Some Aspects of C-Axis Coupling and Transport in the Copper Oxide Superconductors

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Although there is widespread agreement that the fundamental explanation of the phenomenon of high-temperature superconductivity in many copper oxide compounds is to be sought in the interactions of the electrons within an individual CuO_2 plane, many properties of these materials are affected by the coupling and contact BETWEEN neighboring planes. In the normal state, the "traditional" view as applied to these materials would be that a three-dimensional Fermium surface exists but is strongly cylindrical in shape, so that on y a very small fraction of the electrons can contribute to the c-axis transport. I point out that to date there is no experimental evidence for this hypothesis, and give an a priori argument against it; instead, I propose that the normal-state c-axis transport is by "dynamically denoted hopping". As to the superconducting state, most of the evidence points to the idea that the Cooper pairs essentially form within a single CuO_2 plane, and that the main role of ,he inter plane coupling is to stabilize the condensate against fluctuations which can lead to a finite resistivity even in the condensed phase. I consider the possible mechanisms of coiipling among vortex "pancakes" in neighboring planes, and point out that in addition to the well-known Ampere and Josephson couplings mechanisms there exists another one, namely the higher-order effects of the Coulomb interaction.

I. Introduction

As is well known, in the past six years a wide variety of new miterials has been discovered which show the phenomenon of superconductivity at temperatures ranging from 30 to 125 K (for a review of these material and sonie of their properties, see ref. 1). One striking characteristic which all these materials have in common is that they contain well-separated planes of Cu and O atoms with the arrangement shown in figure 1 (CuO_2 planes); the separation of these planes (or groups of them, see below), along the axis normal to them, which we shall follow convention in calling the c-axis, ranges from 12 Å for $YBa_2Cu_3O_{7-\delta}$ (YBCO) to 37 Å for $Bi_2Sr_2Ca_2Cu_3O_{10}$ (BSCCO 2223), and may in fact be arbitrarily large for the so-called "intercalated" material (see below). To be sure, not all material possessirig such well-separated CuO_2 planes are high-temperature superconductors, even when they are metallic (and many, of course, are not); for example, the compound $Bi_2Sr_2Cu\,O_6$, which has this property, becomes superconducting only below 10 K. However, it seems clear that it is not an accident that every material discovered to dite which has a transition temperature appreciably above 30 K falls into this class.

The behavior of these new "copper oxide" superconductors s highly anomalous in both the normal and the superconducting phase, and at present there is no widely agreed theory which describes them. One hypothesis, however, would probably receive fairly widespread agreement, namely that the fundamental mechanism of superconductivity is to be sought in the interactions of the electrons moving in the CuO_2 planes, and the principal role of the atoms interposed between these planes (e.g. Y and Ba atoms and the "chain" Cu and O atoms in YBCO, the Bi and other atoms in BSCCO, etc.) is to act as donors or acceptors and thus control the number of electrons in the planes. As a result, most of the theoretical work on the behavior of the high-temperature superconductors has tended to focus on the properties of a single CuO_2 plane, and to treat the effects of the interactions between these planes, if at all, only as an afterthought.

Nevertheless, it is clear the that obtain a complete theory of the properties of the high-temperature superconductors (hereafter HTS) it is essential to consider the ways in which contact and transport between neighboring planes affects them. This is obvious as regards transport properties such as the c-axis electrical resistivity; it is a priori less obvious as regards apparently "nondirectional" properties such as the onset of superconductivity itself, but we shall see below (section IV) that inter-plane contact must in fact be an essential ingredient in controlling these also. Therefore, in this

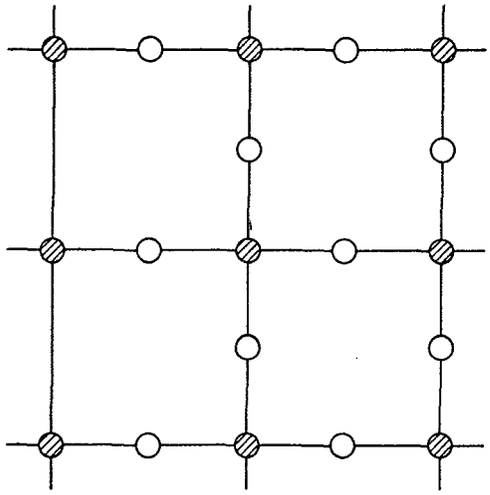

Figure 1.: The arrangement of Cu and O atorns in the CuO_2 plane of a typical high-temperature superconductor. Shaded circles represent Cu atoms, open circles O atoms.

paper I will assess some of the experimental information currently available on the question of c-axis transport and contact, and consider its theoretical implications. This is certainly not intended as a complete review of the subject; in particular, I shall not attempt to discuss those theoretical papers (a minority, but still a considerable number!) which have invoked "inter-plane" processes as an ingredient in the microscopic theory of in-plane superconductivity. The plan of the paper is as follows: in section II review some of the experimental information available on c-ais transport and contact in the normal phase, and in section III consider its implications for theoretical models of that phase. Similarly, sections IV and V cover respectively the experimental and theoretical aspects of the superconducting-state behavior. Section VI is a brief conclusion.

A few general remarks before we start. First, it is well known that in YBCO the unit cell contains not one but two CuO_2 planes, separated by a distance (~ 3Å) which is small compared to the c-ais lattice spacing (12 Å). Similarly, the compounds BSCCO 2212 and 2223 contain respectively pairs and triples of such closely-spaced planes, and similarly with the Tl compounds. For the purpose of this paper I will always regard such pairs or triples of CuO_2 planes as equivalent to a ***single*** plane, and the c-axis "transport" and "contact" will refer to that between neighboring groups (i.e. neighboring unit cells).

Secondly, while the really dramatic anisotropy of properties of the HTS such as resistivity refers to the difference between the c-axis and ab-plane (CuO_2 plane) behavior, that fact that all currently known HATS are of orthorhombic rather than tetragonal crystal symmetry means that there rnay be an appreciable anisotropy of the behavior even in the ab-plane; for example, in untwinned YBCO single crystals the resistivity is found[2] to be anisotropic within this plane by a factor of ~ 2. For the purposes of this paper I shall always neglect this phenomenon and speak simply of the "ab-plane" properties as contrasted with the "c-axis" properties; if necessary, the former should be interpreted as an angular average.

