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Information on the Gauge principle from a (2,0) - Supersymmetric Gauge Model

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Abstract A gauge theory exhibiting two supersymmetries is formulated and discussed. To describe it, an additional Grassmannian spinorial co ordinate is added to the N = 112 superspace. Reading off the resulting superfield model in terms of component fields, the presence of two vector potential fields transforming under a single U(l) gauge group is explicitly identified.

1. Introduction

For physics, the next decade corresponds to a time in which particles will be created from the new machines as an artifact. Then, this perspective, probably based on the engineering of particles, is challenging the horizon of the theoretical methods. Therefore, we are now living a period in which speculations should be welcome to enrich the theoretical possibilities for accomodating these several particles that could appear.

Thus the most immediate requirements is about the development of methods in quantum field theory for generating quanta. There are three main methods: gauge theories[1], string theories[2] and the S-matrix[3]. While the first creates a limited number of quanta from a gauge parameter $\alpha(x)$, strings are based on the fact that those states can be regarded as modes of a relativistic one-dimensional system, and the last one is able to locate poles from the analytical properties of the S-matrix. Nevertheless, all these techniques are not being enough to answer the present questions. Supersymmetry can also be included as another method[4]. Under such a context, the necessity of creating quanta, a method, still under investigation, is based on the possibility of associating more than one potential field to the same group parameter $\alpha(x)$,

$$\begin{aligned} A_\mu(x) &\rightarrow A_\mu(x) + \partial_\mu\alpha(x) \\ B_\mu(x) &\rightarrow B_\mu(x) + \partial_\mu\alpha(x) \\ &\vdots \\ N_\mu(x) &\rightarrow N_\mu(x) + \partial_\mu\alpha(x) \end{aligned} \tag{1.1}$$

There are two types of arguments in order to prove whether (1.1) represents a method for generating N-distinct quanta. Their characteristics differ as to their relation to the Lagrangian expression. A first kind of arguments is developed by not requiring the Lagrangian explicit form, while in the second approach, (1.1) is advocated in a less subtle way. This means that, from a given Lagrangian, L, it tests the independence of the N-quanta, through explicit calculations. For instance, by evaluating the degrees of freedom and the masses associated to each of these fields.

This work intends to substantiate the first kind of argument. This means to complement a geometrical origin for (1.1) already stablished from the Kaluza-Klein approach[5] with reasons from supersymmetry. Intuitively, for proving (1.1), a gauge theory is welcomed that carries an abundance of degrees of freedom. Therefore supersymmetry becomes a rich laboratory to investigate about these degrees of freedom transforming as (1.1). Then, the strategy was to study first the $N = 1/2 - D = 2$ and $N = 1$, $D = 3$ supersymmetric models[6]. These cases were enough to identify the presence of a second potential field transforming under the same simple single group. However, in both cases, it was necessary to impose conditions for relaxing some constraints on the following gauge symmetry identities:

$$[\nabla_A, \nabla\} = T_{AB}^{C}\nabla_C + F_{AB} \tag{1.2}$$

$$[\nabla_A,[\nabla_B,\nabla_C\}\} + [\nabla_B,[\nabla_C,\nabla_A\}\} + [\nabla_C,[\nabla_A,\nabla_B\}\} = 0 \tag{1.3}$$

where ∇_A are the gauge covariant derivatives.

Thus, a further case for being advocated is to prove that such a second potential field can emerge directly, i.e., without relaxing (1.2)-(1.3). Searching for this possibility, this work is motivated to study a situation where there is an abundance of degrees of freedom, as a model with a second supersymmetry[7] naturally allows.

This work is organized as follows. In Section 2, a description of the (2,0)-supersymmetry is presented in terms of just one Grassmannian coordinate. However, in order to formulate this second supersymmetry in combination with a gauge theory, Section 3 extends the superspace. Sections 4-5 develop a U(1)-gauge theory based upon a (2,0) supersymmetry. Then, from this conjunction, the existence of a second potential field transforming as (1.1) naturally emerges. An Appendix follows where the decomposition of (2,0)-superfields in terms of (1,0)-multiplets is written down.

