Negative Absorption by an Ionized Gas Due to Coulomb Scattering

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A possible effect connected with the process of stimulated Bremsstrahlung emission and absorption by free electrons is the negative absorption of an electromagnetic wave in an electron gas with drift velocity relative the to scattering centers. Starting with the Fokker-Planclr equation some authors did find a negative conductivity for drift velocity higher than the electron thermal velocity. However, others, using classical arguments, show that the negative absorption due to collisions in a drifted electron plasma does not exist, because a drifted Maxwellian does not satisfy the Boltzman equation. Due to this controversy, in this paper we present a re-examination of this problem in terms of the center of mass approach, using classical and quantum arguments and considering the effects of the electron velocity fluctuations of the center of mass. Our conclusion is that for Coulomb centers the electrical conductivity is always positive.

I. Introduction

An interesting and possible effect connected with the process of stimulated Bremsstrahlung ernission and absorption by free electrons is the negative absorption of a linearly polarized electromagnetic wave in the presence of a uniform and sufficiently fast electron motion relative to the scattering Coulomb centers. In this case, the energy for the amplification (a possible plasma laser) could be supplied by the drifting energy. From the quantum-mechanical point of view, using a single-particle approach, this effect was first proposed by Marcuse[1] and has subsequently been considered by a number of authors[2,3]. For a semiconductor plasma, Vinogiadof[4] shows that negative absorption arises only in scattering by Coulomb potential, where, besides Coulomb centers he considers the scattering of semiconductor-current carriers by polar optical and acoustic phonons. However this approach does not suffice to study amplification in a plasma and it is necessary to solve the corresponding kinetic equation.

A treatment of this problem was described by Musha and Yoshida[5] starting with the Fokker-Planck equation which cannot, in principle, describe the electrical conductivity for frequencies in the vicinity of the plasma frequency. Tzoar[6], using a similar method to that given by Kohr and Luttinger [7], has derived the kinetic equation for drifted electron-ion system, which is not in thermal ecuilibrium. These authors did find a negative conductivity for drift velocity higher than the electron thermal velocity. However, Stenflo[a], using classical arguments, show that the negative absorption due to collisions in a drifted electron plasma does not exist, because a drifted Maxwellian does not satisfy the Boltzman equation. This conclusion can also be extracted from the Dreicer's theory[g].

On the other hand, it seems that in a plasma with a sufficient anisotropic electron distribution, induced Brernsstrahlung can lead to super-radiance, and hence to a possible free electron laser, which is related to negative conductivity and wave amplification. However, for optical frequencies, a quantum mechanical calculation is essential to get a conclusion about this process[11]. In this paper, we present a re-examination of stimulated Bremsstrahlung in terms of the center of mass approach, using classical and quantum mechanical arguments and considering the role of the electron distribution and the effects of the electron velocity fluctuations of the center of mass which determine the conditions for amplification of wave and negative absorption. In section II, we introduce the center of mass approach. In section III, we give the derivation of the average energy change and discuss the energy-balance. Finally, in section IV we give the main conclusions on this problem.

II. Formulation of the Center of Mass Approach

11.1. Hamiltonian

In this approach we extend the results on plasma transport obtained by us [11], by including the velocity fluctuations of the center of mass. We restrict

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the present study to a three-dimensional electron system considering N electrons scattered by n_i randomly distributed ions, in the presence of a spatially uniform vector-potencial $\vec{A}(t) = \frac{e}{\omega_0} E_0 \cos\omega_0 t$. The time-dependent Hamiltonian can be written as in refs. [10] and [11].

$$H = \sum_i \frac{\left[\vec{p}_i - e\vec{A}(t)/c\right]^2}{2m} + \sum_{ij,i<j} \frac{e^2}{|\vec{r}_i - \vec{r}_j|} + \sum_{ia} u(\vec{r}_i - \mathrm{R}_\alpha). \quad (1)$$

