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Critical string wave equations ãnd the QCD ($U(N_c)$) string

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Abstract We present a formal proof that the QCD (($U(N_c)$) Migdal–Makeenko Loop Wave Equation has a self-avoiding fermionic string solution.

Introduction

We aim, in this note, to present a formal interacting string solution for the Migdal-Makeenko Loop Wave Equation for the colour group ($U(N_c)$ (Ref. 1 and references therein).

Our main tool to solve the Migdal-Makeenko Loop Wave Equation is based on the remark made in the section 1 of this note, where we address the problem of solving critical string wave equations by string functional integrals without making use of our general covariant procedure exposed in Refs. 1, 2. We thus apply the results of section 1 to present a string functional integral solution for the Migdal-Makeenko Loop Wave Equation for the colour group $U(N_c)$.

1. The critical area-diffusion string wave equations

Let us start this section by briefly reviewing our general procedure to write diffusion string wave. equations for bosonic non-critical strings[2]. The first step is by considering the following fixed area string propagator in Polyakov's string quantization framework.

$$\mathcal{G}[C^{\text{out}}, C^{\text{in}}, A] = \int D^c[g_{ab}] D^c[X_\mu] \times \delta\Big(\int_D d\sigma d\tau \sqrt{g(\sigma,\tau)} - A \Big) \times \exp(-I_0(g_{ab}, X_\mu, \mu^2 = 0)) \tag{1}$$

Here the string surface parameter domain is taken to be the rectangle D = $\{(\sigma,\tau)\ ;\ -\pi \leq \sigma \leq \pi\ ,\ 0 \leq \tau \leq T)$. The action $I_0(g_{ab},\ X_\mu,\ \mu^2 = 0)$ is the

Brink-Di Vecchia-Howe covariant action with a zero cosmological term and the covariant functional measures $D^c[g_{ab}]D^c[X_\mu]$ are defined over all cylindrical string world sheets without holes and handles with the initial and final string configurations as unique non-trivial boundaries: i.e. $X_\mu(\sigma,0) = C^{\text{in}}$; $X_\mu(\sigma,\text{T}) = C^{\text{out}}$.

In order to write an area diffusion wave equation for Eq. (1), we exploited an identity which relates its area variation (the Mandelstam area derivative for strings) to functional variations on the conformal factor measure when one fixes the string diffeomorphism group in Eq. (1) by imposing the conformal gauge $g_{ab}(\sigma,\tau) = \rho(\sigma,\tau)\delta_{ab}$ (see Refs. 1, 2). This procedure yields, thus, the following area diffusion string wave equation

$$\begin{aligned}\frac{\partial}{\partial A}\mathcal{G}[C^{\text{out}},C^{\text{in}},A] = \int_{-\pi}^{\pi} d\sigma\Big(&- \frac{\delta^2}{2e^2_{\text{in}}(\sigma)\delta C^{\text{in}}_\mu(\sigma)\delta C^{\text{in}}_\mu(\sigma)} \\ &\frac{1}{2}(C'^{\text{in}}_\mu(\sigma)^2 + \frac{26-D}{24\pi}\lim_{\tau\to 0^+}[R(\rho(\sigma,\tau)) + C_\infty]\Big) \\ &\times \mathcal{G}[C^{\text{out}},C^{\text{in}},A] \end{aligned} \tag{2}$$

At this point a subtle difficulty appears when the theory described by Eq. (1) is at its critical dimension $D = 26$ since the conformal field $\rho(\sigma,\tau)$ decouples from the theory, making it subtle to implement the fixed area constraint in Eq. (1). It is instructive to point out that for a cylinder surface without holes and handles with non trivial boundaries, the argument that the fixed area constraínt is simply fixing the modulus λ of the (torus) conformal gauge $g_{ab}(\sigma,\tau) = \rho(\sigma,\tau)\ ((d\sigma)^2 + \lambda^2(d\tau)^2)$ is insufficient to cover the case of "string creation" from the vacuum as we will need in section 2. This is because in this case $\lambda = 0$ and the string world sheet still has a non-zero area. Note that the topology of this string world sheet creation process is now a hemisphere which again makes impossible the use of the modulus λ as an area parameter.