Thirdly, while it is clear that an obvious way to examine the effects of c-axis contact is to vary the CuO_2 plane separation in a systematic way (cf. in particular section IV), it should be emphasized that such a process has a number of different effects. The most obvious is to vary the single-electron transition matrix element from one plane to the next; however, in addition it rnay vary the chemistry[3] and hence the number of in-plane carriers, it will certainly change the effective screening of the in-plane Coulomb interaction due to inter-plane effects[4] and rnay do other things as well. Thus caution is necessary in interpreting the effects of such variation.

Finally, it is a natural and obvious question to what extent the anomalous properties of the HTS, or perhaps a subset of them such as the c-axis transport properties, are a consequence *only* of the "two-dimensional" nature of these materials and are independent of the detailed properties of the CuO_2 planes as such. In principle, this question could be examined by seeking analogs for the characteristic HTS behavior in other approximately 2D structures such as intercalate graphite or artificially engineered superlattices of Nb with an insulating material. I shall not attempt to discuss this question here, merely remarking that while limited similarities can be found, they do not seem to me particularly helpful in understanding the properties of the HTS since their explanation in the "analog" materials is itself controversial.

II. Normal-state Properties: Experimental

In assessing the available experimental data on the c-axis properties of the HTS it should be borne in mind that the influence of impurities or disorder on these properties is likely to be even more severe than on the ab-plane ones. Crudely speaking, this is because on *any* model of the c-axis transport, contact between the planes is "difficult" compared to contact between different atoms in a single CuO_2 plane, and therefore any kind of disorder or impurity which can make it any easier is likely to have a disproportionate effect. It is therefore essential that experiments be done with single crystals, and moreover the most informative experiments rnay be those done with stoichiometric material ~ e.g. $YBa_2Cu_3O_7$; in the nonstoichiometric compounds $YBa_2Cu_3O_{7-\delta}$ with nonzero 6 the disorder of the "chain" oxygens, which rnay act as a "bridge" between neighboring planes, rnay play an important role.

The most widely measured c-axis normal-state transport property of the HTS is the resistivity, which in accordance with convention we denote p,. The salient

fact is that in *all* the HTS the room temperature value of ρ_c is very much greater them that of the ab-plane resistivity ρ_{ab}, the ratio ρ_{ab}/ρ_c ranging from ~ 0.03 for LSCO to $[10^{-5}]$ for BSCCO (To the best of my knowledge ρ_c has not beem measured for YBCO-PrBCO intercalates (see e.g. ref. 5), for which one would expect the ratio might be even smaller.). This very severe anisotropy of course means that any kind of crystalline disorder may influence the observed ρ_c substantially, by providing mechanisms for the much lower ab-plane resistivity to "short out" the true c-axis resistivity.

The temperature-dependence of the c-axis resistivity is controversial. Digressing for a moment, we recall that in the best samples, with the possible exception of these samples which are furthest from stoichiometry, the *ab-plane* resistivity appears to be almost linear in T, with zero offset[6], over a range which typically extends from a few hundred degrees all the way down to the superconducting transition temperature (which for Bi 2201 is below 10 K). The numerical value of ρ_{ab} at room temperature is $\sim 150 - 300\ \mu\Omega$ cm, and, remarkably, varies by only a factor of 2-3 over a very wide range of the HTS. It is amusing (and may or may not have some fundamental significance) that if we interpret this in terms of the "sheet resistance" $R_\Box$ of a single CuO_2 plane (i.e. the resistance of a square piece of the plane, assumed to be the only conducting region) then $R_\Box^{ab}$ at room temperature is of the order of a few $k\Omega$, i.e. of the order of the "fundamental quantum unit of resistance" $R_Q \equiv h/e^2 \cong 25k\Omega$.

Returning to the c-axis resistivity ρ_c, we find that in most measurements to date ρ_c has appeared to be approximately linear in temperature at high temperatures, with small or zero offset, but to rise substantially above the linear graph at lower temperatures, with $d\rho_c/dT$ often becoming negative as T_c is approached. (In the literature such behavior ($d\rho_c/dt < 0$) is often characterized as "semiconducting", while that with $\rho_c \propto T$ is called "metallic"; however, I believe this terminology is best avoided since it begs the question as to the mechanism). However, this behavior appears to be extremely sensitive to disorder[7]; generally speaking, the greater the disorder the more substantial the upturn as T_c is approached. Although the situation is perhaps not yet completely clear, it appears that in the purest stoichiometric YBCO crystals, at least, ρ_c is linear in T with a small but nonzero offset[7]. In the case of BSCCO, a very intriguing result has recently been reported by Xiang et al[8], who find that while the original crystal shows a sharp upturn in ρ_c above T_c, when it is intercalated with a single layer of iodine the behavior becomes perfectly linear in T. (It is also remarkable that in this experiment, while the intercalation increases ρ_{ab}, the increase can be explained entirely in terms of the (measured) decrease of the density of CuO_2 planes, the sheet resistance $R_\Box^{ab}$ of a single plane remaining unaffected. This would seem to indicate that the "chemistry" relevant to the CuO_2 planes is not affected by the intercalation.) Another relevant observation is that as we vary the stoichiometry so as to approach the insulating transition, both ρ_c and ρ_{ab} diverge, but the ratio ρ_c/ρ_{ab} also diverges[9], with the c-axis upturn becoming more prominent. An obvious question is whether e.g. the increasing departure of ρ_c from linearity with departure from stoichiometry in YBCO is a consequence of the increasing disorder in the arrangement of the chain oxygens (which may provide a bridging mechanism between the CuO_2 planes) or rather to the fact that the ab-plane behavior is itself approaching an insulating transition. At present this question seems moot; the only clear statement one can make is that there do exist samples (pure untwinned crystals at stoichiometry) where the behavior of ρ_c is purely linear. This observation alone would seem to rule out theories such as that of Anderson and Zou[10] in which the "true" c-axis resistivity is proportional to 1/T and the linear term is a result of contamination by the ab-plane resistivity.

Two other d.c. c-axis properties which have been measured experimentally are the Hall effect and the thermoelectric power. The interesting component of the Hall coefficient in the present context is that for which the magnetic field lies in the ab-plane, so that for one of the (antisymmetric) coefficients the directions of current flow is along the c-axis. In contrast to the "ab-plane" Hall coefficient ($\underline{H} \parallel \underline{c}$), which is usually positive and shows an unusual temperature dependence, the c-axis coefficient is negative and only weakly temperature-dependent[9], as in many simple metals. As to the thermoelectric power, this appears to be positive, at least in YBCO[7a].

