Relativistic Absorption in Plasma by an Ultrastrong Laser Field

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We formulate the problem of the interaction of an ultrastrong laser pulse within the framework of the quasifree state of an electron. Starting from a quantum mechanical approach the inelastic cross-section between charged particles is calculated. From this, a simple and useful approximation is obtained for the multiphoton energy transfer processes which accompany the relativistic scattering of a charged particle and an effective collision frequency is found. The amplitude of the electron oscillatory velocity v_0 in the radiation field is assumed to exceed the electron thermal velocity v_{te} and $v_0 \sim c$. Under these conditions, the absorption rate is independent of the initial distribution function and depends only on the photon flux intensity. The effective absorption coefficient is scaled as I^r, where the r-parameter is in the range of $0.5 \leq r \leq 0.75$. Our results are compared with those of other formalisms and experimental results.

I. Introduction

The interaction of ultrahigh-power beams[1] with plasma is rich in a variety of wave-particle phenomena. These phenomena (relativistic optical guiding of the laser beam, excitation of coherent radiation at harmonics of the fundamental laser frequency, generation of large-amplitude plasma waves, resonant absorption and non linear inverse Bremsstrahlung) become particular interesting and involved when the laser power is sufficiently intense to cause the electron oscillation (quiver) velocity to became highly relativistic. At present, the investigation of the relativistic absorption of intense electromagnetic radiation through electron-ion collision processes in a fully ionized plasma is still incomplete. The major difficulty with the existing theories at high radiation intensities is because of large numbers of photon which must be included and, in these cases, the direct evaluation of the contributions from many photons to obtain the total absorption coefficient is very difficult.

Recently[2], (hereafter referred to as I), through a simple quantum mechanical approach, the inverse Bremsstrahlung problem for CO_2 laser-plasma interaction was studied by calculating the transition probability[3] per unit time and the rate at which energy is absorbed by electrons with a specified energy distribution, where we find that $\gamma_{eff} \sim I^p$, $(p \leq 1/2)$. However this scale factor is usefull to model CO_2 laser-plasma experiments when the density gradient length L, does not depend on the laser intensity.

In some papers, the non-relativistic absorption theory was developed at different quantum limits. For the strong interaction $v_0 \gg v_{te}$, they show different dependentes: while Seely[4] indicates that the effective collision frequency, γ_{eff}, is proportional to $I^{-3/2}$ (I is the laser intensity), Schlessinger[5] shows that $\gamma_{eff} \sim I^{-5/2}$ and Nicholson[6] claims that $\gamma_{eff} \sim I^{-1/2}$. In ref. [7] it is also shown that the effective collisional frequency is enhanced due to the non-ponderomotive confining force. Thus, the different non-relativistic formalisms lead to contradictory results for the heating of plasma by multiphoton absorption. Also, at relativistic intensities when the energy of a free electron oscillating in a strong laser field, can exceed its rest energy, m_0c^2, an analysis of the processes by which photons are absorbed, by which the heating occurs and by which the energy is redistributed between electrons and ions presents still some difficulties. The aim of this paper is to generalize the theory presented in ref I, which allows to model CO_2 as well as Nd-Yag laser-plasma experiments, when $L = I^s$, being $s < 1$ at high laser intensities. Our relativistic quantum-mechanical approach is less complex than Rashid's treatment[8] and Krstic's quantum viewpoint[9]. Additional insight is obtained specifically when the plasma has shielding and difraction effects are included.

In this paper, we reproduce some calculations of ref I. Section II is dedicated to the development of a formalism to obtain the inelastic cross-section from transition rates. In Section III we calculate the effective collision frequency and the absorption coefficient and we com-

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pare it with those of published results. In section IV we give the main conclusions.