Here $\vec{r}_i$ and $\vec{p}_i = -i\hbar\nabla_i$ are the coordinate and momentum operator of the i^{th} electron with mass m and charge e. The second term on the right hand side is the Coulomb interaction between i^{th} and j^{th} electrons and the term $u(\vec{r} - 8,)$ denotes the potential at $\vec{r}$ due to the ion at $\vec{R}_\alpha$, which is randomly located (RPA). The Schrodinger equation for i^{th} electron is

$$\left\{ \frac{\left[\vec{p}_i - e\vec{A}(t)/c\right]^2}{2m} + \sum_j \frac{e^2}{|\vec{r}_i - \vec{r}_j|} + \sum_\alpha u(\vec{r}_i - \vec{R}_\alpha) \right\} \psi(\vec{r}_i, t) = i\hbar \frac{\partial}{\partial t} \psi(\vec{r}_i, t). \quad (2)$$

We perform a unitary transformation on equation (2), that is, transformation of the type

$$\psi(\vec{r}, r) = U_1 U_2 \psi'(\vec{r}, t), \quad (3)$$

where

$$U_1 U_2 = e^{i\frac{\vec{p}\cdot\vec{a}(t)}{\hbar}} e^{iS(t)} \quad (4)$$

The function $\vec{a}(t)$ produces a translation in space and is given by

$$\vec{a}(t) = \frac{e\vec{E}_0}{m\omega_0^2} \sin\omega_0 t. \quad (5)$$

Under a unitary transformation the Schrodinger equation for ψ' will have a modified Hamiltonian $\hat{H}_i$, which is independent of the function $S(t)$. The total Harniltonian $\hat{H} = \sum_i \hat{H}_i$ is

$$\hat{H} = \sum_i \frac{p_i^2}{2m} + \sum_{i,j} \frac{e^2}{|\vec{r}_i - \vec{r}_j|} + \sum_{i,\alpha} u(\vec{r}_i - \vec{R}_\alpha - \vec{a}(t)). \quad (6)$$

In this form we can study the nonlinear effects on the transport phenomena when a strong high frequency electric field is applied. But, instead of using the Schrodinger picture, we use the Heisenberg picture which treats the dynamical variables as time-dependent. In Heisenberg picture, the concept of center of mass appears as a simple method for studying the transport properties of a many body system in the isothermal approximation[12]. The basis of this method lies in separating the center-of-mass motion from the relative motion of electrons in the Hamiltonian (equation (6)). In these terms we can express the Hamiltonian (6) as

$$\hat{H} = \hat{H}_{CM} + \hat{H}_e + \hat{H}_{ee} + \hat{H}_{ei}, \quad (7)$$

$$\hat{H}_{CM} = \frac{\hat{P}^2}{2M}, \quad (8)$$

$$\hat{H}_e + \hat{H}_{ee} = \sum_i \frac{(p_i')^2}{2M} + \sum_{i,j} \frac{e^2}{|\vec{r}_i' - \vec{r}_j'|} = \sum_k E_k c_k^\dagger c_k + \frac{1}{2} \sum_q V(\vec{q})(\rho_q^\dagger \rho_q - N), \quad (9)$$

where $\hat{H}_{CM}$ is the center of mass Hamiltonian and $\vec{P}$ is the center of mass momentum operators. N is the total number of electrons and $\mathrm{M} = N m$ is the total mass of electrons. H, is the free-electron Hamiltonian in relative coordinates, $E_k = \hbar^2 k^2/2m$ with $\hbar\vec{k}$ the momentum of electron in state $\vec{k}$. $c_k^\dagger$ and c_k are electron creation and annihilation operators in relative coordinates. H,, is the Coulomb interaction in relative coordinates. The electron-ion interaction is described by

$$\hat{H}_{ei} = \sum_{\vec{q},\alpha} U(q) e^{i\vec{q}\cdot(\vec{R} - \vec{r}_\alpha - \vec{a}(t))} \hat{\rho}_q + cc, \quad (10)$$

where $\hat{\rho}_q = \sum_k c_{k+q}^\dagger c_k$ is the electron density operator, $\vec{R}$ is the coordinate of the center of mass and $U(q) = 4\pi Z e^2/q^2$. A completely homogeneous one-component plasma, like a homogeneous electron gas, cannot absorb light. For absorption to take place the Harniltonian must contain a term which does not commute with $\vec{P}$. In this case it is $\hat{H}_{ei}$.

