \quad (4)$$

where N is a normalization constant and $l_B = \sqrt{c\hbar/eB}$ the magnetic length which equals $l_B = 256.56\text{Å}/\sqrt{B(T)}$ where the magnetic field is expressed in Tesla. The momentum k_y and the center of the cyclotron orbit x_o are related through $x_o = l_B k_y^2$. Naively each cyclotron orbit occupies an area πl_B^2 which tells us that in an area of $L_x L_y$ we can fit about $L_x L_y/\pi l_B^2$ orbits and consequently there are L, $L_y/(2\pi l_B^2)$ states per Landau level (the two originates from spin). Each Landau level contains $N_L = eB/hc$ states. Notice that N_L does not depend on sample parameters, the Landau level number n or any broadening of these levels.

For a given Fermi energy (E_F) the number of fully occupied Landau levels is n = $int(E_F/\hbar\omega_c)$ = $int(n_e/(2eB/h))$. The Fermi energy of a 2DEG is given by $E_F = \pi\hbar^2 n_e/m^* = 3.57 \times 10^{-11} n_e(cm^{-2} meV)$ and n_e is the electron density.

A common way to define the state of the system at high magnetic fields is in terms of the occupancy factor v, which counts the number of filled levels

$$\nu = \frac{n_e}{N_L} = \frac{h}{eB} n_e, \quad (5)$$

where we counted spin states separately.

In the quantum limit, defined by $\omega_c\tau > 1$ the free-electron model breaks down as the Landau level structure of the single electron density of states becomes resolved. This structure is much more dramatic in two-dimensional systerns, in which energy gaps exist in the density of states for ideal 2D systerns. The number of Landau levels filled, v, is proportional to n_e/B. For this reason, many physical phenomena which depend upon the DOS at the Fermi level, such as specific heat, magnetization and magneto-transport exhibit oscillations periodic in n_e or $1/B$.

V. Electrons on a liquid helium surface

Electrons above a liquid helium surface form the cleanest example of a two-dimensional electron gas. The electrons float in vacuum and are scattered only by the heliurn gas atorns above the surface and by possible liquid helium surface excitations (called ripplons). This leads to large electron mobilities, large scattering times and large mean free paths. Because the electrons float in vacuum there are no complications due to band structure effects. The simplicity of this system has made it a testing ground for different theories.

The reason why electrons can float above a liquid helium surface and will not fill the three dimensional space can be understood as follows. An electron a distance z above the helium surface polarizes the helium atoms of the liquid which results in an attractive force towards the helium surface. Mathematically this polarization can be described by an image charge R at a distance z inside the helium. $\Omega = (\epsilon_{He} - \epsilon_v)/2(\epsilon_{He} + \text{E,}) = 0.014$ where $\epsilon_{He} = 1.0572$ is the dielectric constant for He and e, = 1 the one of vacuum. A repulsive barrier of strength V, $\sim 1.0eV$ prevents the electron to penetrate into the liquid helium. The image potential becomes $V(z) = -\text{Re}^2/2\text{z}$ the potential of a one-dimensional hydrogen atom with a strongly reduced nuclear charge of $\Omega/2 = 1/144$.

The interest in this system stems from the interest in 2D phase transitions. In 1934 Wigner[13] suggested that electrons in a three-dimensional Fermi system crystallize if the electron density can be made sufficiently small such that the Coulomb energy dominates the Fermi energy. Such an electron crystal is called a Wigner *crystal*. No clear observation has been made which demonstrates the existing of such a three-dimensional Wigner crystal. In 1971 Crandall and Williams[14] realized that such a Wigner lattice could probably be realized more easily in the 2D system of electrons on a helium surface. Such a Wigner crystal of electrons was observed by Grimes and Adams[15] in 1979. There is an essential difference between Wigner crystallization in two dimensions with respect to Wigner crystallization in three dimensions, namely in 2D the crystal is formed when the electron density is increased while in 3D the electron density has to decrease in order to reach the Wigner crystal phase. This shows clearly the importance of the dimensionality of the electron gas.