II. Transition Rate and Cross Section

The transition probability and the inelastic collision cross-section of fast electrons with atoms or ions can be obtained by the first Born approximation, with a simple heuristic "golden rule" calculation using the screened electron-ion interaction, which is modified by the laser beam. When the ion-ion correlations are considered, we must construct the generalized matrix element of the potential, (ref I)

$$\Phi(k) = \int\int\int \psi_{if}^* e^{-i\vec{k}_f\cdot\vec{r}} g e^{i\vec{k}_i\cdot\vec{r}} \psi_{ii}\, d^3r \,. \qquad (1)$$

The operator g gives the strength of the interaction, and ψ_{ii} and ψ_{if} are the initial and final ion wave functions. The electron wave functions are taken to be normalized plane waves. A meaningful discussion of the operator requires a complete relativistic description.

We must consider the concept of a quasifree state of an electron using the Volkov state formulation[3]. From the Volkov solution of the minimally coupled Dirac equation, we obtain the time-average quasifour-momentum $p^{*\mu}$ given by (h = c = 1)

$$p^{*\mu} = p^{\mu} + \left[\frac{e^2A_0^2}{4k_i p}\right] k_i^{\mu} \,, \qquad (2)$$

so that the effective mass m* can be defined by

$$m^{*2} = m^2 + \frac{1}{2}(eA_0)^2 \,, \qquad (3)$$

where p^{μ} is the four-rnomentum of the electron, without laser field. When the quiver energy of the electron is comparable to or larger than its rest mass energy, we see from equations (2) and (3), that the radiation field could give rise to relativistic effects for a nonrelativistic electron. Hence, once this relativistic contribution has been taken into account in a time-averaged manner, we can always go back to the nonrelativistic picture if the bare electron energy is nonrelativistic.

We start by defining the plasma potential of an energy perturbation optnator as

$$\Phi(k) = \int e^{-i\vec{k}\cdot\vec{r}} \Phi^{P}\, d^3r \,, \qquad (4)$$

where $\Phi_p = \Phi_{eff}(\vec{r}) = \int \psi_{if}^* \Phi^I(\vec{r}_i)\psi_{ii}\, d^3r$ is the effective potential when the ions are not at rest; k = $(\vec{p}_f^* - \vec{p}_i^*)/\hbar$, and $\Phi_p(\vec{k})$ is given by the Poisson equation

$$k^2\Phi_p(\vec{k}) = \frac{4\pi\rho(\vec{k})}{\varepsilon^*(\vec{k})} \,, \qquad (5)$$

where $\rho(\vec{k})$ is the Fourier component of the quasi-static charge and $\varepsilon^*(\vec{k})$ is the dielectric constant which, for close collissions k >> k_D, can take into account some quamtum wave effectsin electron-ion cross section. The expression for $\Phi_p(\vec{k})$, does not corresponds to the Jones and Lee's result $\Phi(\vec{k}, t)$ [10], because Φ_p is a time-average manner of the Poisson-equation, where $\vec{k}$ is dressed by the laser field. Then, considering dressed states, equation (5) becomes

$$\Phi_p(\vec{k}) = \frac{4\pi e^2}{k^2}\frac{F(\vec{k}, I)}{\varepsilon^*(\vec{k})} \,, \qquad (6)$$

where the form factor F, defined as

$$F(\vec{k}, I) = \int e^{-\vec{k}\cdot\vec{r}} \rho(\vec{r}) d^3r \,, \qquad (7)$$

takes into account the presence of the laser field I. This parameter can represent different collisional regimes found in laser-plasma interactions. Using equations (4), (6) and (7), the transition probability becomes [11]

$$T(p_i^* \to p_f^*) = \frac{2\pi}{\hbar}\left(\frac{4\pi e^2}{k^2}\right)^2 \sum_n \left|\frac{F(\vec{k}, I)}{\varepsilon^*(\vec{k})}\right|^2 J_n^2(x)\, \delta(W_f - W_i - nhw) \,. \qquad (8)$$

where $W_{fi} = p_{fi}^{*2}/2m$ and $x = (eE_0/\hbar m\omega^2)\Delta p_{\perp}$; the remander of the symbols have obvious meanings.

This equation can be used to obtain the kinetic energy for the electrons, so that the change in average kinetic energy is:

$$\frac{d<\varepsilon>}{dt} = \int d^3v_f\, \frac{mv_f^2}{2}\, \frac{\partial f}{\partial t}(v_f) \,. \qquad (9)$$

The right hand side of equation (9) may be compared with $\gamma_{eff} e^2 E_0^2/2m\omega^2$ to obtain the effective collision frequency (see ref.I).