Some measurements have been made of the ac conductivity $\sigma(\omega)$ in the c-direction. At low frequencies (1 GHz) $\sigma(\omega)$ appears to be essentially identical to $\sigma(0)$ as a function of temperature, while in the optical regime the behavior of $\sigma_c(\omega)$ is qualitatively similar to that of $\sigma_{ab}(\omega)$.

Finally, I note that any experiment which claims to measure the shape of the (three-dimensional) Fermi surface is in principle relevant to the nature of the c-axis transport. It seems more appropriate to discuss such experiments in the context of the theoretical implications of experiment in this area, and I therefore postpone it to the next section.

III. Normal-state Properties: Implications

The simplest model of electrical conductivity, and the one which describes the behavior of most simple metals, is of course the well-known Bloch-Sommerfeld model: the electrons are described in a basis of the Bloch-wave states appropriate to the crystalline lattice in question, and are conceived as moving freely under the influence of the electric field in the intervals between

collisions with static impurities, phonons or other unspecified excitations. In the simplest form of the model, in which the Fermi surface is an ellipsoid, (figure 2), the resulting formula for the electrical resistivity along a principal axis i of the crystal is given by

$$\rho_i(T) = \frac{m_i^*}{ne^2\tau_i(T)} \tag{3.1}$$

where m_i^* and τ_i are respectively the effective mass and phenomenological relaxation time (due to collisions) appropriate to the axis in question (actually these quantities are in general averages over the Fermi surface of the relevant quantities), and n is the number of electrons per unit volume.

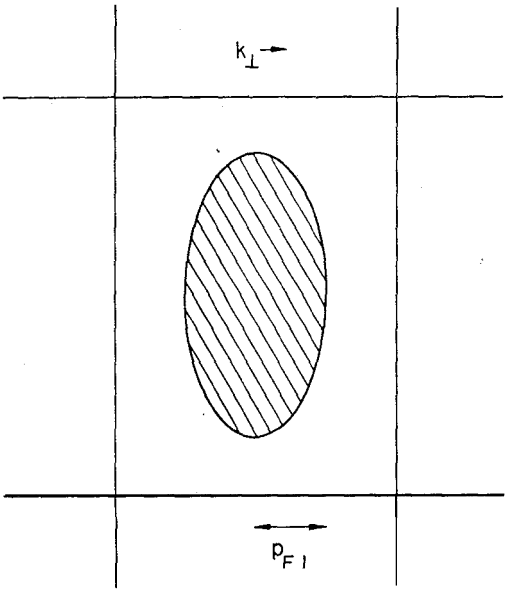

Figure 2.: Simplest possible Fermi surface in the Bloch-Sommerfeld model.

It is very straightforward to see[7a] that this "simplest" form of the Bloch-Sommerfeld model cannot realistically describe the c-axis resistivity of the HTS, at least in the case of the "most anisotropic" ones such as BSCCO. Since we have $\tau_i \sim \ell_i/v_{Fi}$ and $m_i^* \sim p_{Fi}/v_{Fi}$ where ℓ_i, p_{Fi} and v_{Fi} are suitably averaged values of the relevant mean free path, Fermi rnomentum and Fermi velocity appropriate to the relevant direction (here the c-axis), we can rewrite formula (3.1) in the form $\rho_i(T) \sim p_{Fi}/ne^2\ell_i(T)$. Since p_{Fi} on this model cannot exceed $2\hbar/a$ where a is the relevant lattice spacing (otherwise the Fermi surface intersects the edge of the Brillouin zone, see below), we find

$$\rho_i(T) \lesssim 2\hbar/(ne^2 a \cdot \ell_c(T)) \tag{3.2}$$

It is convenient to re-express this result by dividing by ρ_{ab} and introducing the "quantum unit of resistance" $R_Q = h/e^2$ and ab-plane sheet resistance $R_\square^{ab}$ discussed above: this gives

$$\rho_c/\rho_{ab} \lesssim (\pi/na^3) \cdot \frac{R_Q}{R_\square^{ab}} \cdot \frac{a}{\ell_c(T)} \tag{3.3}$$

Now since for all except those materials closest to the insulating transition the number of conduction electrons per unit cell, which is of order na^3 (or less) is ~ 1, while as we have seen $R_Q/R_{ab}^\square$ is also of order unity, the above formula, when applied e.g. to BSCCO ($\rho_c/\rho_{ab} \sim 10^5$) must imply that the c-axis free path $\ell_c(T)$ is tiny compared to the c-axis lattice spacing a. Under these conditions the concept of a "mean free path" of Bloch waves propagating in the c-direction clearly makes no sense.

However, the above argument starts from the premise that the Fermi surface does not intersect the face of the Brillouin zone, and this is anyway a priori implausible. In fact, microscopic band-structure calculations based on the local density approximation (see e.g. refs. (11, 12)) predict that the dispersion of the bands in the c-direction[13] is so weak that, very schematically, the shape of the Ferrni surface is more similar to figure 3, where $k_\perp$ denotes the c-axis component of the wave vector. (In real life, of course, there may be several different bands which intersect the Fermi surface, so the picture is more complicated.[11,12]) In so far as the results of the LDA calculations can be described by a tight-binding model, the effective hopping matrix element along the c-axis, $t_,$, is a small fraction of that in the ab-plane: typically, t_c/t_{ab} varies from ~ 0.07 for YBCO to ~ 0.015 for BSCCO.[14] It should be noted for future reference that this fraction, though small, is much larger then the ratio ($\sim 10^{-5}$) of the corresponding conductivities. However, in the present context the salient point is that such values of t_c/t_{ab} already give rise to highly "cylindrical" Ferrni surfaces like that in figure 3.

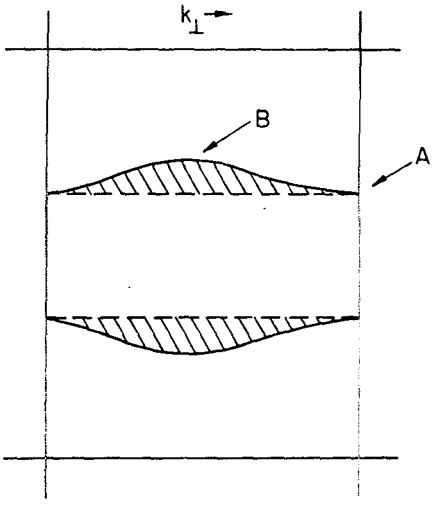

Figure 3.: More plausible Fermi surface for the HTS (schematic).