Another reason for the interest in this system is that it is a simple example of a field theoretical problem with variable coupling constant. An electric field perpendicular to the helium surface can push the electrons closer to the helium surface. In this way the interaction between the electrons and the ripplons can be tuned. This interaction is analogous to a field theoretical problem in which a scalar particle (the electron) interacts with a field (the ripplons) in which the interaction strength can be varied continuously. In the present case one can go from weak to the strong coupling limit in which the electron is *self-trapped*[16]. Experiinentally this transition is observed[17] as a dramatic change of the mobility: a drop of the mobility with at least 5 orders of magnitude.

From Table II it is apparent that the 2DEG on a liquid helium is in a different region in the physical parameter space than the 2DEG in semiconductor systems. Electrons on He are a *classical* 2DEG while electrons in semiconductor systems behave typically as a *quantum* 2DEG. The electron density above a He film is limited because of the formation of a surface instability at high electron densities. Only when the He film thickness is decreased the surface becomes more stable and higher densities are possible. Electrons on thin liquid *He-films* may bridge the gap between the two different regions.

Table II: Some physical parameters for a two-dimensional electron gas in different systems.

	Electrons on He	Si-MOS	$GaAs/Al_xGa_{1-x}As$
$E_B(meV)$	0.7	5-50	20-40
$< z >(Å)$	114	30	50-100
$n_e(cm^{-2})$	10^5-10^9	10^{11}-10^{13}	10^{11}-10^{12}
$E_F(meV)$	10^{-7}-10^{-3}	1-50	20-100
m^*/m_e	1.0	0.19	0.066
τ(sec)	10^{-7}	10^{-12}	10^{-12}-10^{-9}
$\mu(cm^2/Vs)$	10^7	10^3	10^4-10^6
new effects discovered	Wigner lattice Self-trapping	QHE	FQHE Wigner lattice

VI. Quantiim Hall effect

A very important experiment to characterize semiconductors is the Hall effect. A schematic view of such an experiment is given in Figure 8. A current is driven through the sample in the presence of a magnetic field which is perpendicular to the 2DEG. One measures the Hall voltage $V_H \sim E_y$ and the longitudinal voltage $V \sim E_x$.

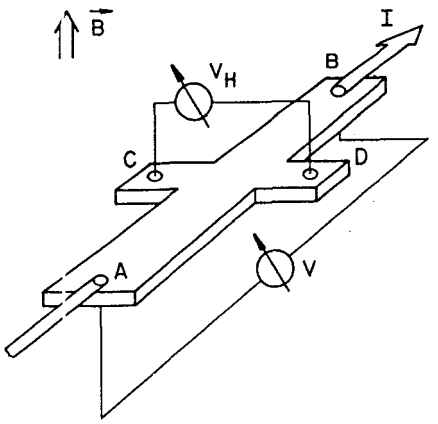

Figure 8.: Principle of the Hall effect. A current $\mathbf{I} = \mathbf{I}_x$ is drawn through the sample which is placed in a magnetic field $\vec{B} = B\vec{e}_z$. A voltage $V \sim E_x$ and a Hall voltage $V_H \sim E_y$ is measured.

In the presence of a magnetic field the resistivity and conductivity become tensors. The resistivities ρ_{xx} and ρ_{xy} are related to the conductivities σ_{xx} and σ_{xy} via the relations

$$\rho_{xx} = \frac{E_x}{I_x} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2}, \tag{6a}$$

$$\rho_{yx} = \frac{E_y}{I_x} = \frac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2}. \tag{6b}$$

At high fields $E_y/E_x = \sigma_{xy}/\sigma_{xx} = \mu B \gg 1$ and we can write $\rho_{xx} \sim \sigma_{xx}/\sigma_{xy}^2$ and $\sigma_{yx} \sim 1/\sigma_{xy}$. Thus in a high magnetic field we have the remarkable result that the resistivity is proportional to the conductivity: zero conductivity implies zero resistivity!

VI.1. Integer quantum Hall effect

In 1980 Klaus von Klitzing[3] at the Max-Planck Institute in Grenoble, discovered that for certain magnetic field ranges the Hall resistance is constant and quantized to the values (he received the Nobel prize for this discovery in 1985)

$$R_H = \frac{1}{n}\frac{h}{e^2} = \frac{1}{n}25.8128k\Omega. \tag{6}$$

The resistance for n = 1 is also called the one *Klitzing* which is the unit of resistance.