However, it makes sense to consider the limit of the parameter D = 26 directly in our string diffusion Eq. (2) which reproduces the usual critical string wave equations (Eq. (2) with D = 26 and $\rho(\sigma,\tau) = 1$).

In this short section we intend to show that the following critical string propagator:

$$\begin{aligned}\mathcal{G}[C^{\text{out}},C^{\text{in}},A] = \int D^F[X^\mu(\sigma,\tau]\times \\ X^\mu(\sigma,0) = C^{\text{in}}_\mu(\sigma); \quad , \quad X^\mu(\sigma,\text{A}) = C^{\text{out}}_\mu(\text{o}) \\ \exp\Big\{-\frac{1}{2}\int_0^A d\tau\int_{-\pi}^{\pi}\Big[\Big(\frac{\partial X^\mu}{\partial\sigma}\Big)^2 + \Big(\frac{\partial X^\mu}{\partial\tau}\Big)^2\Big](\sigma,\tau)\Big\}\end{aligned} \tag{3}$$

where the intrinsic string time parameter T is identified with the area difusion variable, satisfies the string critical diffusion wave equation.

To show this simple result we evaluate the A - derivative of Eq. **(3)** by means of Leibnitz's rule

$$\frac{\partial}{\partial A}\mathcal{G}[C^{\text{out}},C^{\text{in}},A] = \frac{-1}{2}\lim_{\tau\to A^-}\left\langle\int_{-\pi}^{\pi}d\sigma\left[\left(\frac{\partial X^\mu}{\partial\sigma}\right)^2+\left(\frac{\partial X^\mu}{\partial\tau}\right)^2\right](\sigma,\tau)\right\rangle_s \tag{4}$$

where the surface average $<>_s$ is defines by the bosonic path-integral in Eq. **(3).**

In order to translate the path integral relation Eq. (4) into an operator statement, we use the usual Heisenberg Commutation Relations for two-dimensional $(2D)$ free fields on D

$$[\Pi_\mu(\sigma,\tau),X_\nu(\sigma',\tau)]=i\delta(\sigma-\sigma')\delta_{\mu\nu} \tag{5}$$

and its associated Schrödinger representation for $\tau = A$

$$\Pi_\mu(\sigma,A)=\lim_{\tau\to A}\left\langle\frac{\partial}{\partial\tau}X^\mu(\sigma,\tau)\right\rangle_s = i\frac{\delta}{C_\mu^{\text{out}}(\sigma)} \tag{6}$$

$$\left|\frac{dC_\mu^{\text{out}}(\sigma)}{d\sigma}\right|^2=\lim_{\tau\to A}\left(\frac{\partial X^\mu(\sigma,\tau)}{\partial\tau}\right)^2 \tag{7}$$

After substituting Eqs. (6)-(7) into Eq. (4) we obtain the desired result

$$\frac{\partial}{\partial A}\mathcal{G}[C^{\text{out}},C^{\text{in}},A] = -\left(\int_{-\pi}^{\pi}d\sigma\left[-\frac{\delta^2}{2\delta C_\mu^{\text{out}}(\sigma)\delta C_\mu^{\text{out}}(\sigma)}+\frac{1}{2}|C_\mu^{\prime\text{out}}(\sigma)|^2\right)\times \mathcal{G}[C^{\text{out}},C^{\text{in}},A] \tag{8}$$

Let us point out that general string wave functionals (the Schrodinger representation for the theory's quantum states) may be formally expanded in terms of the eigenfunctions of the quantum string Hamiltonian (the string wave operator in Eq. (8))

$$-\Delta_c\psi_E[c]=-\left\{\int_{-\pi}^{\pi}d\sigma\left(-\frac{\delta^2}{2\delta C_\mu(\sigma)\delta C_\mu(\sigma)}+\frac{1}{2}|C_\mu'(\sigma)|^2\right)\right\}\psi_E(c)=E\psi_E[c] \tag{9}$$

$$\psi[c]=\sum_{\{E\}}\rho(E)\psi_E[c] \tag{10}$$

The functionals endowed with the (formal) inner product given by

$$< \psi[c]|\Omega[c] >= \int D^F[c] \cdot \psi^\star[c] \cdot \Omega[c] \tag{11}$$

constitute a Hilbert space where the string Laplacian $-\Delta_c$ is formally a Hermitean operator.