For a general Bloch-Sommerfeld model of this type, the relevant expression for the diagonal component of

the conductivity tensor $\mathbf{a}, \equiv \rho_i^{-1}$ is

$$\sigma_i(T) = e^2 \int^{F.S} \frac{dS}{v_{F(\hat{\mathbf{n}})}} \quad (\hat{\mathbf{n}})\tau_i(\hat{\mathbf{n}}) \qquad (3.4)$$

where $v_F(\hat{\mathbf{n}})$ is the magnitude of the Ferrni velocity ($\equiv \hbar^{-1} \nabla_k \epsilon(k)$) at point $\hat{\mathbf{n}}$ on the Fermi surface and $v_{Fi}(\hat{\mathbf{n}})$ its i-th component. The integral in eq.(3.4) is to be taken only over the "open" parts of the Fermi surface; thus, the states between the dashed lines in figure **3** cannot contribute to the c-axis conductivity, as is intuitively obvious since they effectively constitute a set of one-dimensional filled bands. If one writes $v_{Fi}(\hat{\mathbf{n}})\tau_i(\hat{\mathbf{n}}) \equiv \ell_i(\hat{\mathbf{n}})$ and uses the fact that for any but pathological geometries the quantity $\int dS \sin\theta_i(\hat{\mathbf{n}})$ is a measure of the fraction of the Fermi surface outside the dashed lines (i.e. the shaded area in figure 3), it is clear that the net result is to replace, in formula (3.2), the total number of conduction electrons by a quantity n_{eff} which is of the order of the number in the shaded region, i.e. in the "effective" part of the Fermi sea. Thus it is clear that thre is no obvious inconsistency, even for BSCCO, in describing the c-axis transport by a Bloch-Sommerfeld model with $\ell_c(T) \gtrsim$ a, provided that the ratio n_{eff}/n is small enough ($\lesssim 10^{-5}$). However, since the quantity n_{eff}/n in a tight-binding model is evidently of order t_c/t_{ab}, it is clear that existing band-structure calculations would then have to overestimate the latter by a arge factor.

We will nou give an a priori argument against the validity of a 3D Bloch-Sommerfeld model **as** described above, at least for extreme cases such as BSCCO. The definition of a 'Bloch wave" with respect to the c-axis requires that an electron be able to hop *coherently* from one plane to the next, which in turn requires that to a good approximation the energies (neglecting hopping) of the two states in question are degenerate. If the energies in question differ by an amount much larger than the hopping matrix element ("detuning"), as happens for example in an amorphous semiconductor due to the differences in static potential energy, then no coherent hopping is possible and the process of transport must be described by a completely different model, e.g. Mott variable-range hopping. Now in a pure stoichiometric HTS there is perfect translational symmetry between one plane and the next, and therefore there should be no question of *static* detuning. However, the crucial point is that because of very highly layered nature of the material, there should be large and in many cases more or less independent *zn-plane* fluctuations in each of the planes, and hence *there is a very substantial degree* of *"dynamical" detunzng,* which may lead to what in a different cortext has been christened[15] "dynamical destruction of the band " In such a situation one would expect that while *wzthzn* a given a-b plane the transport might still be of Bloch-wave type, the transport along the c-axis would have to be described by a totally different picture. Such a situation has not to my knowledge been previously studied in'any solid-state context, and indeed the copper oxide superconductors may be the first experimental realization of it. Note that such a picture need not imply "localization" in the c-direction, and hence the familiar objection that c-axis localization cannot coexist with ab-plane delocalization (because of the theorem[16] that all strictly 2D electron systems with any finite impurity scattering will localize) does not apply to it, at least prima facie.

Let us try to put in a few numbers to estimate the plausibility of this scenario. First, a quite general consideration: the fluctuation of the relative energy of two corresponding points in neighboring planes (figure 4) can scarcely be less then the energy fluctuations within a single plane (unless the planes are much more tightly correlated than we have any a priori reason to expect). Now it is well known that if the Rloch-Sommerfeld model is used to describe the in-plane electrical resistivity, the relevant electron lifetime $\tau_{ab}(T)$ against collisions (with entities which in any given model may be identified, e.g. as spin fluctuations) is exactly of order $\hbar/kT$. It is plausible in the present context to take the "in-plane energy fluctuation" to be of order $\hbar/\tau_{ab}(T)$; if anything this is liable to understimate it, since the $\tau_{ab}(T)$ which appears in the resistivity may be lengthened due to e.g. partial conservation laws. Thus, we may estimate the "detuning" energy $\Delta\epsilon$ (figure 4) to be $\gtrsim kT$. This then should be compared with the c-axis tunneling matrix element t_c. If we use for the latter the theoretically estimated (by band structure calculations) value, then we find that for BSCCO t_c is comparable with kT at room temperature, and is somewhat larger for most of the other HTS; thus, on the argument we should not expect substantial detuning. However, 'this conclusion is misleading: as we have seen, the "experimental" value of t_c necessary to justify a Bloch-Sommerfeld picture of the c-axis transport is three orders of magnitude smaller, so taht we have $kT >> t_c$ for *all* temperatures of interest. Thus, at least in the case of BSCCO, *the Bloch-Sommerfeld rnodel is internally inconsistent* even in its more general form.

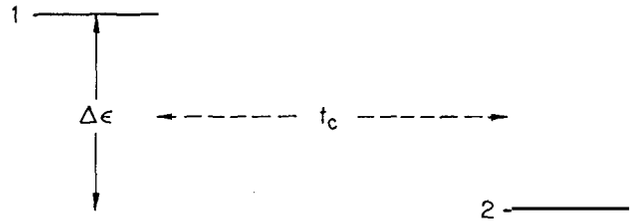

Figure 4.: An electron hopping hetween planes 1 and 2 in the presence of a "detuning" $\Delta\epsilon$.