The most fascinating property of the quantum Hall effect[18] (QHE) is the fact that frorn a relatively simple experiment on a semiconductor a new type of electrical resistor $R_n = 25.8128k\Omega$ can be deduced which is independent of the microscopic properties of the semiconductor and reproducible at a level of better than 10^{-6}. In 1986 the quantized Hall resistance was accepted as a new resistor standard and h/e^2 is considered as a new

fundamental physical constant. The surprising result is that the ratio h/e^2 can be measured directly on a macroscopic system.

Another application is the measurement of h/e^2 wliich is identical to another very fundamental constant in physics: the fine structure constant. The inverse fine structure constant is given by $\alpha^{-1} = \frac{h}{e^2}\frac{2}{\mu_o c}$ with $\mu_o = 4\pi \times 10^{-5} H/cm$ the permeability of vacuum and c = 2.99792458 x $10^{10} cm/s$ the velocity of light. When μ_o and c are known accurately we note that an accurate measurement of h/e^2 gives us the fine structure constant which resulted into α^{-1} = 137.035968 which agrees very well with the result obtained from the gyromagnetic ratio of the proton. An accurate knowledge of this fundamental constant is important for different areas in physics. Once you have a new value for α it will influence the values of other fundamental constants like the electron mass, the Planck's constant, the electron charge.

A qualitative picture of the QHE will be given here. Essential for the excistence of the QHE is the presente of energy gaps. In a high mobility sample already at relatively low magnetic fields closed cyclotron orbits with discrete energy levels are present. This Landau quantization together with the size quantization of the 2D system leads to the desired energy gaps $\boldsymbol{AE}$. The quantum Hall effect becomes more pronounced if the ratio $\Delta E/kT$ is made larger, which implies low temperatures (small thermal broadening) and high magnetic fields because A E $\sim \hbar\omega_c$.

Another essential ingredient is localization. In the presence of disorder electrons in the tail of the Landau level are localized (see top of Figure 9) and do not contribute to conduction. Thus as long as the Fermi energy is in the area of localized states $\sigma_{xx} = 0$, because only states at the Fermi energy contribute to $a_{..}$ For σ_H the number of free propagating electrons are relevant. This number does not change if the Fermi energy moves through an area with localized states and thus σ_H is constant. Thus we have that R_H is constant when $\rho_{xx} \sim a_{,} \sim 0$ as seen experimentally (see Figure 10). The existence of broad zero resistance minima in ρ_{xx} suggests that the Fermi energy is pinned in between Landau levels for a finite range of magnetic fields. For this to be the case, there must exist electron states in the gaps of the density of states. In order for these states not to affect transport phenomena, these midgap states must be localized.

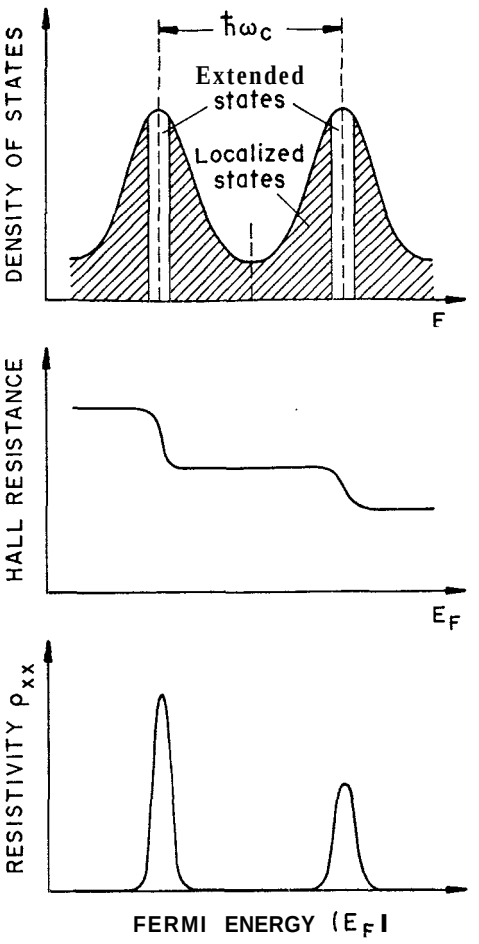

Figure 9.: The density of states, the Hall resistance and the longitudinal resistivity p, as function of the position of the Fermi level.