II.2. Quantum-kinetic equations for the drift velocity of electrons and energy change

Starting from the quantum-mechanical Liouville equation, we can derive the kinetic equations for a set of macroscopic observables, $\{O_\ell(t)\}$. Here we choose $\{O_\ell(t)\} = \vec{v}, f_k(t), W(t)$. The corresponding operators are $\hat{O}_\ell = [\frac{\hat{P}}{M}, c_k^\dagger c_k, \hat{H}_{CM}]$ which are connected with observables by

$$O_\ell(t) = Tr\{\hat{O}_\ell \hat{\rho}(t)\}, \quad (11)$$

where $\hat{\rho}$ is the statistical density matrix. If we solve the Liouville equation to the lowest order, the rate of

change of any function S of $O(t)$ is given by [10]:

$$\frac{d}{dt}(S(O(t))) = \frac{i}{\hbar} < [H_0(t), S(O(t))] > + \frac{i}{\hbar} < \left[\hat{V}(t), S(\hat{O}(t))\right] > - \left(\frac{i}{\hbar}\right)^2 \lim_{\varepsilon \to 0} \int_{-\infty}^{t} dt' e^{\varepsilon(t-t')} < \left[\dot{S}(\hat{O}(t)), \hat{V}(t')\right] >, \quad (12)$$

where $\dot{S}(\hat{O}(t)) = -\frac{i}{\hbar}\left[S(\hat{O}(t)), \hat{V}(t)\right]$, $\left[\hat{A}, \hat{B}\right]$ is the commutator of operators $\hat{A}$ and B, $\hat{V} = \hat{H}_{ee} + \hat{H}_{ei}$ and

$$\hat{V}(t') = \exp\left(\frac{i}{\hbar}\int_t^{t'} \hat{H}_0(s)ds\right)\hat{V}(t) \exp\left(-\frac{i}{\hbar}\int_t^{t'} \hat{H}_0(s)ds\right), \quad (13)$$

where $\hat{H}_0 = \hat{H}_{CM} + \mathrm{H}_{,}$. $< \ldots \cdot > \equiv Tr[\hat{\rho}_0(\cdot\,\cdot)]$ with $\hat{\rho}_0$ the unperturbed density matrix, and Tr means trace. Thus, the frictional force operator is

$$\hat{F} = -\frac{i}{\hbar}\left[\vec{P}, \hat{V}\right] = -i\sum_{q,\alpha} \vec{q} U(\vec{q}) e^{i\vec{q}\cdot(\hat{\vec{R}} - \vec{R}_\alpha - \vec{a}(t))}\hat{\rho}_q + cc \quad (14)$$

and the rate of thange of the center of mass energy operator is

$$\dot{H}_{CM} = -\frac{i}{\hbar}\left[\hat{H}_{CM}, \hat{V}\right] = -i\sum_{q,\alpha} \vec{q}\cdot\dot{\vec{R}} U(\vec{q}) e^{i\vec{q}\cdot(\hat{\vec{R}} - \vec{R}_\alpha - \vec{a}(t))}\hat{\rho}_q + cc. \quad (15)$$

The operators in equation (12) are in the interaction representation, so $\vec{R}$ changes with time as the coordinate of a free particle with a large mass M. Equations (14) and (15) illustrate the microscopic evolution of the momentum and eriergy of the center of mass of electrons, respectively The relative electron coordinates are described by the statistical density matrix $\rho(t)$. The center of mass variable enters into the density matrix, while the system of relative electrons acts as a heat-bath, which is coupled to the center of mass through electron-ion interactions. The heat-bath variable $\hat{\rho}_q$ defined in the second quantized form may be elirninated from the equations using the retarded Green's function $\hat{\Pi}(q,t)$ [12]. By averaging equations (14) and (15) over the ensemble (center of mass and heat-bath and ion system), we will obtain the macroscopic momentum and energy transport equations. This is the same as to take the corresponding trace of $\vec{P}$ and $\hat{H}_{CM}$, respectively.

