Revista Brasileira de Física, Vol. 21, nº 4, 1991

Efficiency simulation for untapered free-electron laser oscillators with space charge effect

A. Serbeto
Instituto de Física, Universidade Federal Fluminense, 24020, Niterói, RJ, Brasil

and

B. Levush and T. M. Antonsen Jr.
Laboratory for Plasma Research, University of Maryland, 20742, College Park, Maryland, USA

Received September 5, 1991

Abstract A nonlinear simulation for an untapered Compton FEL oscillator with space-charge effect has been performed. The Poisson equation, together with the pendulum equation, are solved by Fourier analysis with m harmonics. It has been shown that a large number of Fourier harmonics are crucial to simulate the single-particle efficiency. As the particle bunching increases the normalized efficiency also increases. In order to have a well defined efficiency saturation, the number of Fourier modes has to be increased as the number of plasma oscillations in the system increases.

The generation of high-power coherent radiation using a relativistic electron beam moving in a periodic magnetostatic field has attracted considerable interest lately. The device based on this effect, named free-electron laser (FEL), is capable of operating in a broad band of the electromagnetic spectrum even at wavelengths not accessible to conventional lasers. This tunable feature of the FEL is due to the fact the wavelength of the coherent radiation λ_r is mainly determined by the relativistic electron beam energy and the wavelength of the wiggler or undulator magnetic field ℓ_w, which satisfy the approximate resonant condition

$$\lambda_r \cong \ell_w/2\gamma_R^2$$

where γ_R is the normalized resonant electron beam energy[1-4]

The goal of this paper is to determine by numerical simulation the single-particle efficiency saturation for low gain Compton FEL oscillators when space-charge effects caused by the particle bunching are present. To consider the space-charge effects, it is necessary to add the Poisson equation, which gives us the electrostatic potential, to the particle's dynamics equations. Antonsen and Latham[5] have recently solved this set of equations in a linear approximation for low and high gain regimes of operation for a sheet-electron beam FEL, where a single electrostatic mode is present. In our one-dimensional model this set of nonlinear equations is solved for a single-pass electromagnetic mode in a thick beam FEL. In order to describe the FEL interaction we assume that the wiggler strength is small enough that one can consider the unperturbed beam velocity as uniform.

To calculate the nonlinear efficiency we assume that a large number of Fourier (electrostatic) modes due to the phase particle bunching are present.

In an untapered FEL oscillator the motion of an axially streaming electron is affected by the combined action of two vector potentials. The first is he wiggler potential

$$\vec{A}_w = \hat{x}\; A_{w0}\cos(k_w z), \tag{1}$$

which guides the electron through N_w periodic oscillations as it travels the length L of the magnet, and the second is the coherent radiation vector potential

$$\vec{A}_r = \hat{x} A_{r0}(z,t)\exp[i(k_r z - \omega_r t)] + c.c., \tag{2}$$

with the given fields which propagate along the z-direction; the forces in this direction acting on a particle are the ponderomotive force, resulting from the beating between the wiggler and radiation fields, and a collective force due to the axial space-charge field created by the particle bunching, which begin to influence the interaction between the individual electron and the radiation field. By averaging the forces over a wiggler period, the particle's dynamics can be described by the well-known pendulum equations[6]

$$d\psi/d\xi = P = \partial H/\partial P, \tag{3.a}$$

$$dP/d\xi = -\partial H/\partial\psi, \tag{3.b}$$

which govern the electron dynamics in the phase space, where

$$H(\psi, P) = P^2/2 + V(\psi) - A_0 \cos\psi, \tag{4}$$

is the interaction Hamiltonian, with

$$P = \delta\gamma\omega_r L/[c(\gamma_R\beta_R)^3] \tag{5}$$

being the normalized canonical momentum, which replaces the resonant energy deviation 67, β_R is the normalized resonant electron beam velocity and ω_r is the coherent frequency. In this case, the axial distance is normalized to the length of the interaction region $\xi = z/L$ and the ponderomotive wave amplitude is normalized according to

$$A_0 = (\omega_r L/c)^2(a_{w0}a_{r0})/(\gamma_R\beta_R)^4, \tag{6}$$

where $a_{w0} = qA_{w0}/mc^2$ is the wiggler parameter and $a_{r0} = qA_{r0}/mc^2$ is the normalized radiation field amplitude. The normalized electrostatic potential is defined as