VI.2. The Fractional Quantum Hall Effect

In 1982 Tsui and Stormer[4] at Bell Laboratories in Murray Hill (New Jersey) investigated a very high mobility sample grown by Gossard in an attempt to observe the Wigner lattice. An electron crystal state which has long been predicted to exist in an ideal 2D electron system in very high magnetic fields. Instead, they discovered the fractional quantum Hall effect (FQHE) at filling factors $\nu = \frac{1}{3}$ and $\frac{2}{3}$. Later observations at lower temperatures on higher mobility samples discovered new structures at other fractions (see Figure 11). Because the value of n in Eq.(7) is no longer an integer but a fraction, this effect is called the *fractional* quantum Hall effect in contrast to the integer quantum Hall effect discover by von Klitzing. It turns out that the origin of the FQHE is totally different from the one of the QHE.

Phenomenologically the FQHE resembles very much the integral QHE. Once again the Hall resistivity is quantized and the diagonal resistivity vanishes concornitantly (see Figure 11). The crucial difference is found in the associated quantum numbers which in the FQHE are rational numbers. In spite of the superficial relatedness of both quantum Hall effects the underlying physics is very different. Whereas the IQHE rests on the quantization conditions for degenerate, noninteracting carriers in the presence of a magnetic field, the FQHE is a many-particle effect, and the result of strong carrier-carrier correlation. It is presently under-

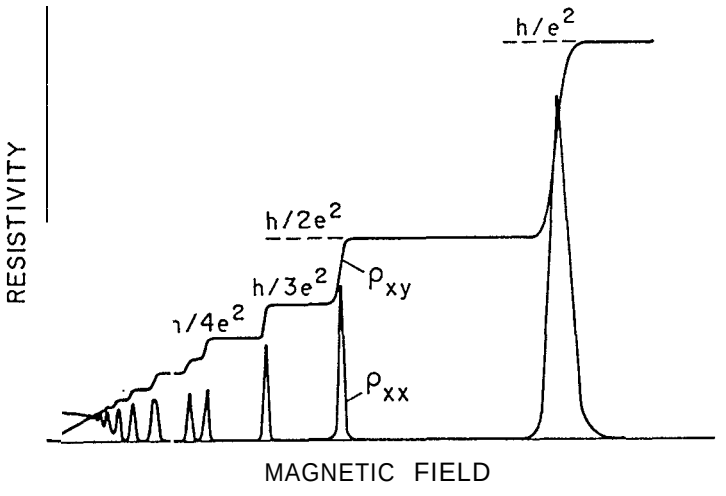

Figure 10.: The magneto (ρ_{xx}) and the Hall (ρ_{xy}) resistivity as function of the magnetic field at low temperatures.

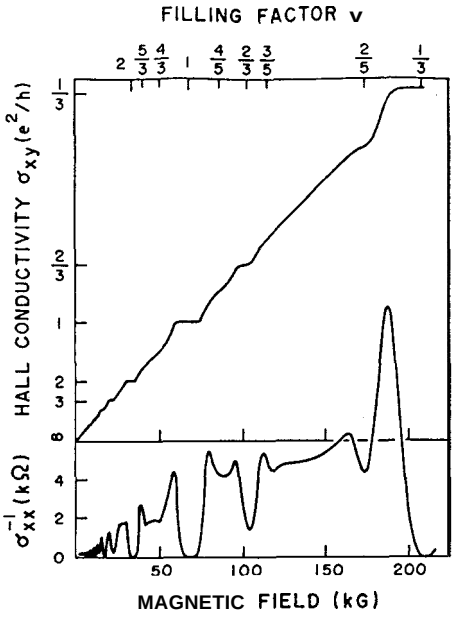

Figure 11.: The fractional quantum Hall effect at T = 0.511' manifests itself as plateaus in the Hall conductance at fractional values of e^2/h (top figure) and dips in the resistivity at fractional filling (v) of the lowest Landau level. The integer quantum Hall effect occurs for v equal to an integer.

stood in terms of a novel electronic quantum fluid which exists exclusively at primitive odd-denominator fractions $v = 1/q$ and $v = 1 - 1/q$. The wavefunction describing this state has been constructed by Laughlin[19].