Here we are interested in the "steady state" high frequency transport, where the fluctuation terms of the transport equations disappear, but the velocity fluctuations of the center of mass remain in the quantum-kinetic equations.

We begin with the requirement

$$O_\ell(t) = Tr\left[\hat{O}_\ell(t), \hat{\rho}_0(t)\right]. \quad (16)$$

Then, by substituting equations (13), (14), (15), and (16) into equation (12), we can obtain (to the lowest orden in $|U(\vec{q})|^2$) the following equations for drift velocity $\vec{R}$ and energy W_{CM} of the center of rnass:

$$\frac{Nm}{vol}\frac{d\vec{R}(t)}{dt} = -i\sum_{q,n} \vec{q} \lim_{\varepsilon \to 0^+} \int_{-\infty}^{t} dt' e^{-\varepsilon(t-t')} \exp\left(i\vec{q}\cdot\int_{t'}^{t} \dot{\vec{R}}(s)ds\right) n_i |U(\vec{q})|^2 J_n^2(\vec{q}\cdot\vec{a}) e^{in\omega_0(t-t')}\hat{\Pi}(\vec{q},(t-t')) + cc, \quad (17)$$

$$\dot{W}_{CM} = \frac{< \dot{H}_{CM} >}{vol} = -i\sum_{q,n} \vec{q}\cdot\dot{\vec{R}} \lim_{\varepsilon \to 0^+} \int_{-\infty}^{t} dt' e^{-\varepsilon(t-t')} \exp\left(i\vec{q}\cdot\int_{t'}^{t} \dot{\vec{R}}(s)ds\right) n_i |U(\vec{q})|^2 J_n^2(\vec{q}\cdot\vec{a}) e^{in\omega_0(t-t')}\hat{\Pi}(\vec{q},(t-t')) + cc. \quad (18)$$

The corresponding memory function is

$$\mu = \sum_q \vec{q}\vec{q}\int_{-\infty}^{t} dt' \exp\left(i\vec{q}\cdot\int_{t'}^{t} \dot{\vec{R}}(s)ds\right) n_i |U(\vec{q}|^2 \exp\left(i\vec{q}\cdot(\vec{a}(t') - \vec{a}(t))\right) \cdot\left(\frac{1}{\hbar}(E_{k+q} - E_k)\right)^{-1}. \quad (19)$$

In a similar way we obtain the energy rate for the system of relative electrons

$$\dot{W}_e = \frac{< \dot{H}_e >}{vol} = -i\sum_{k,q,n} \vec{q}\cdot\frac{i}{\hbar}(E_{k+q} - E_k) \lim_{\varepsilon \to 0^+} \int_{-\infty}^{t} dt' e^{-\varepsilon(t-t')} \exp\left(i\vec{q}\cdot\int_{t'}^{t} \dot{\vec{R}}(s)ds\right) n_i |U(\vec{q})|^2 J_n^2(\vec{q}\cdot\vec{a}) e^{in\omega_0(t-t')}\Pi(\vec{k},\vec{q},(t-t')) + cc, \quad (20)$$

where

$$\hat{\Pi}(\vec{q},t) = \sum_k \Pi(\vec{k},\vec{q},t)$$

and

$$\hat{\Pi}(\vec{q},(t-t')) \equiv < -\frac{i}{\hbar}[\hat{\rho}_g(t),\hat{\rho}_{-q}(t')] > = -\frac{i}{\hbar}\sum_k \frac{f_{k+q}-f_k}{\varepsilon(\vec{q},(t-t'))} \exp\left[\frac{i}{\hbar}(E_{k+q}-E_k)(t-t')\right], \quad (21)$$

and $\hat{\Pi}(\vec{q},\omega)$ is the Fourier transformation of the retarded Green's function $\hat{\Pi}(\vec{q},t)$

$$\hat{\Pi}(\vec{q},\omega) = \int_{-\infty}^{\infty} e^{i\omega t}\hat{\Pi}(\vec{q},t)dt. \quad (22)$$