$$V(\psi) = q\omega_r kL^2/[mc^3(\gamma_R\beta_R)^3]\Phi(\psi), \tag{7}$$

where $\Phi(\psi)$ is the collective potential given by the Poisson equation

$$\partial^2\Phi/\partial\psi^2 = -4\pi q/k^2 n(\psi), \tag{8}$$

where the particle's phase ψ which replaces the axial position is defined by

$$\psi = kL\delta\xi$$

and

$$k = k, + k_w$$

is the space-charge wave number. The normalized electron position deviation is given by

$$\delta\xi = kL(\xi - v_R t/L),$$

with $v_R = \omega/k$ being the resonant velocity of the charge particle perturbation, which is assumed constant for a weak wiggler strength ($a_{w0} << 1$). Then, in order to derive the contribution of the space-charge potential to the pendulum equation, we can represent the phase bunched particle density as

$$n(\psi) = 2\pi n_0/N \sum_{j=1}^{N} \delta(\psi - \psi_j), \tag{9}$$

where N is the number of particles in the beam. Introducing this expression into eq. (8) and expanding in Fourier series in ψ, we have for the collective potential

$$\Phi(\psi) = 4\pi q n_0/k^2 \sum_{m=-\infty}^{m=\infty} \langle e^{-im\psi}\rangle G_m e^{im\psi}, \tag{10}$$

where n_0 is the average beam density and

$$\langle e^{-im\psi}\rangle = 1/N \sum_{j=1}^{N} e^{-im\psi}j, \tag{11}$$

is the electron bunching parameter.

Substituting eq.(10) into eq.(7), one gets for the normalized potential

$$V(\psi) = \omega_p^2 \sum_{m=-\infty}^{\infty} \langle e^{-im\psi}\rangle G_m e^{im\psi}, \tag{12}$$

where m represents the number of Fourier modes in the beam,

$$\omega_p = (\omega_b/v_R)L, \tag{13}$$

is the normalized relativistic plasma frequency with

$$\omega_b^2 = 4\pi q^2 n_0/(m_0\gamma_R^3)$$

and

$$G_m = 1/m, \tag{14}$$

is the Fourier coefficient for thick electrom beam. Introducting eq. (12) and eq. (14) into eq. (3), and working out the space-charge term, the basic nonlinear system of equations for the particle phase dynamics becomes

$$\partial\psi/\partial\xi = P \tag{15.a}$$

$$\partial P/\partial\xi = A_0 \sin\psi + 2\omega_p^2 \Im\left\{ \sum_{m=1}^{\infty} (\langle e^{-im\psi}\rangle e^{-im\psi})/m \right\} \tag{15.b}$$

where $\Im$ refers to "the imaginary of".

Equations (15) are integrated numerically for an ensemble of electrons whose initial canonical momenta $P(\xi = 0)$ have been set equal to the injected momenta $P_{\rm inj}$. The normalized efficiency is determined from the averaged canonical momentum for the exiting electrons and is given by

$$\Delta P = P_{\rm inj} - < P(\xi = 1) >, \tag{16}$$

where the angular brackets represent an average over the initial phases. The actual efficiency is determined from eqs. (5) and (16):

$$\eta = \{[\gamma_R(0)\beta_R(0)]^3/k_r L\}\Delta P/[\gamma_R(0) - 1]. \tag{17}$$

The nonlinear energy gain can be defined as

$$G(P, \omega_p) = \Delta P/A_0. \tag{18}$$

As we can see, when the bunching particle force is considered, the energy extracted from the beam is distributed between the radiation and the space-charge waves; otherwise in the case where the space-charge force is neglected, the energy loss by the electron is completely transferred to the coherent radiation field. Similarly, following the usual procedure to obtain the small radiation energy gain without space-charge[7–9], we find that the small-gain with a single electrostatic mode (Fourier mode) is given by

$$G(P_{\rm inj}, \omega_p) = -1/4\omega_p\{[\cos(P_{\rm inj} + \omega_p) - 1]/(P_{\rm inj} + \omega_p)^2 - \\ - [\cos(P_{\rm inj} - \omega_p) - 1]/(P_{\rm inj} - \omega_p)^2\}. \tag{19}$$

In the limit as $\omega_p \to 0$ (no space-charge) this equation reduces to the well-known line width expression for a free-electron laser:

$$G(P) = \frac{1}{2}\partial/\partial P_{\rm inj}[(\cos P_{\rm inj} - 1)/P_{\rm inj}^2]. \tag{20}$$

Figure 1 gives the simulation of the gain function for a single electrostatic mode as a functions of the injected canonical momenta. This result agrees with the analytical result in eq. (19) which is similar to that presented by Shih and Yariv[8]. One sees that as the bunches particle density increases, the maximum gain reduces in comparison with the value for $\omega_p = 0$ and moves toward large $P_{\rm inj}$.