2. An N = 1/2 formulation of the (2,0)-supersymmetry

The basic feature for implementing a second supersyrnmetry in the N = 1/2 superspace is to derive generators Q^1 and Q^2 obeying the following algebra

$$\{Q^i,Q^j\} = -2i\partial_-\delta^{ij} \tag{2.1}$$

Considering that for N = 1/2,

$$\{Q, D) = 0 \quad \text{and} \quad D^2 = -i\partial_- \tag{2.2}$$

the most immediate possibility is to choose as generators

$$Q^1 = Q \quad \text{and} \quad Q^2 = D \tag{2.3}$$

where Q and D are respectively the generator and covariant derivative operator for (1,0) supersymmetry.

Thus, in order to build up an action with n bosonic and fermionic superfields $(i = 1,...,n)$,

$$S = \int d^2x d\theta [(D\Phi^i)(\partial_+\Phi^i) + \Psi^i(D\Psi_i) + m_{ij}\Phi^i\Psi^j] \tag{2.4}$$

and invariant under the following supersymmetric transformations,

$$\delta_I \Phi^i = i \, \epsilon \, Q\Phi^i$$
$$\delta_I \Psi^i = i \, \epsilon \, Q\Psi^i \tag{2.5}$$

and

$$\delta_{II} \Phi^i = f^i{}_j \, \xi \, D\Phi^j$$
$$\delta_{II} \Psi^i = g^i{}_j \, \xi \, D\Psi^i, \tag{2.6}$$

where $f^i{}_j$ and $g^i{}_j$ are constant matrices, we have to impose constraints.

Closure conditions as,

$$[\delta_I, \delta_{II}]\Phi^i = 0$$
$$[\delta_{II}, \delta_{II}]\Psi^i \sim i\partial_- \Phi^i \tag{2.7}$$

with the invariant condition,

$$\delta_{II}\delta = 0 \tag{2.8}$$

and with the reality condition implie that,

$$f^2 = -\mathbb{1}$$
$$f^t = -f \quad , \quad f \text{ real}$$
$$g^2 = -\mathbb{1}$$
$$g^t = -g \quad , \quad g \text{ real} \tag{2.9}$$

and also the following restriction for the mass-matrix,

$$fm + mg = 0 \tag{2.10}$$

Now, in order to comprehend some implications of such N = 1/2 - SUSY (2,0), we are going to study the physical masses. For this, we have to calculate the poles of a two point Green's functions. Writing in components $\Phi = \{\phi, \psi\}$ and $\Psi = \{\beta, F\}$, one gets for the scalar part

$$S_{\text{scalar}} = \int d^2x \left[\frac{1}{2} U^T K U\right]$$

where

$$U = \begin{pmatrix} \phi \\ F \end{pmatrix} \quad , \quad \phi = \begin{pmatrix} \phi^1 \\ \vdots \\ \phi^n \end{pmatrix} \quad , \quad F = \begin{pmatrix} F^1 \\ \vdots \\ F^n \end{pmatrix} \tag{2.11}$$

Then, propagators $< T(\phi(x), \phi(y)) >$, $< T(\phi(x) F(y)) >$ and $< T(F(x) F(y)) >$ are read off from the inverse matrix

$$K^{-1} = \begin{bmatrix} \left(\Box - \frac{1}{2} m m^T\right)^{-1} & -\frac{1}{2} m \left(\Box - \frac{1}{2} m^T m\right)^{-1} \\ -\frac{1}{2} m^T \left(\Box - \frac{1}{2} m m^T\right)^{-1} & \frac{1}{2} \Box \left(\Box - \frac{1}{2} m^T m\right)^{-1} \end{bmatrix} \tag{2.12}$$

where $R = -2\partial_+ \partial_-$.

(2.12) shows that the masses of the scalars fields are the eigenvalues of the matrix mm^T and that the auxiliary fields are not independent,

$$< T(F(x) F(y)) > = \Box < T(\phi(x) \phi(y)) >$$

$$< T(\phi(x) F(y)) > = \frac{1}{2} m < T(\phi(x) \phi(y)) > \tag{2.13}$$

Another possibility for showing a non-physical meaning for the auxiliary fields is by eliminating them through the equation of motion. It gives,

$$S_{\text{scalar}} = \int d^2x \left[\frac{1}{2} \phi^T \left(\Box - \frac{1}{2} m m^T\right) \phi\right] \tag{2.14}$$

where **(2.14)** verifies that the pole structure remains unchanged.