It is worth remarking that an explicit expression for the Green's Function

$$(-\Delta_c)^{-1}(C^{\text{out}}, C^{\text{in}}) = \sum_{\{E\}} \psi_E[C^{\text{out}}]\psi_E^\star[C^{\text{in}}]/E$$

of the string Laplacean in terms of the cylindrical string propagator Eq. (3) may be easily obtained.

In order to deduce this expression we integrate both sides of Eq. (8) with respect to the A-variable. Considering now the Asymptotic Behaviors

$$\lim_{A\to\infty} \mathcal{G}[C^{\text{out}}, C^{\text{in}}, A] = 0 \tag{12}$$

$$\lim_{A\to 0} \mathcal{G}[C^{\text{out}}, C^{\text{in}}, A] = \delta^F(C^{\text{out}} - C^{\text{in}}) \tag{13}$$

we obtain the relationship

$$\delta^F(C^{\text{out}} - C^{\text{in}}) = -\Delta_c\Big(\int_0^\infty \mathcal{G}[C^{\text{out}}, C^{\text{in}}, A]\Big) \tag{14}$$

leading thus to the following identity

$$(-\Delta_c)^{-1}[C^{\text{out}}, C^{\text{in}}] = \int_0^\infty \mathcal{G}[C^{\text{out}}, C^{\text{in}}, A]\Big) \tag{15}$$

2. A four coupling Fermion on a self-interacting Bosonic Random surface as solution of QCD($U(N_c)$) Migdal-Makeeko Loop Equation

Let us start this section by considering the (non-renormalized) Migdal-Makeenko Loop Equation satisfied by the Quantum Wilson Loop in the form of Ref. 3 for the colour group $U(N_c)$

$$\begin{aligned} - \Delta_c < W_{k\ell}[C_{X(-\pi)X(\pi)}] >= (g^2 N_c) \int_{-\pi}^{\pi} d\sigma \frac{dX^\mu(\sigma)}{d\sigma} \cdot \frac{dX^\mu(\sigma')}{d\sigma'} \\ \times \delta^{(D)}(X_\mu(\sigma) - X_\mu(\sigma') < W_{kp}[C_{X(-\pi)X(\sigma)}]W_{p\ell}[C_{X(\sigma)X(\pi)}] > \end{aligned} \tag{16}$$

The Quantum Wilson Loop is given by

$$
\begin{aligned}
&< W_{k\ell}[C_{X(-\pi)X(\pi)}, A_\mu(x)] >= \\
&= \frac{1}{N_c}\Big\langle T_R^{(c)}\Big(\exp - \int_{-\pi}^{\pi} d\sigma(A_\mu(X_\mu(\sigma)) \cdot X'^\mu(\sigma))\Big)\Big\rangle_{k\ell}
\end{aligned}
\tag{17}
$$

As usual, $A_\mu(x)$ denotes the usual $U(N)$ colour Yang-Milss field which possesses an additional, not yet specified intrinsic global "Flavor" group $O(M)$ represented by matrix indices (k,ℓ). The average <> is given by the $U(N)$ - colour Yang - Milss field theory.

A useful remark concerning Eq. (16) is that trivial loop self-intersect points $(X_\mu(\sigma) = X_\mu(\sigma')$ with a # a') still contribute to the right - hand side (see Ref. 1 - Appendix A).