In contrast to the QHE where disorder was essential, which in essence is a one particle effect, the FQHE is a consequence of the electron-electron interaction. Very pure samples with very small amounts of disorder are required tc observe the effect. Due to the electron-electron interaction many-particle effects are very important and a new highly correlated ground state is possible in 2D. This new electron state constitutes an *zncompressible quantum fluid* which has a number of rernarkable properties including a gap for the excitation of fractionally charged quasi-particles and holes, and a collective excitation gap. The FQHE effect results from the transpcrt of fractionally charged quasi-particles in the many-electron system.

Laughlin[19] has written down an explicit multiparticle wavefunction that accounts for most of the peculiar properiies observed in the fractionally quantized Hall effect. The correlation probability resulting from this wavefunction goes to zero as a power law when the separation between two electrons tends to zero. This extraordinary tendency of the electrons to avoid one another results in a very low Coulomb energy of this state, much lower than the competing Wigner lattice. Unlike other quantum liquids, superfluid helium for example, the fractional quantum Hall states are essentially incompressible. To raise the density above the fixed rational fractions of the filled Landau density one must overcome an energy gap of a few Kelvin (this is the reason why the FQHE can only be observed at low temperature). This energy gap corresponds to the cost of generating the elementary excitations of the quantum liquid which are localized quasiparticles of fractional electric charge. The excitations of the $v = 1/3$ ground state, for example, are quasi-particles of charge 1/3. The quantum-liquid states are incompressible because changing their volume is tantamount to injecting quasi-particles. In contrast to superfluid helium and superconductivity, the fractional Hall states are not coherent macroscopic quantum states. They exhibit no phase transition at finite temperature, but they are quantum liquids of a very novel sort.

VII. One and zero dimensional structures

VII.1. Quantum wires

In many cases two-dimensional electronic systems have been further confined by various lithographic, etching, gating and related techniques. High-mobility GaAs/AlGaAs heterostructures have become the system of choice for these experiments because of the lower effective mass and the larger inelastic mean free path. The reduction of scattering and the preservation of the phase over long distances have yielded a myriad of information on the fundamental aspects of electronic transport and confinement in artificial structures.

The understanding of the physics of noise, transport phenomena and basic processes such as doping in these lower dimensionality structures will be the subject of

intensive research in future years. The recent observation of the quantized ballistic resistance of a two-dimensional electron gas confined to a short and narrow channel has generated considerable excitement[20] (see next subsection).

Quantum interference semiconductor devices have recently started to attract the attention of several investigators[21]. A very interesting idea is the possibility of achieving transistor action by adjusting, via an applied voltage, the relative phase of ballistic electrons propagating in two parallel channels. Quantum interference phenomena in mesoscopic physics, such as the Ahronov-Bohm effect, have of course been the subject of considerable investigation in metallic structures such as rings and are also well-known in the context of Josephson devices. The observation of related phenomena in semiconductor heterostructures is contributing to bringing together these neighboring fields of investigation.

VII.2. Quantum point contacts

These are short and narrow constrictions (see Figure 12) in a two-dimensional electron gas with a width of the order of the Fermi wavelength $\lambda_F \approx 400\mathring{A}$. Because of the high mobility, elastic impurity scattering and inelastic (phonon) scattering are of secondary importance in the ballistic and adiabatic transport regimes. The transport mean free path is about $10\mu m$. Instead, scattering is determined by the *geometry* of the sample boundary. Transport phenomena in the ballistic and adiabatic regime can be viewed as scattering or transmission experiments with modes in an electron waveguide. Quantization, i.e. the discreteness of the mode index, is essential for some phenomena but not for others.