For the fermionic part,

$$S_{\text{fer}} = \int d^2x [i \chi^T R \chi]$$

where

$$R = \begin{bmatrix} \partial_+ & \frac{1}{2} m \\ \frac{1}{2} m^T & \partial_- \end{bmatrix} \quad , \quad \chi = \begin{pmatrix} \psi \\ \beta \end{pmatrix} \tag{2.15}$$

Thus, (2.12), **(2.14)** and **(2.15)** confirm that the masses for the bosonic and fermionic fields, with or without auxiliary fields, are the same. Analysing the off-shell degrees of freedom, one can readily see that there are 2n fermionic and 2n bosonic, whereas on-shell there are only n in both sectors. As an example, to explore the meaning of such a second supersymmetry, we are going to take a case with two bosonic and fermionic superfields. From (2.9) constraints, one gets

$$f = g = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{2.16}$$

$$f = -g = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{2.17}$$

Substituting (2.16) and (2.17) in (2.10), it gives two possible mass matrices

$$m = \begin{pmatrix} m_1 & m_2 \\ m_2 & -m_1 \end{pmatrix} \tag{2.18}$$

and

$$m = \begin{pmatrix} m_1 & m_2 \\ -m_2 & m_1 \end{pmatrix} \tag{2.19}$$

where both prescriptions results in

$$mm^T = m^T m = (m_1^2 + m_2^2)\mathbb{1} \tag{2.20}$$

Consequently, (2.20) shows that the fields ϕ^1, ϕ^2, ψ^1, ψ^2 and β^1, β^2 contain the same physical mass. Thus, this second supersymmetry is well implemented. However, the mass spectrum reveals a difference between these two supersymrnetries. In the first case all masses are possible, while the second supersymmetry restricts the spectrum just to real masses as in (2.20). This means that its presence avoids tachyons.

For the interaction contribution, two terms can be proposed with a dimensionless coupling constant

$$S_{\text{int}} = \lambda_{ijk} \int d^2x d\theta \phi^i (D\Phi^i)(\partial_+ \Phi^k) \tag{2.21}$$

and

$$S_{\text{int}} = G_{ijk} \int d^2x d\theta (D\Phi^i)(\partial_+ \Phi^j)(D\Psi^k) \tag{2.22}$$

The conditions over λ_{ijk} and G_{ijk} for implementing supersymmetry (2,0) are

$$\begin{aligned} &\lambda_{ikm} f^k_{\ j} + \lambda_{kjm} f^k_{\ i} = 0 \\ &\lambda_{ijk} f^k_{\ m} - \lambda_{kjm} f^k_{\ i} = 0 \end{aligned} \tag{2.23}$$

and

$$\begin{aligned} &G_{ijk} f^j_{\ m} + G_{jmk} f^j_{\ i} = 0 \\ &G_{ijk} f^j_{\ m} + G_{imj} g^i_{\ k} = 0. \end{aligned} \tag{2.24}$$

However, for the purpose of this work, to substantiate the presence of more than one potential field transforming as (1.1), (2.4)-(2.21)-(2.22) are not enough. It will be necessary to study a gauge theory involving such a supersymmetry. Nevertheless, for the degrees of freedom of a su.sy. (2,0) to be a source to explain such a pluriformity of potential fields that a gauge group develops, it would be more efficient to work with an extended superspace.

3. An extended superspace

In order to formulate a gauge theory containing a second supersymmetry it is advisable to extend the superspace formulation. The action invariance is not satisfied by (2.4), when complex matter and gauge superfields are introduced. Thus, we have to deal with a superspace element given by the supercoordinate $Z^A \equiv (x^+, x^-; \theta_1, \theta_2)$, where θ_1 and θ_2 are two independent L-handed Majorana-Weyl spinors. Considering the superspace transformations as in ref. 7,

$$\begin{aligned} x^{+'} &= x^+ \\ x^{-'} &= x^- + i\epsilon_1\theta_1 + i\epsilon_2\theta_2 \\ \theta_1' &= \theta_1 + \epsilon_1 \\ \theta_2' &= \theta_2 + \epsilon_2 \end{aligned} \tag{3.1}$$

one gets the supercharges Q_1 and Q_2,

$$Q_1 = -i\left(\frac{\partial}{\partial\theta_1} + i\theta_1\partial_-\right) \tag{3.2}$$

$$Q_2 = -i\left(\frac{\partial}{\partial\theta_2} + i\theta_2\partial_-\right) \tag{3.3}$$

where **(3.2)** and **(3.3)** satisfy (2.1).