Let us consider the following critica1 non-linear interacting Fermionic String theory first considered in Ref. 4

$$
\begin{aligned}
&S[X_\mu(\sigma,\tau),\psi_{(k)}(\sigma,\tau)] = \frac{1}{2}\int_0^A d\tau \int_{-\pi}^{\pi} d\sigma\Big[\Big(\frac{\partial X^\mu}{\partial\varsigma}\Big)^2 + \Big(\frac{\partial X^\mu}{\partial\sigma}\Big)^2\Big](\sigma,\varsigma) \\
&+\frac{i}{2}\int_0^A d\varsigma\int_{-\pi}^{\pi} d\sigma[\bar\psi_{(k)}(\gamma^a\partial_a)\psi_{(k)}](\sigma,\tau) \\
&\beta\int_0^A d\varsigma\int_{-\pi}^{\pi} d\sigma\int_0^A d\varsigma'\int_{-\pi}^{\pi} d\sigma'(\psi_{(k)}\bar\psi_{(k)})^2(\sigma,\varsigma)\times \\
&T^{\mu\nu}(X_\alpha(\sigma,\varsigma)\cdot\delta^{(D)}(X_\alpha(\sigma,\varsigma) - X_\alpha(\sigma',\varsigma'))T_{\mu\nu}(X_\alpha(\sigma',\varsigma'))
\end{aligned}
\tag{18}
$$

The notation is as follows: the string vector position is described by the $2D$-fields $X_\mu(\sigma,\varsigma)$ with the Dirichlet boundary condition $X_\mu(\sigma,A) = C_{X(-\pi),X(\pi)}$; i.e., the surface S $= \{X_\mu(\sigma,\varsigma),\ -a \le \sigma \le a,\ 0 \le \varsigma \le A)$ has as unique boundary the *fixed* Loop $C_{X(-\pi)X(\pi)}$ of Eq. (16). The surface orientation tensor where it is defined is given by

$$
T_{\mu\nu}(X_\mu(\sigma,\tau)) = \frac{\epsilon^{ab}}{\sqrt{2}}\frac{\partial_a X^\mu\partial_b X^\nu}{\sqrt{h}}
$$

with h $=$ $det\{h_{ab}\}$ and $h_{ab}(\sigma,\tau) = \partial_a X^\mu\partial_b X^\mu$. Note that S possesses self-intersecting lines of the form $X_\mu(\sigma,\tau) = X_\mu(\sigma',\tau')$ with a $\neq$ a' and $\tau = \tau'$ (see ref.1) and around these lines $X_\mu(\sigma,\tau)$ (and consequently $T_{\mu\nu}(x(\sigma,\tau))$ is a a-multivalued function. Additionally we have introduced a set of single-valued intrinsic Majorana 2D-spinors on the surface domain parameter D $= \{(\sigma,\tau),\ 0 \le \varsigma \le A;\ -\pi \le$ o $\le r)$. They are choosen to belong to a real representation of the flavor group $SU(2)\otimes SU(3)\approx O(22)$ since for this group we have cancelled exactly the theory's conformal anomaly (26=4+22), which in turn leads to the vanishing of the kinetic term associated to the conformal factor $\rho(\sigma,\varsigma)$ (see Ref. 1). We further impose as a boundary condition on these Fermions the vanishing of the Fermion energy - tensor projected on the Loop $C_{X(-\pi),X(\pi)}$. Let us point out that a bilinear term of the form $(\bar\psi_{(k)}\psi_{(k)})$ in the interaction action as considered earlier in Ref. 1 should be ruled out in Eq. (26), by our requirement of the theory's Weil symmetry at the classical level. Let us point out that the Weil symmetry makes sense to speak in conformal anomaly in our theory Eq. (16) which preservation at quantum level by its turn will determine the string flavor group to be Weinberg-Salam group $SU(2)\otimes SU(3)\approx O(22)$ (see ref.1).