In equations (17), (18) and (20) the complex conjugate terms are added because equations (14) and (15) must be Hermitian so their average values must be real. In these expressions, we have taken the factor related to the center of mass motion as

$$< \exp(i\vec{q}\cdot\hat{R}(t))\exp\left(-i\vec{q}\cdot\hat{R}(t')\right) >= \exp\left(i\vec{q}\cdot\int_{t'}^{t}\dot{\vec{R}}(s)ds\right), \quad (23)$$

with $\vec{R}(s) = \vec{P}(s)/M$ as the drift velocity of the electron system. Here the fluctuation of the coordinates of the center-of-mass of electrons may be included. We note that this factor reflects the non linear dependence of the kinetic equations on the electron drift velocity and equation (23) is clearly consistent with the large-N approximation.

The factor related to the ion dynamics, considering the presence of the laser field is given by[10] :

$$\sum_{q,\alpha,\alpha'} e^{i\vec{q}\cdot(\vec{R}_\alpha+\vec{a}(t'))}e^{i\vec{q}\cdot(\vec{R}'_\alpha-\vec{a}(t))}U(\vec{q})U(-\vec{q}) = n_i\, vol\sum_n J_n^2(\vec{q}\cdot\vec{a}_0)\mid U(\vec{q})\mid^2 e^{in\omega_0(t-t')}. \quad (24)$$

We can see that the transport equations contains a memory function in a non-Markov form which envolves a surn over multiphoton terms and has a non linear dependence on the velocity of the center of mass, $\vec{R}$.

The kinetic equations (17), (18), and (20) correspond to generalized Langevin equations. One can study a variety of transport properties such as electrical conductivity, stopping power and inverse Bremsstrahlung process[11]. As the mernory funtion is associated to the density-density response function, we find that when the fluctuation of the coordinates of the center of mass, $\delta\vec{R}$, is included, the structure of the response function may change significantly.

III. Plasma Transport Equations and Electrical Conductivity

Equations (17), (18), and (20) are still in microscopic form, from the view point of the center of mass. The momentum and energy equations are quite rigorous and general. Although we have restricted our treatment to the lowest-order in the electron-ion interaction, our results are valid for any value of the laser electric field. However the evaluation of the steady-state transport equations, including the fluctuation parts of the velocity $\vec{R}(t)$, can be carried out only approximately. In general, we can denote the velocity $\vec{R}$ as the sum of the drift velocity $\vec{V}_{CM}$ and its fluctuation $\delta\vec{R}$, with $\overline{\delta\vec{R}} = 0$, where a bar denotes the ensemble average over the center of mass cooodinates. The factor affected by this average is equation (23), so that using the cumulant approximation[13] we have

$$\overline{\exp\left(i\vec{q}\cdot\int_{t'}^{t}\vec{V}(s)ds\right)} = \exp\left((i\vec{q}\cdot\vec{V}_{CM}-Dq^2)(t-t')\right) \quad (25)$$

then, in the random phase approximation we have

$$\hat{\Pi}(\vec{q},\omega) = \frac{\Pi_0(\vec{q},\omega)}{1-v(\vec{q}),\Pi_0(\vec{q},\omega)}, \quad (26)$$

in which $v(\vec{q}) = 4\pi e^2/q^2$ and $\Pi_0(\vec{q},\omega)$ is given by

$$\Pi_0(\vec{q},\omega) = 2\sum_k \frac{f(\vec{k}+\vec{q})-f(\vec{k})}{(\varepsilon_{k+q}-\varepsilon_k-\omega-i\delta)}, \quad (27)$$

with $\delta = Dq^2$, where D is the diffusion constant for the center of mass coordinate[13]. In this form we have a generalised dielectric function, which incorporates velocity fluctuation effects

$$\bar{\varepsilon} = \varepsilon_{RPA} = 1 - v(\vec{q})\Pi_0(\vec{q},\omega). \quad (28)$$