The reader may object that it is not clear that $\hbar/\tau_{ab}$ is in fact a reasonable estimate of Ac, in particular

since the frequency distribution of the fluctuations may be crucial. Let me therefore briefly sketch a second phenomenological argument which leads to the same qualitative conclusion. Let is consider a small square piece of a single CuO_2 plane and focus on the fluctuations of the electrostatic potential across it. By the usual fluctuation-dissipation theorem, the mean-square voltage fluctuation within a frequency range from 0 to $\Delta\omega(<< kT/\hbar)$ is $4kT \cdot R_{\square}^{ab} \cdot$ Aw. This, then, is a reasonable estimate of the voltage between two such squares on neighboring planes, and the corresponding mean-square fluctuation of the electron energy is $4e^2kT \cdot R_{\square}^{ab}$. Aw. Suppose now that we think of the motion of the electron between these two regions as a "spin-boson" problem (see e.g. ref. (17)): then, since the spectrum of the fluctuations is "ohmic" in the classification of ref. (17), we can identify the dimensionless dissipative parameter a of that reference as $4R_{\square}^{ab}(T)/R_Q$, which is $\gtrsim 1$ at least for temperatures of the order of or above room temperature. Now the dynamics of the spin-boson problem with ohmic dissipation has been intensively studied, and it is known that coherent ("underdamped") motion is predicted to be observed (for $kT >>$ A) only in the regime of parameters defined by $\alpha kT/\Delta \lesssim 1$, where **A** is the renormalized hopping matrix element, here to be identified with $t_{\perp}$. Thus, this argument also leads to the conclusion that no coherent hopping is possible if t_c is fitted to a Bloch-Sommerfeld model, i.e. that the latter model is internally inconsistent as applied to the c-axis transport. Althoiigh the argument as it stands needs to be taken with a pinch of salt (in particular because it assumes zero correlation between planes in the electrostatic fluctuations, which is probably unrealistic), it should be possible to sharpen it up, and it is highly suggestive that we must look for a completely different model of the c-axis resistivity, which takes into account the "detuning" from the start. Such a model is being currently developed by L.-Y. Shieh and the author[18].

Of course, quite independently of these theoretical considerations, the question of whether the c-axis transport is of coherent (Bloch-Sommerfeld) type or not can in principle be settled definitely by experiment; in particular, reliable observation of a truly 3D Fermi surface would be conclusive evidence in favor of coherent transport. Probes which are routinely used to measure the **(3D)** Fermi surface in more familiar materials include angularly resolved photoemission spectroscopy (ARPES), 2-dimensionally resolved positron annihilation spectroscopy (ACAR), and, most usefully of all, de Haas-van Alphen oscillations (dHvA). Unfortunately, to date none of these techniques has definitively answered the question of interest to us. In the case of ARPES, this is because almost all experiments[14] have been done in a geometry where the relevant surface of the sample is (for reasons of experimental practicality) the ab-plane. Since it is only the components of momentum parallel to the surface which are conserved in the process of escape from the crystal, such experiments give no information on the c-axis dispersion, except for the qualitative information that *if* a Bloch-wave picture is applicable, it is very small. Similarly ACAR experiments with the usual geometry[20] measure only the momentum distribution $n(\underline{p})$ integrated with respect to $p_{\perp}$, and hence again prima facie cannot distinguish between coherent and incoherent models. The most interesting recent data in the present context are the dHvA results of Fowler et al.[21] on YBCO (with field $\| \underline{c}$); these show evidence for three (and only three!) dHvA periods, which in the standard interpretation[22] are ascribed to extremal orbits on the Fermi surface. One might at first sight think that the mere existence of more than one period should be evidence for a 3D Fermi surface as in figure **3,** with for example two of the periods associated with points A and B; however, it is clear, even apart from detailed band-structure calculations, that provided the transport between the pair of CuO_2 planes within the YBCO unit cell is coherent, this already must provide two extremal orbits (corresponding to the even- and odd-parity bands), and a third will arise from the "chain" band, so that even in a purely 2D picture it is possible to accommodate the observations. Finally, in the context of a question which is different but related, we should note that inelastic neutron scattering experiments on YBCO[23] show a modulation along the c-direction which clearly reflects coherence in the ***magnetic*** structure between the pair of planes within unit cell, but no evidence for any coherence between planes in neighboring cells. (In any case, even if such coherence were to be seen, it is not clear that it would necessarily imply coherence in the single-electron transport).

To sum up, the usual Fermi surface probes have so far given no clear answer to the question of the nature of c-axis transport in the HTS. In part this may be because very little work has been done on the experimental consequences of alternatives to the 3D Bloch-Sommerfeld picture. In this context one can make one qualitative prediction without detailed calculation, namely that if the c-axis transport is totally incoherent then any component of the magnetic field in the ab-plane should be very nearly irrelevant[24], and that hence the dHvA behavior should be simply a function of the component of the field along the c-axis. Unfortunately, to date no dHvA experiments have been done in which the field direction is varied.

IV. The Superconducting State: Experiment

The influence of inter-layer coupling on the superconducting state of the HTS is in many ways much more dramatic then its effects in the normal state. From a theoretical point of view this is not altogether surprising, since it is known that the off-diagonal long-range order associated with bulk 3D superconductivity cannot

occur in a strictly 2D system[25], and in such a system the phase transition to a superconducting state is of a different type (the so-called Kosterlitz-Thouless[25] or topological type). One might expect intuitively that if one starts from completely uncoupled planes an gradually switches on the interplane interaction, the properties of the system would change gradually from Kosterlitz-Thouless to bulk 3D-a qualitatively change. Thus, in some sense one might expect that many of the most fundamental aspects of superconductivity are controlled by the interplane contact, and this is borne out by experiment, particularly (but not only) in the presence of high magnetic fields. It is impossible in the space of a short paper such as this to review all the relevant data, so I shall concentrate on a few selected experiments which show up the role of the inter-layer contact most directly.

An important qualitative difference between the HTS and the conventional "old-fashioned" superconductors is that in the case of the former the very definition of "superconductivity" is a nontrivial question. At the transition temperature T_c in a conventional superconductor, the electrical resistivity $\rho(T)$ falls from a finite value typical of the normal state to zero over a range ΔT_c which is usually unmeasurably small and always very small compared to T_c itself. By contrast, in a typical HTS the decrease of $\rho(T)$ is much less abrupt and ΔT_c, particularly in high magnetic fields, may be comparable to T_c: see for example the graphs in the paper of Briceño et al[27]. Thus, rather than introducing a single parameter T_c, it is conventional to define an "onset temperature" $T_{\rm onset}$ at which the resistivity starts to deviate appreciably from its (usually slowly varying) normal-state form, a "midpoint" temperature $T_{\rm mid}$ at which $\rho(T)$ is half its normal-state value and/or a "zero-resistance" temperature T_{co} at which $\rho(T)$ becomes unmeasurably small. It should be emphasized that in general these temperatures may be *different* for ρ_{ab} and ρ_c (cf. below).