Normally the construction of the nonrelativistic collision term, $\partial f(v_f)/\partial t$, for electrons with momentum $\vec{p}_f$ is very complex (Seely[4], Kim[12]). For the relativistic case, when the electrons in the plasma are in states specified by the quasi-four momentum, it is possible to obtain a modified Boltzman equation ([13]). However, the problem analysed by Rashid corresponds to a cold plasma where the energy of the photon is much greater than the mean kinetic energy.

In this paper, we are interested in a hot plasma with a mean electron energy much greater than the photon energy. The quantum-mechanical calculation to obtain the operator $\partial f/\partial t$ will be more complex than the non-relativistic case. To overcome such problem and another which arises from the elastic formalism, we propose a simple formalism to calculate the effective collision frequency: equation (8) is transformed to inelastic collision cross-section (Landau and Lifchitz [1]), assuming inelastic scattering at high energies.

The differential cross-section for inelastic scattering to the final state n is obtained from equation (8) by Fermi's golden rule[14]

$$\frac{d\sigma_n}{d\Omega}=\frac{2\pi}{\hbar v_i}\left(\frac{4\pi e^2}{k^2}\right)^2\left|\frac{F(\vec{k},I)}{\varepsilon^*(k)}\right|^2 J_n^2(x)\rho(W_f)\,, \qquad (10)$$

where $\rho(W_f)$ is the density of the final state per unit energy and unit solid angle, which equals to $p_f^{*2}[dp_f^*/(2\pi\hbar)^3\,dW_f]$. The initial and final velocity are v_i and v_f respectively. With $dW_f/dp_f^*=v_f$ and the expression $p_f^*=(W_f/c^2)\,v_f$ [15], we obtain for the differential cross section

$$\frac{d\sigma_n}{d\Omega}=\left(\frac{2e^2W_f}{\hbar^2c^2k^2}\right)^2\frac{v_f}{v_i}J_n^2(x)\left|\frac{F(\vec{k},I)}{\varepsilon^*}\right|^2 . \qquad (11)$$

Since $\vec{k}=\vec{k}_i-\vec{k}_f$; $\cos 0=\vec{k}_i\,.\,\vec{k}_f/k_ik_f$ and $d\Omega=\sin\theta d\theta d\Phi$, (11) becomes

$$d\sigma_n=2\pi\left(\frac{2e^2}{\hbar v_i}\right)^2\frac{W_f}{W_i}\overline{J_n^2(x)}\left|\frac{F(\vec{k},I)}{\varepsilon^*}\right|^2\left(\frac{k_i}{k}\right)\frac{dk}{k^3}\,. \qquad (12)$$

where $\overline{J_n^2(x)}$ is an angular averaged Bessel function, namely $\frac{1}{4\pi}\int J_n^2(x)d\Omega$. For the strong field case, $F(k,I)\frac{W_f}{W_i}$ is a function which has maximum values near $\frac{eE_0}{m_e\omega}\Delta p_\perp^*=(p_f^{*2}-p_i^{*2})/2m$ so, for hot electrons, we have approximately

$$F(k,I)\frac{W_f}{W_i}\approx F(k)\frac{W_f}{W_i}\approx R(I)F(k)\,. \qquad (13)$$

Indeed at $v_0\sim$ c, it is necessary to calculate F as function of $\vec{k}$ and I, $F(\vec{k},\mathrm{I})$, taking into account two kinds of relativistic effects, which appear simultaneously. One correction concerns the average relativistic motion of the scattered electrons and the very rapid oscillation of the electrons in the powerful laser field. Without solving the Dirac equation or the Klein-Gordon equation it is possible to show that[3]

$$<|\Phi(k)|^2>|_{\frac{2\pi}{\omega_0}}\doteq\left[1+\left(\frac{v_0}{c}\right)^2\right]^2|\Phi(k)|^2\,, \qquad (14)$$

so

$$F(k,I)\approx\left[1+\left(\frac{v_0}{c}\right)^2\right]^2F(\vec{k})\approx R(I)F(\vec{k})\,. \qquad (15)$$