Here, D can be treated as a control parameter, which introduces a damping effect on the electron response and the dispersion relation. Finally, by substituting equations (25), (26), (27) and (28) into equations (17) and (18), and taking the average over the $\delta\vec{R}$ distribution, we obtain

$$\bar{\vec{f}} = \frac{Nm}{vol}\frac{d(\vec{V}_{CM}+\delta\vec{R})}{dt} = \sum_{q,n}\vec{q}n_i\frac{\mid U(\vec{q})\mid^2}{v(\vec{q})}J_n^2(\vec{q}\cdot\vec{a})\, Im\left\{\frac{1}{\bar{\varepsilon}(\vec{q},\delta,n\omega_0-\vec{q}\cdot\vec{V}_{CM})}\right\} \quad (29)$$

and

$$\frac{< H_{CM} >}{vol} = \overline{W_{CM}}$$

$$= \sum_{q,n} \left(\vec{q}\cdot(\vec{V}_{CM}+\delta\vec{R})\right) n_i \frac{|U(\vec{q})|^2}{v(\vec{q})} J_n^2(\vec{q}\cdot\vec{a}) \; Im\left\{\frac{1}{\bar{\varepsilon}(\vec{q},\delta,n\omega_0-\vec{q}\cdot\vec{V}_{CM})}\right\}. \quad (30)$$

In a similar way we obtain the energy rate for the relative electrons

$$\frac{<\dot{H}_e>}{vol} = \overline{\dot{W}_e} = \sum_{q,n} \left(n\omega_0+\vec{q}\cdot\vec{V}_{CM}\right) n_i \frac{|U(\vec{q})|^2}{v(\vec{q})} J_n^2(\vec{q}\cdot\vec{a}) \; Im\left\{\frac{1}{\varepsilon(\vec{q},\delta,n\omega_0-\vec{q}\cdot\vec{V}_{CM})}\right\}. \quad (31)$$

In this form, equations (28) - (31) are the generalized expressions of the plasma transport equations, which have included the diffusion constant of the center of mass. In general D is field dependent, but in the classical limit, we can use the Einstein relation $D = \frac{k_B T}{N m \nu}$, where $\frac{1}{\nu}$ is the moinentum transport time. With Dq^2, in the large-N approximation, clearly for non degenerated plasma, the correction on the memory function, $\overline{\mu}$, and on the energy loss function, $Im[1/\bar{\epsilon}]$, can be neglected. However, for degenerated plasma, specially for quasi-one and two-dimensional electron gas, the correction due to the dampirig Dq^2 may be significant for smaller systems and smaller densities. We note that our theory have total validity for 1-D and 2-D electron gas, when the "volume" is redefined. In our case, if D = 0, expression (29) reduces to the frictional force with multiphoton interactions given by Jones and Lee[14]. If D = 0 and $V_{CM} = -v_i$, equation (30) agrees with the recent result given by Arista et al.[15]. If D = 0 and $\vec{q}\cdot\vec{V}_{CM} << n\omega_0$, equation (31) corresponds to the Rae and Burnnet's result[16]. Also, the linearization of the average memory function, μ, with $D = 0$, leads to the linear prediction of the high-frequericy electrical conductivity[17].

In connection with transport coefficients, the energy loss-method has also been proposed to calculate the d.c. resistivity ($\rho \sim 1/\sigma$) of a many body system [18]. The electrical resistivity ρ is defined as

$$\rho_{dc} = \lim_{J\to 0} \frac{<\vec{J}\cdot\vec{E}>}{J^2}, \quad (32)$$

where $\vec{J} = n_e e \vec{V}_{CM}$; however, this method is not appropriate in the presence of laser field and the last coefficient must be seen more carefully. For a revision of adiabatic and isothermal resistivity, see refs.[19,20]. However at high frequency, ($\nu_{coll}/\omega_0 \ll 1$), the derivation of the total energy rate is quite direct if we take H_{CM} and a similar expression for $\dot{H}_e$. After averaging the ion, the center of mass and relative electron coordinates, we have (if we neglect the particle recoil)