The most obvious question one can raise is: Is a finite degree of interplane contact essential to the existence of superconductivity (zero resistance)? In one sense this question is definitively answered in the negative by the experiment of Terashima et al[28], who find that a single layer (i.e. a layer one unit cell thick) of YBCO sandwiched between the semiconducting material PrBCO shows zero ab-plane resistance, though at a temperature ($\sim 30K$) far below the (fairly well-defined) T_c of bulk YBCO ($\sim 90K$). It is significant that the depression of the onset temperature is much smaller ($\sim 10K$); since the measured Hall number of the single layer is smaller than for bulk YBCO, Terashima et al. conjecture that this depression is simply due to the decreased carrier density in the CuO_2 plane rather than to the absence of interplane coupling, and that if by a suitable choice of matrix the carrier density of the single layer could be made equal to the bulk then $T_{\rm onset}$ would take the bulk value. Other work[29,30] had demonstrated that when YBCO is intercalated with PrBCO T_{co} is very sensitive to the intercalation, dropping rapidly with the spacing of YBCO layers and eventually saturating around 10-20 K, while $T_{\rm onset}$ is much less sensitive. The most natural conclusion from this group of experiments is that the basic transition responsible for superconductivity occurs in the individual layers, at a temperature $T_{\rm onset}$ which is essentially the bulk value for the relevant value of carrier concentration, but than when the layer is sufficiently decoupled from its neighbors fluctuations keep the resistivity finite down to a temperature which is a small fraction ($\sim 0.1 - 0.2$) of $T_{\rm onset}$. The principal role of the interlayer coupling is then to stabilize the individual layers against the fluctuations and thereby decrease the width of the resistive transition.

It should be emphasized that all the above remarks refer to the ab-plane resistivity; to the best of my knowledge the c-axis resistivity has not been measured for any intercalated structure (nor, of course, for a single layer!). However, recent experiment[27] on the c-axis resistivity of pure BSCCO show a very intriguing behavior, particularly as regards the effect of a magnetic field applied *parallel* to the c-ais. For the single crystal used in the experiments the normal-state values of $\rho_{ab}(T)$ and $\rho_c(T)$ are insensitive to field, and show, respectively, the usual linear dependence on T and the familiar upturn a little above T_c ($\sim 90K$ for this sample). In zero magnetic field, $T_{\rm onset}(= T_c)$ is the same for ρ_{ab} and ρ_c, and the temperature-dependence below T_c is virtually identical; both $\rho_{ab}(T)$ and $\rho_c(T)$ drop rapidly, reaching "zero resistance" around 85 K. Application of a magnetic field of a few Tesla along the c-axis dramatically changes these results: in the ab-plane the resistive transition has the same onset temperature but is much broader, with $T_{co} \sim 15K$ for $H \sim 7$T (see figure 5). This in itself is not particularly surprising, since qualitatively similar behavior is seen when the field is in the ab-plane (see e.g. ref. *31);* the obvious qualitative explanation in each case is in terms of phase slips mediated by the vortices created by the field. What is more intriguing is the behavior of the c-axis resistivity (figure 5). In a field of 7T this simply extrapolates its normal-state "upturn" behavior past T_c, continuing to rise down to $\sim 60K$, where it reaches a sharp maximum and thereafter drops steeply to reach the zero-resistance axis at $\sim 15K$ (similarly to ρ_{ab}). In other words, the effective onset temperature of superconductivity along the c-axis is a factor of $\sim 30\%$ lower than that for the ab-plane! A possible explanation for this intriguing behavior has been proposed by Kim et al[32] (see also ref. 27).

I now turn to a set of measurements of a rather different nature, whose goal is to understand whether the themnodynamic (as distinct from the transport) properties correspond to 2D or 3D behavior. Provided one works sufficiently close to the transition temperature

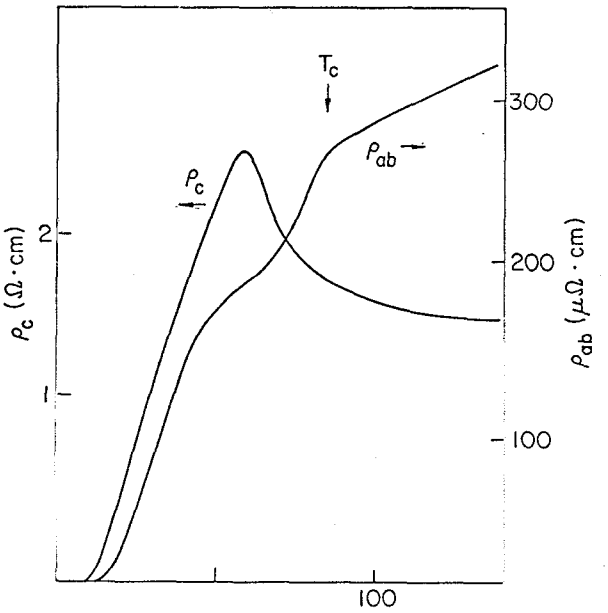

Figure 5.: Temperature-dependence of ρ_{ab} and ρ_c for BSCCO in a field of 7T (schematic, after ref. 27). Note the different scales. T_c is the approximate onset temperature in zero field.

(on either side), a good description of the thermodynamics should be given by Ginzburg-Landau theory (see e.g. ref. 33, and below, section V). It is then possible to calculate the effect of critical fluctuations of the order parameter on quantities such as the specific heat, the diamagnetic susceptibility etc., and the theory predicts that the temperature-dependence of these quantities should depend on the effective demensionality of the fluctuations. For example, the diamagnetic susceptibility should diverge as the temperature approaches T_c from above: for effective dimensionality $d = 3$ the divergence should be of the form $(T_c - T)^{-1/2}$, while for 2D it is proportional to $(T_c - T)^{-1}$. Analysis of the experiments shows[34] that the 3D formula holds in the limit $T \rightarrow T_c$, while for larger values of $T_c - T$ the 2D formula takes over. This is consistent with theoretical considerations (see section V). Analysis of the specific heat data is also consistent with this picture though in this case there are substantial experimental complications.[34]

V. Superconducting State: Theoretical Considerations

Since there has been a certain tendency in some of the literature to assume that all the concepts of BCS and Ginzburg-Landau theory can be taken over unmodified to the case of the HTS, I will start with a brief discussion of relevant aspects of the former, restricting myself for simplicity to the case of a pure superconductor.