Here we have considered an adiabatic coupling with the laser field, where only kinematically linked starred quantities, p* and W*, have to be considered. In this case the factor $\left(e^2A_0^2/4k_ip\right)k_i^\mu$ in the last term of equation (2) does not go to zero as the electron decouples from the laser field. Thus, we can model $F(k,\mathrm{I})$ as equation (15). We also assume that the sum rule for multiphoton quasi-free-free transition does hold; $\sum\overline{J_n^2(x)}=1$, that is the Kroll and Watson result supports the idea of looking at the electron in the laser field as a dressed one. Then, from equations (12) and (13) or (15), we have for multiphoton absorption

$$\sigma=\sum_n\sigma_n=2\pi\left(\frac{2e^2}{\hbar v_i}\right)^2R(I)\int\frac{dk}{k^3}\left(\frac{k_i}{k}\right)\left|\frac{F(k)}{\varepsilon^*}\right|^2\,. \qquad (16)$$

When $\hbar\rightarrow0$, the quiver distance $r_0=\frac{eE_0}{m\omega^2}$ is large compared to the thermal de Broglie wave length $\lambda_T=\hbar/(mT)^{1/2}$, and if $\varepsilon^*(k)$ and $F(\vec{k})$ can be calculated classically, then, the total cross section is

$$\sigma=2\pi\left(\frac{2e^2}{mv_i^2}\right)^2\int\left|\frac{F(k)}{\varepsilon}\right|^2\left(\frac{k_i}{k}\right)^3\frac{dk}{k}R(I)\,, \qquad (17)$$

where $R(I)$ is a correction factor due to the strong field and $\varepsilon(k)$ is the usual response function for electrostatic oscillations.

A similar expression for R(I) [15], was recently found by Krstić and Milošević[9] (equation 4.14) and Rashid" (equation 3.12) using different solution of the Dirac equation. In both cases, the algebra is very complex and does not elucidate the role of the electron spin. Also, from the exact solution of the Klein-Gordon equation Ehlotzky[16] gives a correction factor which is approximately equal to R(I). All these confirm our equation (15), when $v_0\leq$ c.

Since the potential (or $F(\vec{k})$ and $\varepsilon_{\|}(\vec{k},\omega)$), was not specified explicitly the results of equations (15) and (16) are quite general, therefore, it can be applied to a variety of cases. For pure Coulomb potential $F(k)=1$ and $\varepsilon(k)\rightarrow1$. For screened Coulomb potential $F(k)\rightarrow1$, $\varepsilon(k)\rightarrow1+k_D^2/k^2$ and for a plasma with quantum diffraction effects, $F(k)\rightarrow1$, $\varepsilon(k)\rightarrow1+k^2/k_T^2$. When $\mathrm{I}\rightarrow0$, n $\rightarrow0$, we have the elastic case and equation (10) gives

$$\frac{d\sigma}{d\Omega}=\left(\frac{e^2}{2mv_i^2}\right)^2\frac{1}{\sin^4(\theta/2)}\,, \qquad (18)$$

that is, the result derived from classical mechanics by Rutherford.

In summary, equation (16) can be used to calculate the total cross section in the presence of an intense laser field, where $F(k)$, $\varepsilon_{\|}(k)$ or $\Phi(k)$, must be determined carefully according to the dominant process. In general, $\Phi(k)$, when calculated quantum mechanically, is not the same to that when calculated classically; so the classical limit must be carefully taken through the correct dielectric response function from its quantum mechanical counterpart[10].