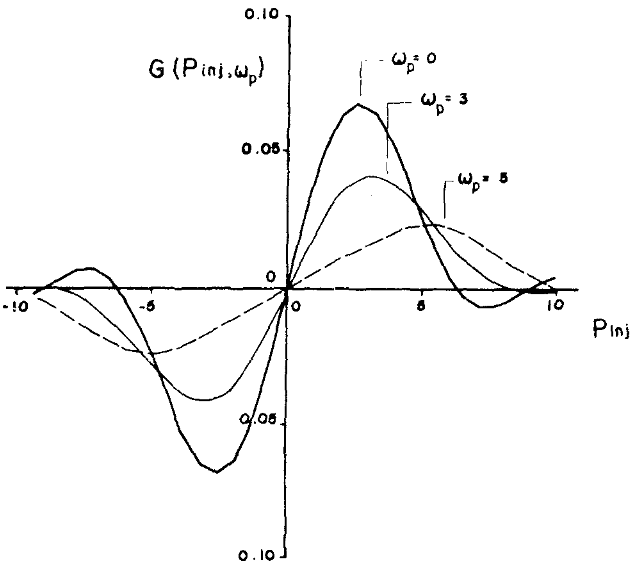

Fig. 1 – Gain curve $G(P_{\rm inj}, \omega_p)$ as function of $P_{\rm inj}$ for different values of normalized plasma frequency w_p.

Figure 2 shows the behaviour of the normalized efficiency A P for normalized plasma frequency $\omega_p = 3$ and for optimum injected canonical momentum $P_{\rm inj} =$ 5.14 [10]. One can see that, in order to have a well-defined efficiency saturation one needs three Fourier harmonics (m = 3) to simulate the correct energy extraction from the beam. Actually the efficiency is a function of three parameters, i.e., $\Delta P = \Delta P(P_{\rm inj}, A_0, m)$. The increase of the efficiencyin comparison with the case without space-charge is due to the increase of the magnitude of the electrostatic electric field caused by the particle bunching. This field removes some of the particles from the ponderomotive bucket in a competitive way such that part of the energy transferred to the radiation comes from the space-charge wave, i.e., the maximum efficiency is reached when a large number of the electrostatic modes and particle transfer their energy to the coherent electromagnetic wave.

Figure 3 shows the results of the efficiency simulation for $\omega_p = 5$ and injected canonica! momentum $P_{\rm inj} = 5.14$. The efficiency saturates for the Fourier harmonics m = 5 in comparison with m = 1; we can note that the maximum efficiency for

$m = 5$ is large than the maximum efficiency for m = 1. Otherwise, the simulation is well defined for m = 3 with a error of 2% in comparison with m = 5, since the maximum efficiency reaches almost the same value for $m = 5$. The advantage of calculating the electronic efficiency in this case for m = 3 is to reduce the simulation time for a free-electron lasers in a Compton regime of operation when space-charge effect is present.

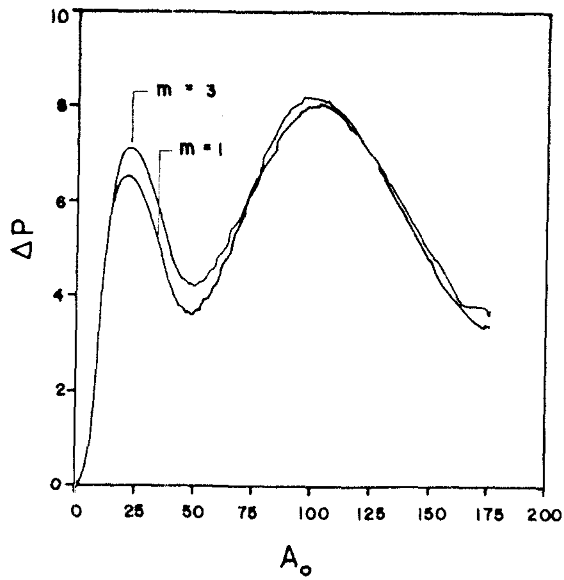

Fig. 2 – Efficiency A P versus beat wave amplitude A_0 for normalized plasma frequency $\omega_p = 3$, and injected energy $P_{inj} = 5.14$ with different values of the Fourier harmonics m.