$$<\dot{H}_e> + <\dot{H}_{CM}> = <\vec{J}\cdot\vec{E}>, \quad (33)$$

whith $<\vec{J}\cdot\vec{E}>$ given by

$$\frac{<\vec{J}\cdot\vec{E}>}{v_0} = \sum_{q,n} n_i \frac{|U(\vec{q})|^2}{v(q)} n\omega_0 \cdot Im\left\{\frac{1}{\varepsilon(\vec{q},\delta,n\omega_0-\vec{q}\cdot\vec{V}_{CM})}\right\}, \quad (34)$$

where $\vec{J}$ is the current density of the system. Due to the presence of $\vec{V}_{CM}$, it is necessary to distinguish within $<\vec{J}.\vec{E}>$ the irreversible dissipation $<P_a>$ from the reversible kinetic flux. Otherwise, the electrical conductivity given by $\sigma_{ac} = 2\frac{<\vec{J}\cdot\vec{E}>}{E^2}$ can be negative if $V_{CM} > v_t$ (thermal velocity of electrons). Early papers[5,21] show this possibility for a fully ionized plasma, overlooking the presence of the particle kinetic flux. From equation **(34)** taking n = 1 and $\omega_0 \ll \vec{q}\vec{V}_{CM}$, we obtain that $a_r < 0$, which corresponds to the results of these papers. Also, the same conclusion can be obtained for a semiconductor plasma, where the claim of Vinogradov[4] seem to be wrong. In this sense, it is of primary importance to achieve a correct energy balance. Fundamentally, the rf-absorption must be equal to with the energy moment of the quasilinear term obtained from the kinetic theory. The knowledge of an adequate approximation for the high frequency current is not sufficient to establish the correct expression for the local power absorption. Indeed, $\vec{J}\vec{E}$ differs from P_a by the reversible kinetic flux part, which is due to the coherent sloshing motion of the electrons.

On local power absorption in tokamak geometry in hot inhomogeneous plasmas, the problem of the reversible kinetic flux has also been recently considered by Smithe[22] and Brambilla and Krüchen[23], following the method suggested by McVey et al.[24]. Here, the subjet of local wave energy in plasmas is treated via quasilinear theory. As our approach is also within the quasilinear theory, we can obtain $<P_a>$ using a heuristic argument: only the relative electron system absorbes energy and the center of mass can be considered as a particle which gains or loses energy depending on the energy exchange with the radiation field. Thus, the differential quantity for $<\vec{J}\cdot\vec{E}>$ is

$$<\vec{J}\cdot\vec{E}>_{q,n} = \frac{d}{dq}<\vec{J}\cdot\vec{E}>_n = ni\frac{|U(\vec{q})|^2}{v(q)} n\omega_0 \; Im\left\{\frac{1}{\varepsilon(\vec{q},\delta,n\omega_0-\vec{q}\cdot\vec{V}_{CM})}\right\}.$$

Now making the ratio between $<\vec{J}\cdot\vec{E}>_{q,n}$ and $<P_a>_{q,n}$

$$\frac{<P_a>_{q,n}}{<\vec{J}\cdot\vec{E}>_{q,n}} = \frac{n\omega_0}{n\omega_0+\vec{q}\cdot\vec{V}_{CM}}, \quad (35)$$

we see that the absorption power is given by

$$< P_a >= \sum_{q,n} n_i \frac{|U(\vec{q})|^2}{v(q)} (n\omega_0)^2 J_n^2(x) \frac{1}{(n\omega_0 - \vec{q}\cdot\vec{V}_{CM})} Im\left\{\frac{1}{\bar{\varepsilon}(\vec{q},\delta,n\omega_0 - \vec{q}\cdot\vec{V}_{CM})}\right\} . \quad (36)$$