III. Effective Collision Frequency and Absorption Coefficient

The typical calculation of the effective collision frequency is based on a model of the electron kinetic equation[4,12]. The different classical results for γ_{eff} are due to: i) equation (8) is used without momentum conservation of electron-photon system; ii) different quantum limits are considered with the same instantaneous approximation for the electron-ion scattering (scattering time is smaller than the period of laser field); iii) the initial distribution function is approximated by a time-independent $f(\vec{v_e})$ which may be doubtful for high laser flux, specially when the electron temperature rises by several orders of magnitude over the initial value. It is possible that the alternative formalism presented here, can overcome some of these problems. As it was shown in ref.I, for high fluxes, the absorption rate is independent of the initial distribution function. For CO_2 laser-plasma interactions, it was assumed that the incident electron beam is represented by a delta function, $\delta(v_{te}, I)$. Here, in order to include Nd-Yag-laser-target interaction, we assume that the normalization factor of f(v) does not depend of I.

$$f(v) = \frac{n_e\delta[v-(v_{te}^2+\alpha I)^{1/2}]}{4\pi[v_{te}^2]} , \qquad (19)$$

where we set $v_0^2 = \alpha I$; v_{te} is the thermal electron velocity and $\alpha^{-1} = I/v_0^2$. This approximation is useful for $v_0/v_{te} = 0.4\lambda\sqrt{I}/\sqrt{T_e} \geq 1$ and $(v_0/c)^2 < 1$, λ is given in micron, I in 10^{15} W/cm^2 and T_e in keV, [3].

The effective collision frequency is calculated as

$$\gamma_{eff} = \int v\sigma(v)f(v)d^3v . \qquad (20)$$

Taking $\varepsilon \rightarrow 1$ and $|\Phi(k)|^2 \rightarrow 1$ (normalized Coulomb potential), γ_{eff} was scaled as

$$\gamma_{eff} \sim \frac{8\pi e^4 n}{m^2(v_{te}^2+\alpha I)^{1/2}}R(I)\ln\left(\frac{k_{max}}{k_{min}}\right) , \qquad (21)$$

Where, $\ln(k_{max}/k_{min}) \sim \ln(v_0/\omega r_c)$ for $v_0/\omega \gg r$, and $Ze^2 \gg \hbar v_0$ or $\ln(k_{max}/k_{min}) \sim \ln(mv_0^2/\hbar\omega)$ for $(\hbar\omega/m)^{1/2} << v_0$ and $r_c = 4e^2/mv_0^2$; $v_0 = eE_0/m\omega$ (see Silin [17]).

From equation 2.i we can obtain several asymtotic results, for the weak-field case we have

$$\gamma_{eff} \sim \frac{8\pi e^4 n}{m^2 v_{te}^3}\ln\Lambda . \qquad (22)$$

Equation (22) agrees with Silin's and Seely's expression[17,4].

At high beam intensity, when elastic collisions are considered, $R(I) = 1$ so γ_{eff} is scaled as $\gamma_{eff} \sim I^{-1/2}$ which is similar to the scale founded by Nicholson[6]. However for inelastic collision, $R(I) \sim 1 + \alpha I/v_{te}^2$ and γ_{eff} becomes approximately $\gamma_{eff} \sim \frac{I^{1/2}}{T_e}$. Since $v_0/v_{te} \gg 1$ we have an enhanced inverse Bremsstrahlung which can be compared with the early result of Silin ($\gamma_{eff} \sim I^{-3/2}$) [17].

In the relativistic regime, (CO_2), $v_0 \leq$ c, $R(I) \sim \left[1+(v_0/c)^2\right]^2$, so extending this result for $v_0 \equiv \frac{eE_0}{m\omega} >$ c, we have

$$\gamma_{eff} \sim \frac{8\pi e^4 n}{m^2(v_{te}^2+\alpha I)^{1/2}}\ln\Lambda\left[1+\left(\frac{v_0}{c}\right)^2\right]^2 \sim I^p . \qquad (23)$$

Where $1 < p < 1.25$, here we have considered that the hot electron temperature is scaled as $T_{hot} \sim I^{1/2}$ [18], [19]. Equation (23), is the main result of this paper, because clearly it shows an enhanced collision frequency which is usefull to estimate the absorption coefficient.