Associated to the non-linear string's theory Eq.(18) we consider the following Fermionic propagator for a *fixed* surface

$$\begin{aligned}
&\bar W_{k\ell}[C_{X(-\pi)X(\pi)};X_\mu(\sigma,\varsigma),A]\\
&=\int D^F[\psi_k(\sigma,\tau)](\psi_{(k)}(-\pi,0)\bar\psi_{(\ell)}(\pi,0)\\
&\times\exp\left\{\frac{i}{2}\int_0^A d\tau\int_{-\pi}^{\pi}d\sigma(\bar\psi_{(k)}(\gamma^a\partial_a)\psi_{(k)})(\sigma,\tau)\right\}\\
&\times\exp\left\{\beta\int_0^A d\tau\int_{-\pi}^{\pi}d\sigma\int_0^A d\tau'\int_{-\pi}^{\pi}d\sigma'(\bar\psi_{(k)}\psi_{(k)})^2(\sigma,\tau)\times\right.\\
&\left.T^{\mu\nu}(X_\alpha(\sigma,\tau))\delta^{(D)}(X_\alpha(\sigma,\tau)-X_\alpha(\sigma',\varsigma'))T_{\mu\nu}(X_\alpha(\sigma',\tau'))\right\}
\end{aligned}\tag{19}$$

The basic idea of our string solution for QCD $(U(N_c))$ is a technical improvement of Ref. 1 and consists in showing that the surface averaged propagator Eq. (19)

$$<\bar W_{k\ell}[C_{X(-\pi)X(\pi)},X_\mu(\sigma,\tau),A]>_s=\mathcal{D}_{k\ell}(C_{X(-\pi)X(\pi)},A),$$

when integrated with respect to the A-parameter as in Eq. (15), now satisfies the full $U(N_c)$ non-linear Migdal-Makeenko Loop Equation (16) instead of the more restrictive Loop equations associated to the T'Hooft limit $N_c\to\infty$.

The surface average $<>_s$ is defined by the free bosonic action piece of Eq. (18) as in section 1. In this context we consider $\mathcal{G}_{k\ell}(C_{X(-\pi)X(\pi)},A)$ as the non-linear string propagator describing the "creation" of the Loop $C_{X(-\pi)X(\pi)}=C^{\text{out}}$ from the string vacuum, which is represented here by a "collapsed" point - like string initial configuration $C^{\text{in}}\equiv\{x\}$ (x denotes an arbitrary point of the surface which may be considered as such initial string configuration).

Let us thus, evaluate the A-derivative of $\mathcal{G}(C_{X(-\pi)X(\pi)}, A)$

$$
\begin{aligned}
&\frac{\partial}{\partial A} < \mathcal{G}(C_{X(-\pi)X(\pi)}, A) >_s = \\
&\int D^F[X^\mu(\sigma,\tau)] \int D^F[\psi_{(k)}(\sigma,\tau)] \exp[(-S[X_\mu(\sigma,\tau), \psi_{(k)}(\sigma,\tau)]) \\
&\times \psi_{(k)}(\pi,0)\bar{\psi}_{(\ell)}(\pi,0) \\
&\times \frac{\partial}{\partial A}\Big\{ \int_0^A d\tau \int_{-\pi}^{\pi} d\sigma \Big[\frac{1}{2}(\partial X^\mu)^2 + \frac{i}{2}\bar{\psi}_{(k)}(\gamma^a\partial_a)\psi_{(k)}\Big](\sigma,\tau) \\
&+ \beta \int_0^A d\tau \int_{-\pi}^{\pi} d\sigma \int_0^A d\tau' \int_{-\pi}^{\pi} d\sigma' (\psi_{(k)}\bar{\psi}^2_{(k)}(\sigma,\tau) T^{\mu\nu}(X_\alpha(\sigma,\tau)) \times \\
&\delta^{(D)}(X_\alpha(\sigma,\tau) - X_\alpha(\sigma',\tau') T_{\mu\nu}(X_\alpha(\sigma',\tau')\Big\}
\end{aligned}
\tag{20}
$$