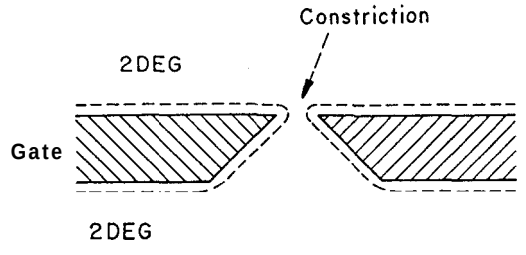

Figure 12.: Point contact in a two-dimensional electron gas. The gate, which is located above the 2DEG, induces a constriction in the 2DEG.

Ballistic transport through point contacts in metals is known for a long time. They are usually fabricated by pressing a metal needle on a metallic single crystal. Point contacts in a 2DEG are defined electrostatically by means of a split-gate on top of the heterostructure. In this way one can define short and narrow constrictions in the 2DEG, of variable width which can made comparable to the Fermi wavelength, i.e. then it is called a *quantum point contact*. The two high-mobility 2DEG regions are thus electrically isolated from the rest of the 2DEG, apart from the narrow constriction, or point contact (see Figure 12). Increasing the negative gate voltage forces the contriction to become progressively narrower.

The experimental result of the conductance (which is proportional to the inverse of the resistance) through the constriction as function of the gate voltage (i.e. the width of the constriction) is depicted in Figure 13. The average increase of the conductance with increasing width is due to the well-known Sharvin[22] effect. For a classical two dimensional gas the conductance through a split of width W is given by

$$G = \frac{2e^2}{h}\frac{k_F W}{\pi} \tag{7}$$

where k_F is the Fermi wavevector of the electron gas. This experimental result in Fig.13 shows a sequence of plateaus and steps. The plateaus are quantized in units of $2e^2/h$. This quantization is reminiscent of the quantum Hall effect, but is observed in the absence of a magnetic field, and thus can not have the same origin. The deviations from exact quantization are typically of the order of 1% while in the QHE an accuracy of 1 part in 10^7 is obtained. The exact shape of the plateaus and the sharpness of the transition between the plateaus vary among devices of identical design, which indicates that the detailed shape of the electrostatic potential defining the constriction is important. The quantization of the observed conductance is a consequence of the quantization of the wavevector of the electrons which are allowed through the constriction. The constriction acts like a quantum well and the electron wave parallel to the constriction has to be a standing wave according to quantum mechanics. The number of different allowed standing waves for electrons with an energy E_F is $N = int(k_F W/\pi)$. As a consequence the previous classical expression is transformed into the quantum result

$$G = \frac{2e^2}{h}N. \tag{8}$$

VII.3. Quantum boxes

Quantum boxes are quasi-zero-dimensional quantum microstructures which exhibit quantum carrier confinement in all three dimensions. Individual atoms are the microscopic limit for very small, confined-electron systerns. As a consequence of the quantum confinement in all three dimensions the energy levels are discrete. That is the reason why quantum boxes are so interesting because they are a new class of artificially structured materials with atomic-like discrete states which is ideal for use in laser structures. They have interesting nonlinear optical properties witli large changes in the optical absorption and index of refraction.

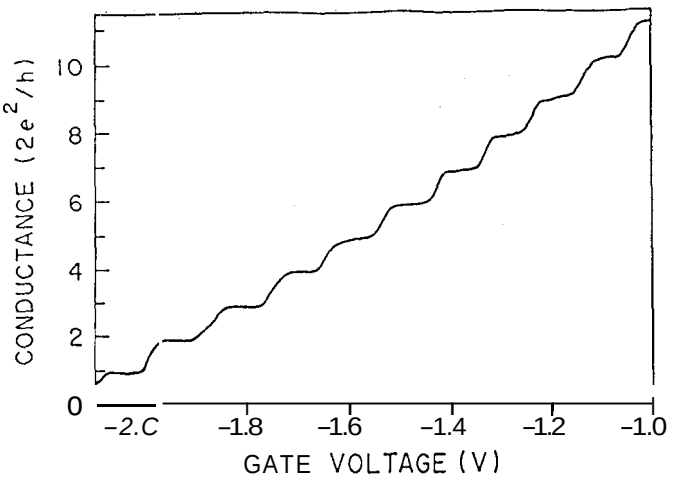

Figure 13.: Conductance of a point contact in the absence of a magnetic field.