In the BCS theory of a translation-invariant electron system, the ground state in zero external field in the particle-number-conserving representation takes the form[35]

$$\Psi_N = A\phi(\vec{r}_1 - \vec{r}_2)\phi(\vec{r}_3 - \vec{r}_4)...\phi(\vec{r}_{N-1} - \vec{r}_N), \quad (5.1)$$

where A is the antisymmetrization operator and I have omitted the spin indices for simplicity of notation. Note that the relative wave function ϕ is the same for all pairs, so that the wave function (5.1) is qualitatively similar to that of a Bose condensate of diatomic molecules, the crucial difference of course being that the range of ϕ is large compared to the typical inter-electron distance. It is often more convenient to use the "number-nonconserving" representation introduced by BCS[36], and in particular to introduce the "anomalous average"

$$F(\vec{r} : \vec{R}) \equiv < \Psi_\uparrow^\dagger(\vec{R} + \vec{r}/2)\Psi_\downarrow^\dagger(\vec{R} - \vec{r}/2) >, \quad (5.2)$$

which plays the role of an effective wave function for the Cooper pairs. In the ground state (or more generally in thermodynamic equilibrium) in zero external field $F(\vec{r} : \vec{R})$ is a function only of the relative coordinate $\vec{r}$ and is then related to the function ϕ by a simple integral equation[35]; however, in the presence of e.g. a magnetic field or in nonequilibrium situations it may depend also on the center-of-mass coordinate $\vec{R}$. Considering the case where F is a function of $\vec{r}$ only, we may define the BCS zero-temperature coherence length ξ_0 as the effective "range" of the function F, which characteristically falls off as e^{-r/ξ_0} for $r >> \xi_0$; thus, in an intuitive sense ξ_0 is the "radius" of the Cooper pair at zero temperature. This radius is only weakly dependent on temperature and in particular is finite and of order ξ_0 even for $T \rightarrow T_c$.

In the Ginzburg-Landau theory[33], one introduces a phenomenological complex order parameter $\Psi(\vec{R})$, which with hindsight may be identified, apart from normalization, with the quantity $F(0 : \vec{R})$ defined in BCS theory; i.e. $\Psi(\vec{R})$ is effectively the wave function of the *center of mass* of a Cooper pair. One then expresses the free energy of the system, for T close to T_c, as a functional of $\Psi(\vec{R})$ in the standard way[33]. As a result there appear two characteristic lengths: the London penetration depth $\lambda(T)$, which is the length over which an external magnetic perturbation is screened out, and the "correlation length" or "temperature-dependent coherence length" $\xi(T)$, which effectively measures the length over which one has to bend the phase of the order parameter through an angle $\sim \pi$ in order that the bending (kinetic) energy associated with this process equals the condensation energy of the (stationary) superconducting phase. As is well known, the lengths $\lambda(T)$ and $\xi(T)$ both diverge as $(T_c - T)^{-1/2}$ in the limit $T \rightarrow T_c$, and their ratio determines whether the system

is a type-I or a type-II superconductor. If it is type-II, then $\lambda(T)$ may be obtained from the slope of the lower critical field $H_{c1}(T)$ and $\xi(T)$ from that of the upper critical field $H_{c2}(T)$. (Crudely speaking we have $H_{c2}\xi^2 \sim H_{c1}\lambda^2 \sim hc/2e$). Further, since $\lambda(T)$ is proportional to the zero-temperature coherence length λ_0 and $\lambda_0 = (m^*/ne^2)^{1/2}$, we can extract from $\lambda(T)$ the effective mass m^*. It is also a remarkable property of BCS theory (for a pure system) that the prefactor ξ_0' in $\xi(T) = \xi_0'(1 - T/T_c)^{-1/2}$ is, apart from a numerical factor, simply the zero-temperature BCS coherence length, ξ_0, i.e. the pair radius.

Now let us consider how the above ideas need to be modified in order to apply them to a highly anisotropic layered system such as a typical HTS. In the first place, it is clear that even for the ground state the form of eq.(5.1) is inadequate: in it we must replace $\phi(\vec{r}_1 - \vec{r}_2)$ by $\phi(\vec{R}+\vec{r}/2, \vec{R}-\vec{r}/2)$, where the dependence of ϕ on the z-component of the center-of-mass variable $\vec{R}$, at least, is likely to be substantial. (Intuitively, we expect ϕ to be large when $\vec{R}$ lies close to a CuO_2 plane and small elsewhere). This fact already means that we should be skeptical about simply taking over the relationship between BCS and GL concepts which holds in "old-fashioned" superconductors.

Suppose, however, that we for the moment ignore this caveat and simply proceed to write down an anisotropic Ginzburg-Landau theory; the only difference from the simple case is then that the coefficient of the gradient terms depends on whether the variation of Ψ is within the ab-planes or along the c-axis. We can now proceed just as above to derive *anisotropic* values $\lambda_i(T)$ and $\xi_i(T)$ of the Landau penetration depth and correlation length, and in this way extract λ_{i0} (hence a ratio of effective masses m_i^*) and ξ_{i0}. (Note that in extracting the ratio of the m_i^* we must postulate that the number of carriers n – in general not a *directly* measurable quantity – is the same for all directions). Such an analysis has been carried out in ref. 34, with the result that the "effective mass ratio" for YBCO, $m_\perp^*/m_\parallel^*$, is 37 ± 10, while the in-plane and c-axis correlation lengths $\xi_{\parallel 0}$, $\xi_{\perp 0}$ are approximately 12 and 3 Å respectively. It is sometimes remarked that the last result "should not be taken seriously" because $\xi_{\perp 0}$ is considerably less than the interplane spacing (~ 12 Å). However, this is not an indictment of GL theory as such, since $\xi_{\perp 0}$ has only a formal meaning as the prefactor in $\xi(T)$, which in turn is itself only a formal measure of the relative importance of bending energies. We get into difficulties only if we try to identify $\xi_{\perp 0}$ with the "radius of the Cooper pairs along the c-axis" – a maneuver for which there seems no obvious justification in cases where ϕ depends on $\vec{R}$ as well as $\vec{r}$. It is also worth noting that there is no good reason to identify the "effective masses" $m_\perp^*$ which are extracted from the data with the effective means for c-axis Bloch wave motion in the normal phase; indeed, as we have seen, a Bloch-wave description of the c-axis transport may not even exist.