It was early reported that the coefficient of absorption of CO_2, (Nd-Yag) laser light by spherical targets has a nonlinear dependence on the laser intensity I, for $I \geq 10^{15}W/cm^2$ ($I \geq 10^{16}$), [20], [22]. The observed dependence of the absorption coefficient, a, on the laser intensity was interpreted as an enhancement of resonant absorption due to a distortion of the critical surface. However it is simple to show that the resonant absorption is unable to explain the experimental observations. The maximum energy stored in the resonant field is $W_r = (E_r^2/8\pi)l$, where the length l at wave breaking is approximately $l = v_r/\omega_0 = (2v_0 l\omega_0)^{1/2}$ with $v_r = eE_r/m\omega_r$. If all this energy is lost, it will take a time $t_r \sim (8L/v_0\omega_0)^{1/2}$ to rebuild. Thus the highest absorption rate is $I_{abs}/I_0 \sim k_0 L$. When the incident irradiance, and consequently the ponderomotive force increases, increasingly steep density gradients are established to maintain the dynamic equilibrium, that is, the critical density scale length L decreases with increasing incident radiation. Experimentally, above $I = 10^{14}$, $L \sim I^{1/2}$ [18]. Thus for $v_0/\omega_0 \geq L$ the resonant absorption mechanism is quenched. Also, a change in the scale length, $L(I)$, will cause a change in the scaling with I of the resonant field amplitude and therefore of the hot electron temperature. Experimentally it was found that the hot electron temperature is scaled as $T_{hot} \sim I^{0.5}$. Unfortunately, no calculation has been done to support these qualitative remarks. Here we propose that the observed intensity-dependence of a results from the nonlinear effects in the quasi-relativistic inverse bremsstrahlung by the preheated electrons in the corona.

With a simple calculation, we can show that the absorption coefficient a is proportional to $(\gamma_{eff}/\omega_0)k_0 L$ [11], so from equation (23) we see that a is scaled as I^r, where $1/2 \leq r \leq 3/4$. The same scale factor was obtained theoretically by Brunel[18] using a classical non resonant process. However the dependence of the absorption coefficient with the laser intensity comes from a different mechanism. Experimentally[20,21,22] it was reported that the laser absorption in a gold target increases with laser intensity when $I > 10^{15}$ W/cm^2, and it seems that $\gamma_{eff} \sim I^p$ with $p \geq 0.5$ [20,21]. This a p parently agree with our estimation of α, but this conclusion is not definitive, specially in the case of laser produced-plasmas at 1.0 micrometer.It seems that our result is more appropriate to model CO_2 laser-plasma experiments. Our estimates are based on qualitative arguments where the basic behavior of high energy process envolving relativistic electrons are determined by the total electron energy in the wave field. The precise nature of the plasma dynamics that leads to higher absorption is not fully understood. However from this work, one general observation is pertinant: the absorption increases as the electron becomes relativistic, that is , when $T_{hot}/mc^2 \sim 1$, where m is the electron rest mass and T_{hot} is the superthermal electron temperature in the plasma. For plasma irradiated by CO_2 lasers at $I > 10^{16}$ W/cm^2 [20,21], T_{hot} is in the range of 200 keV. This observation also is confirmed by calculations from the plasma simulation code WAVE [23].

In summary, from a relativistic quantum-mechanical approach, we have calculated a simple expression for the effective collision frequency that takes into account the laser field, $\gamma_{eff} \sim I^{1-1.25}$ and that this can be considered as an enhanced inverse Bremsstrahlung. With this γ_{eff} we are able to obtain the absorption coefficient, which is scaled as $\gamma_{eff} \sim I^{1/2-3/4}$.

IV. Conclusions

The main results of this paper are equations (15), (16), and (23). The first two equations give the correction factor $R(I)$, and the total cross section respectivoly, taking into account the presence of the laser field. The last equation give the effective collision frequency, which are scaled as $\gamma_{eff} \sim$ I*, where $p \geq 1/2$. They provide simple and reasonable approximation for multiphoton absorption at high intensity laser. Contrary to the classical result ($\gamma_{eff} \sim I^{-3/2}$), in the relativistic regime, for $v_0 \leq$ c or $v_0 >$ c we obtain $\gamma_{eff} \sim I^{1-1.25}$ and $\alpha \sim I^{0.5-0.75}$, which approximately agrees with numerical simulation[23,18]. With this formalism other laser-plasma interaction can also be taken into account such as quantum diffraction effects when two particles approach each other closer than the de Broglie thermal wavelength.

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