The results presented here show us that simulation of a Compton FEL with self-electrostatic field when a Fourier solution of the Poisson equation is performed should be carefully described, since the pendulum equations which describe the free-electron lasers' interaction with space-charge effect derived by many authors[11–14] has made use of the space-charge term in this equation assuming that there is a convergency of the numerical calculation for a single Fourier mode (m = 1) in the self-field term. In contrast with those authors we can say that

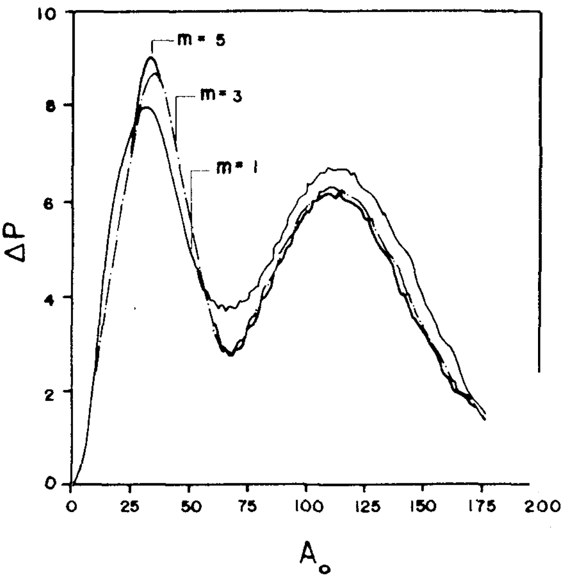

Fig. 3 – Efficieiicy ΔP versus beat wave amplitude A_0 for normalized plasma frequency w_p = 5, and injected energy P_{inj} = 5.14 with different values of the Fourier harmonics m.

a large number of Fourier modes are required to calculate the nonlinear single-particle efficiency, because the motion of the electron is not synchronous at saturation, particularly in the case where the space-charge waves are important. Efficiency simulation for ω_p large than 5 has also been performed and the results have shown the necessity of further increase of Fourier harmonics in the Poisson solution of the self-field, i.e., the number of harmonics increases as the number of plasma oscillations in the system increases.

Acknowledgements

This work was partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) - Brasil and by the U. S. Department of Energy. One of the authors (A. Serbeto) gratefully acknowledge helpful discussions with Drs. K. H. Tsui and A. Tomimura of the Universidade Federal Flurninense (RIo de Janeiro, Brasil).

References

1. N. M. Kroll and W. A. McMullin, Phys. Rev. **A** 17, **300 (1978).**
2. I. B. Bernstein and J. L. Hirschfeld, Phys. Rev. A20, **1661 (1979).**
3. P. Sprangle, C. M. Tang and W. M. Manheimer, Phys. Rev. A21, **302** (1980).
4. T. C. Marshall, ***Free Electron Lasers*** (MacMillan, New York, **1985).**
5. T. M. Antonsen Jr. and P. E. Latham, Phys. Fluids 31, **3379 (1988).**
6. N. M. Kroll, P. L. Morton and M. Rosenbluth, IEEE J. Quantum Electron QE-17, **1436 (1981).**
7. W. B. Colson, Phys. Letters **A64(2), 190 (1977).**
8. C. C. Shih and A. Yariv, Phys. Rev. A22, **2717 (1981).**
9. C. W. Roberson and P. Sprangle, Phys. Fluids **B1, 9 (1989).**
10. A. Serbeto, B. Levush and T. M. Antonsen Jr., Phys. Fluids **B1, 435** (1989).
11. C. M. Tang and P. Sprangle, J. Appl. Phys. 52, **3148 (1981).**
12. P. Prosnitz, A. Szoke and V. K. Neil, Phys. Rev. A24, **1436 (1981).**
13. J. B. Murphy, C. Pellegrini and R. Bonifacio, Opt. Commun. 53, **197** (1985).
14. S. Y. Cai, A. Bhattacharjee and T. C. Marshall, IEEE J. Quantum Electron, **QE-23, 1651 (1987).**

Resumo

E feita a simulação não linear para um laser de elétrons do tipo oscilador com parâmetros constantes levando-se em conta o efeito das ondas de cargas espaciais. A equação de Poisson, juntamente com a equação do pendulo, é resolvida através da análise de Fourier com m liarmônicos. Mostra-se que um grande número de harmônicos é crucial para se simular a eficiência de extração de energia de uma partícula. Vê-se que quando o agrupamento de partículas aumenta a eficiência também aumenta. Para se ter uma ótima eficiência na saturação, o número de modos eletrostáticos (modos de Fourier) do feixe deve ser aumentado de acordo com o número de oscilações de plasma do sistema.