The free Bosonic term in the right-hand ised of Eq. *(20)* leads to the string Laplacean as in Eq. (4) of section 1. The free Fermion term

$$\lim_{\tau\to A^-} \bar{\psi}_{(k)}(\sigma,\tau)(\gamma^a\partial_a)\psi_{(k)}(\sigma,\tau)$$

vanishes as a consequence of our imposed vanished energy - momentum tensor boundary conditions on the intrinsic Fermion field. The evaluation of the more subtle boundary limit on β-term requires explicitly that the surface $\{X_\mu(\sigma,\tau)\}$ does not possesses self-intersections of the type $X_\mu(\sigma,r) = X_\mu(\sigma',\tau')$ with $\tau \neq \tau'$ *as* we showed in Ref. 1 - Appendix B. This condition means that the surface $\{X_\mu(\sigma,\tau)\}$ does not have holes and handles but only boundaries with non-trivial topology. The result of this boundary limit evaluation is given explicitly by the expression below (Ref. 1 - Appendix B) ·

$$
\begin{aligned}
&\int D^F[X^\mu(\sigma,\tau)] \int D^F[\psi_k(\sigma,\tau)] \Big\{ \int_{-\pi}^{\pi} d\sigma \int_{-\pi}^{\pi} d\sigma' \delta^{(D)}(X_\mu(\sigma) - X_\mu(\sigma')\Big\} \\
&\times \frac{dX^\mu(\sigma)}{da} \cdot \frac{dX^\mu(\sigma')}{da'} \Big[\sum_{p=1}^{22} \psi_k(-\pi,0)\bar{\psi}_\ell(-\pi,0) \times \\
&(\psi_p(\sigma,0)\bar{\psi}_p(\sigma,0))^2\Big] \exp(-S[X_\mu(\sigma,\tau), \psi_k(\sigma,\tau)]) \\
&= \beta \int_{-\pi}^{\pi} do \int_{-\pi}^{\pi} d\sigma' \delta^{(D)}(X_\mu(\sigma) - X_\mu(o')\frac{dX^\mu(\sigma)}{da} \cdot \frac{dX^\mu(\sigma')}{d\sigma'} \times \\
&\Big\langle \bar{W}_{kp}[C_{X(-\pi)X(\sigma)}, X_\mu(\sigma,\tau)] \times \bar{W}_{pp}[C_{X(\sigma')X(\sigma)}, X_\mu(\sigma,\tau)] \\
&\times \bar{W}_{p\ell}[C_{(\sigma)X(\pi)}, X_\mu(\sigma,\tau), A]\Big\rangle_s
\end{aligned}
\tag{21}
$$

It is worth pointing out that we have used only the factorization of the Fermionic propagator **Eq.** (19) and not of the bosonic functional measure as in our earlier study[1]