This last point leads to the question: do we actually have any reason to believe that the GL free energy, which approximates the energy associated with bending of the order parameter in the z-direction by a simple term of the form $|d\Psi/dz|^2$, is even qualitatively correct? An alternative point of view is that in such a strongly layered system the order parameter $\Psi(\vec{r})$ should be thought of as defined only on the CuO_2 planes, and that interaction between the values of Ψ on a given plane and its neighbor should be described by a Josephson type of coupling, i.e.

$$\Delta F = -const. \int d\vec{r}_\parallel |\Psi_1(\vec{r}_\parallel)| \cdot |\Psi_2(\vec{r}_\parallel)| \cos(\chi_1(r_\parallel) - \chi_2(r_\parallel)), \quad (5.3)$$

where $\chi_i(\vec{r}_\parallel)$ is the phase of the complex order parameter $\Psi(r_\parallel)$, with $\vec{r}_\parallel$ the in-plane coordinate. This is the so-called Lawrence-Doniach[37] (LD) model.

It is important to note that under many circumstances it does not matter whether we use the GL or LD model: if only the phase of the order parameter varies along the c-axis (a common situation) and the variation is slow on a scale of the interlayer spacing a, then it is easy to see that the **cosine** can be replaced, apart from an irrelevant constant, by a term proportional to $(\vec{\nabla}\chi)^2$, and we recover the phase-bending part of the GL free energy. The difference between the two models is important only if it is possible to bend the order parameter sufficiently to make the above approximation unsatisfactory, while at the same time staying in the superconducting phase (i.e. the kinetic energy involved is less then the bulk condensation energy). It is straightforward to verify that the relevant condition is $\xi_\perp(T) \lesssim a$. Since $\xi(T)$ diverges as $(T_c - T)^{-1/2}$, we conclude that sufficiently close to T_c, where $\xi_\perp(T) >> a$, it is adequate to treat the thermodynamics by a 3D GL approach, whereas at lower temperatures ($\xi_\perp(T) \lesssim a$) it is necessary to use the LD model (which is never wrong!) explicitly, and we should expect some aspects of the thermodynamics, at least, to be governed by 2D rather than 3D fluctuations. This is exactly what is seen experimentally, see section IV. Note that we expect that the region of validity of 3D GL is smaller, the more "extreme" the layered nature.

A very striking piece of evidence in favor of the LD description for BSCCO has been obtained very recently by Kleiner et al[38] They found that the c-axis current-voltage characteristics obtained in their sample had multiple (~ 100) branches, which is exactly what one would expect if each pair of layers formed effectively a Josephson junction, so that the whole system can be thought of as a large number of junctions in series. While alternative explanations of the data of ref.

38 cannot at present be totally excluded, it is certainly very suggestive.

So far we have implicitly assumed that the order parameter is at least approximately uniform within each ab-plane (perhaps with small fluctuations superimposed). But what happens when it is not, for example when a magnetic field is applied along the c-axis and creates vortices? While for relatively "tightly coupled" HTS such as LSCO the traditional 3D vortex-like picture rnay be adequate (with the necessary allowance made for the anisotropy), in the case of extremely weakly coupled systems such as BSCCO or the intercalates a more appropriate zeroth-order model rnay be the "pancake vortex" model[39], which envisages the vortices as forming independently in the individual layers and then coupling together. The possible mechanisms of inter-plane coupling of vortex pancakes are then of great interest, since it is likely that it will dominate not only some of the static properties but also the in-plane transport.

To date, two obvious mechanisms of coupling have received attention, the Ampère (current-current) interaction and the Josephson coupling. The Ampère coupling dominates at short distances and gives rise to an interaction between parallel-oriented vortex pancakes on neighboring Iayers which is attractive but falls off as $1/r$. The Josephson coupling, by contrast, is very weak at short distances; however, it is "confining" for a neutral system, in the sense that to separate two (parallel-oriented) vortex pancakes as neighboring planes to infinity costs infinite energy, since the relative phase $\Delta\chi(r_{\|})$ is then finite for an infinite area. The interplay between the two effects is then nontrivial, and has been studied in a number of papers (see especially ref. 40); the net upshot is that a pair (or "string of beads") of parallel-oriented vortex pancakes always suffer an attractive interaction, which rnay however be rather weak.

Very recently, J.-M. Duan and the author[41] have discovered a third mechanism of coupling, which derives from the higher-order effects of the inter-plane Coulomb interaction. It rnay be shown[41,42] that these effects tend to favor a finite relative velocity of the electrons in neighboring planes, and it therefore gives rise to a net *repulsion* between parallel-oriented vortices. Whether this interaction can, in any realistic system, actually outweight (over a limited range of relative separation, of course) the effect of *both* the Josephson and the Ampère interactions-in which case we might expect interesting effects on the statics and dynamics–is at present unclear, though it seems plausible that for systems with low carrier density and relatively small interplane separation (but weak Josephson coupling) it rnay indeed do so.

VI. Conclusion

We have seen that the question of the nature of c-axis contact and transport is still an open one, both in the normal state and, to a somewhat lesser degree, in the superconducting state. As regards the former, there are strong a priori arguments against the Bloch-Sommerfeld picture, and alternative models based on "dynamically detuned hopping" are being developed: it is a challenge to bring the predictions of such models to the point where meaningful experiments to test them can be done. As regards the superconducting state, the picture which seerns to be emerging is that, at least in the more weakly coupled systems, the basic process of formation of the superconducting state takes place plane by plane, but the interplane coupling, which for the uniform case is best described as of Josephson type, plays a crucial role by stabilizing the individual planes against fluctuations which might increase the resistivity. Of course, there are many open questions concerning the relation between the normal-state c-axis properties and the Josephson (or other) coupling in the superconducting state. Finally, for the non-uniform case the importance of the new "Coulomb" mechanism of coupling of pancake vortices is still an open question.

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