This expression is always positive and corresponds to a system that can only absorb energy (inverse Bremsstrahlung process). From (36) we can always obtain positive scattering probabilities. If $V_{CM} \rightarrow 0$, we reproduce the heating rate obtained classically by Jones and Lee[14], Klimontovich[25] and others[26,27]. The velocity fluctuations of the centre of mass are neglected because we are considering a three dimensional system. When it is assumed that the unperturbed velocity distribution function is a displaced Maxwellian, the system cannot have a resistive instability which should produce a negative conductivity. In a weak externa1 field the distribution function will become isotropic in a time $(\nu_{coll})^1$ where ν_{coll} is the usual collision frequency for a fully ionized plasma. In a strong field the electrons run away in a time $t \leq \nu_{coll}$, so the anisotropy exists in a time too short to be able to account for any instability. This result can be extended to semiconductor plasma where, besides Coulomb centers, we can have acoustic phonons and polar optical phonons.

IV. Conclusions

Using the center of mass approach we are able to show that for an electron gas with a displaced maxwellian the electrical conductivity is always positive and this result agrees with the conclusion given by Stenflo and Dreicer. Thus, the amplification of waves in plasma (maser or laser action) should not be attributed to the collision between electrons and ions but rather to some instability (two-stream instability). Also, for a three dimensional Coulomb plasma in the large-N approximation, we show that the velocity fluctuations of the center of mass are negligible.

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References

1. D. Marcuse, Bell Sys. Tech. Journal, 41, 1557 (1961).
2. F. V. Bunkin, A. E. Kazakov and M. V. Federov, Sov. Phys. Usp, 15, 416 (1974).
3. P. V. Elyutin, Sov. Phys. JETP, **38,** 1097 (1974).
4. A. V. Vinogradof, Sov. Phys. JETP, 43, 531 (1976).
5. T. Musha and F. Yoshida, Phys.Rev. **133A**, 1303 (1964).
6. N. Tzoar, Phys. Rev. 162, **123** (1967).
7. W. Kohn and J. M. Luttinger, Phys. Rev. 108, 590 (1957).
8. L. Stenflo, Plasma Physics, 10, 801 (1968).
9. H. Dreicer, Phys. Rev., 117, 329 (1960).
10. H. Torres-Silva, Interação Multifotônica em Plasmas Criados por Laser Superintenso. PhD thesis, Universidade Estadual de Campinas, Campinas, December 1990.
11. H. Torres-Silva and P. H. Sakanaka, J. of Phys. Soc. Japan 60, 2555 (1991).
12. X. L. Lei and C. S. Ting, Phys. Rev. B 32, 1112 (1985).
13. G. Y. Hu and R. F. O'Connell, Physica **149A**, 1 (1988).
14. R. Jones and K. Lee, Phys. Fluids 25, 2307 (1982).
15. N. Arista, R. M. O. Galvão, and L. C. M. Miranda, J. of the Phys. Soc. of Japan 59, 544 (1990).
16. S. Rae and K. Burnett, Phys. Fluids B 2, 1015 (1990).
17. N. Horing, X. Lei and H. Cui, Phys. Rev. B 33, 6929 (1986).
18. E. Gerlach, J. Phys. C 19, 4585 (1986).
19. R. S. Fishman, Phys. Rev. B 39, 2994 (1989).
20. R. S. Fishman and G. D. Mahan, Phys. Rev. B 39, 2990 (1989).
21. R. Karapetyan, Sov. Phys. Tech. Phys. 21, 152 (1976).
22. D. N. Smithe, Plasma Physics and Controlled Fusion 30, 1083 (1989).
23. M. Brambilla and T. Krucken, Plasma Physics and Controlled Fusion 30, 1083 (1988).
24. B. D. MacVey et al. Phys. Rev. Lett. 55, 507 (1985).
25. Y. Klimontovich, *Kinetic Theoy of Non Ideal Gases and Non Ideal Plasmas.* (Pergamon N.Y., 1982).
26. S. Rand, Phys. Rev. B 136, 231 (1964).
27. F. Albini and S. Rand, Phys. Fluids 8, 134 (1965).