The predictions about the interesting optical properties of quantum boxes are valid for L $< 100\AA$ in which Coulomb effects (which scales as $1/L$) are a weak perturbation to the quantum confinement effects, which scales as $1/L^2$. However, significant improvements in miirofabrication capability is still needed to make structures in this size regime with sufficient uniformity. For optical purposes one usually needs an array of typically more than a million of identical quantum boxes. Present day quantum boxes are in the L $\sim 1000\AA$ regime where Coulomb effects between the electrons are still important (see e.g. Ref. 23).

Acknowledgments

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References

1. For a review see e.g. A. Y. Cho and J. R. Arthur, Progr. Solid State Chem. **10**, 157 (1975).
2. B. L. **A** tshuler, JETP Lett. 41, 648 (1985); P.A. Lee and A.D. Stone, Phys. Rev. Lett. 55, 1622 (1985).
3. K. von Klitzing, G. Dorda and M. Pepper, Phys. Rev. Lett. 45, 494 (1980).
4. D. C. Tsui, H. L. Stormer and A. C. Grossard, Phys. Rev. Lett. 48, 1559 (1982).
5. F. Cappaso and R. A. Kiehl, J. Appl. Phys. 58, 1366 (1985).
6. T. Ando, A. B. Fowler and F. Stern, Rev. Mod. Phys. 54, 437 (1982).
7. L. Esaki and R. Tsu, IBM J. Res. Dev. 14, 61 (1970).
8. S. M. Sze, Physics of Semiconductor Devices, (Wiley, New York, 1981).
9. R. Dingle, H. L. Stormer, A. C. Gossard and W. Wiegmann, Appl. Phys. Lett. 33, 665 (1978).
10. M. Shur, *GaAs* Devices and Circuits, (Plenum Press, N. Y., 1987).
11. Physics of Quantum Electron Devices, Eds. F. Capasso (Springer-Verlag, Berlin, 1990).
12. F. M. Peeters, in The Physics of the *Two*-Dimensional Electron Gas, Eds. J. T. Devreese and F. M. Peeters (Plenum Press, N.Y., 1987), p.393.
13. E. P. Wigner, Phys. Rev. 46, 1002 (1934).
14. R. S. Crandall and R. Williams, Phys. Rev. Lett. A34, 404 (1971).
15. C. C. Grimes and G. Adams, Phys. Rev. Lett. 42, 795 (1979).
16. O. Hipólito, G. A. Farias and N. Studart, Surface Sci. 113, 394 (1982).
17. E. Y. Andrei, Phys. Rev. Lett. 52, 1449 (1984).
18. For an overview of the quantum Hall effect see: The Quantum Hall *Effect*, Eds. R.E. Prange and S.M. Girvin (Springer-Verlag, Berlin, 1987).
19. R. B. Laughlin, Phys. Rev. Lett. 50, 1395 (1983).
20. B. J. van Wees, H. van Houten, C. W. Beenakker, J. G. Williamson, L. P. Kouwenhoven, D. van der Marel and C. T. Foxon, Phys. Rev. Lett. **60,** 848 (1988); D. A. Wharam, T. J. Thornton, R. Newbury, M. Pepper, H. Ahmed, J. E. F. Frost, D. G. Hasko, D. C. Peacock, D. A. Ritchie and G. A. C. Jones, J. Phys. C: Solid State Phys. 21, L209 (1988).
21. See for example: Science and *Engineering of One*- and Zero-Dimensional Semiconductors, Eds. S. P. Beaumont and C. M. Sotomayor Torres (Plenum Press, N.Y., 1990).
22. Yu. V. Sharvin, Sov. Phys. JETP 21, 655 (1965).
23. G. W. Bryant, in Interfaces, quantum *wells*, and superlattzces, Eds. C. R. Leavens and R. Taylor (Plenum Press, N.Y., 1988), p.143.