$$
\begin{aligned}
&\sum_{p=1}^{22} \int \Big(\prod_{\substack{-\pi\le\xi\le\pi \\ 0\le\tau\le A}} d\psi(\xi,\tau)\Big) \exp\Big\{\frac{i}{2}\int_0^A d\tau \int_{-\pi}^{\pi} d\xi (\bar\psi_k(\gamma^a\partial_a)\psi_k)(\xi,\tau)\Big\} \\
&\times \exp\Big\{-\beta\int_0^A d\tau\int_0^A d\tau'\int_{-\pi}^{\pi} d\xi\int_{-\pi}^{\pi} d\xi'(\bar\psi_k\psi_k)^2(\xi,\tau)T_{\mu\nu}(X(\xi,\tau))\times \\
&\delta^{(D)}(X_\alpha(\xi,\tau)-X_\alpha(\xi',\tau'))T^{\mu\nu}(X(\xi',\tau'))\Big\}\times \\
&(\psi_k(-\pi,0)\bar\psi_\ell(\pi,0)\psi_p(\sigma,0)\bar\psi_p(\sigma,0)\psi_p(\sigma,0)W\bar\psi_p(\sigma,0)) \\
&=\sum_{p=1}^{22}\Big[\int\Big(\prod_{\substack{-\pi\le\xi\le\sigma \\ 0\le\tau\le A}} d\psi(\xi,\tau)\Big)\exp\Big\{\frac{i}{2}\int_0^A d\tau\int_{-\pi}^{\sigma} d\xi(\bar\psi_k(\gamma^a\partial_a)\psi_k)(\xi,\tau)\Big\} \\
&(\psi_k(-\pi,0)\bar\psi_p(\sigma,0))\times\exp\Big\{-\beta\int_0^A d\tau\int_0^A d\tau'\int_{-\pi}^{\sigma} d\xi\int_{-\pi}^{\sigma} d\xi' \\
&(\bar\psi_k\psi_k)^2(\xi,\tau)T_{\mu\nu}(X(\xi,\tau))\times \\
&\delta^{(D)}(X(\xi,\tau)-X(\xi',\tau'))T^{\mu\nu}(X(\xi',\tau'))\Big\}\Big]\times \\
&\Big[\int\Big(\prod_{\substack{-\sigma\le\xi\le\pi \\ 0\le\tau\le A}} d\psi(\xi,\tau)\Big)\exp\Big\{\frac{i}{2}\int_0^A d\tau\int_{\sigma}^{\pi} d\xi(\bar\psi_k(\gamma^a\partial_a)\psi_k)(\xi,\tau)\Big\}\times \\
&\exp\Big\{-\beta\int_0^A d\xi\int_0^A d\xi'\int_{\sigma}^{\pi} d\xi\int_{\sigma}^{\pi} d\xi'(\bar\psi_k\psi_k)^2(\xi,\tau)T_{\mu\nu}(X(\xi,\tau)) \\
&\delta^{(D)}(X(\xi,\tau)-X(\xi',\tau'))T^{\mu\nu}(X(\xi',\tau'))\Big\} \\
&\times(\psi_p(\sigma,0)\bar\psi_\ell(\pi,0))\Big]
\end{aligned}
\tag{22}
$$

and its unity normalization condition

$$\bar W_{pp}[C_{X(\sigma)X(\sigma)}, X_\mu(\sigma,\xi), A] = 1 \tag{23}$$

By imposing the identification $g^2N_c = \beta$ between the QCD $(U(N_c))$ gauge coupling constant and our non linear string theory described by **Eq.** (3) we obtain the identification between the QCD $(U(N_c))$ Wilson Loop **Eq.** (17) and the surface averaged Fermion Propagator **Eq.** (19)

$$
\begin{aligned}
&\Big\langle W_{k\ell}[C_{X(-\pi)X(\pi)}, \mathcal{A}_\mu(x)]\Big\rangle_{\text{Yang Mills } U(N_c)} \\
&= \int_0^\infty dA \Big\langle \bar W_{k\ell}[C_{X(-\pi')X(\pi)}, X_\mu(\sigma,\xi), A]\Big\rangle_s
\end{aligned}
\tag{24}
$$

Critical string wave equations and the QCD ($U(N_c)$) string

The above equation is the main result of this note and generalizes to the case of $U(N_c)$ colour group our previous studies made for the t'Hooft limit of Ref. 1.

Finally we remark that by considering an ultra-violet cut-off on the space-time, $|\Delta X^{\mu}(\sigma,\tau)| \geq 1/\Lambda$, our proposed self-avoiding string theory Eq. (18) in the case of non dynamical 2D-Fermions ($< (\psi_k \bar{\psi}_k >= \mu =$ constant) produces the extrinsic string with the topological invariant of string world - sheet self - intersection number as an effective string theory for the proposed QCD string as conjectured in the first Ref. 5 (see ref. 6 for this study).

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Resumo

Apresentamos uma solução do tipo de cordas auto-repulsoras com uma estrutura Fermiônica como solução formal da Equação Migdal-Makeenko associada a Cromodinâmica Quântica com grupo de simetria $U(N_c)$.