Informations can also be obtalned through the complex spinors θ, θ^*

$$\begin{aligned} \theta &= \theta_1 + i\theta_2 \\ \theta^* &= \theta_1 + i\theta_2 \end{aligned} \tag{3.4}$$

For instance, the covariant derivatives

$$\begin{aligned} D &= 2\frac{\partial}{\partial\theta} - i\theta^*\partial_- \\ D^* &= 2\frac{\partial}{\partial\theta^*} - i\theta\partial_- \end{aligned} \tag{3.5}$$

expresses that

$$\begin{aligned} D^2 - (D^*)^2 &= 0 \\ \{D, D^*\} &= -4i\partial_- \end{aligned} \tag{3.6}$$

The next step is to define superfields in order to accomodate the (2,0) degrees of freedom. The complex scalar superfield is defined as

$$\Phi(x;\theta,\theta^*) = \phi(x) + \theta\psi(x) + \theta^*\chi(x) + \theta\theta^* A(x) \tag{3.7}$$

where the vectorial degree of freedom can be eliminated through the constraint

$$D\Phi = 0 \tag{3.8}$$

It yields,

$$\Phi^{(2,0)} = \phi(x) + \theta^*\psi(x) + \theta\theta^*\left(\frac{i}{2}\partial_-\phi(x)\right) \tag{3.9}$$

Similarly, defining the complex left-handed spinorial superfield Ψ as

$$\Psi(x;\theta,\theta^*) = \beta + \theta^* F + \theta G + \theta^*\theta\chi \tag{3.10}$$

and

$$D\Psi = 0 \tag{3.11}$$

one gets

$$\Psi^{(2,0)} = \beta + \theta^* F + \theta\theta^*\left(\frac{i}{2}\partial_-\beta\right) \tag{3.12}$$

For revealing the existence of more than one supersymmetry, (3.9) and *(3.12)* can be rewritten as

$$\Phi^{(2,0)} = \Phi^{(1,0)} - i\theta_2 D_1 \Phi^{(1,0)}$$

$$\Psi^{(2,0)} = \Psi^{(1,0)} - i\theta_2 D_1 \Psi^{(1,0)}$$

where

$$\Phi^{(1,0)} = \phi(x) + \theta_1\psi(x)$$

$$\Psi^{(1,0)} = \beta(x) + \theta_1 F(x) \tag{3.13}$$

See Appendix A.

The corresponding free action is

$$S = \int d^2x d\theta_1 d\theta_2 \left\{\frac{1}{4}\left[\Phi^*\partial_+\Phi - h.c\right] - \frac{i}{2}\Psi^*\Psi\right\} \tag{3.14}$$

From now, on we shall drop the subscripts (2,0) and (1,0) of the superfields appearing in the integrand. *(3.14)* is realized in components as

$$S = \int d^2x[(\partial_-\phi^*)(\partial_+\phi) + i\psi^*\partial_+\psi] + \int d^2x[i\beta^*\partial_-\beta + F^*F] \tag{3.15}$$

4. A Gauge Theory with (2,0) Supersymmetry

Consider superfields transforming under the following phase rotation

$$\Phi'^{(2,0)} = e^{iq_1\Lambda}\Phi^{(2,0)} \tag{4.1}$$

$$\Psi'^{(2,0)} = e^{iq_2\Lambda}\Psi^{(2,0)} \tag{4.2}$$

where $\Lambda(x;\theta,\theta^*)$ is a complex scalar superfield and q_1, q_2 are the respective U(1) charges. Here, for the constraints *(3.8)* and (3.11) to be preserved, the superfield gauge parameter must also be restricted to

$$D\Lambda(x;\theta,0^*) = 0 \tag{4.3}$$

It gives,

$$\Lambda(x;\theta,\theta^*) = \alpha(x) + \frac{\theta^*}{\sqrt{2}}\eta(x) + \theta\theta^*\left(\frac{i}{4}\partial_-\alpha(x)\right) \tag{4.4}$$

Covariantizing

$$\nabla_+\Phi^{(2,0)} \equiv \partial_+\Phi^{(2,0)} + iq_1g\left(\Gamma_+^{(2,0)} + iG_+^{(2,0)}\right) \tag{4.5}$$

with

$$\Gamma_+'^{(2,0)} + G_+'^{(2,0)} = \Gamma_+^{(2,0)} + iG^{(2,0)} - \frac{1}{g}\partial_+\Lambda^{(2,0)} \tag{4.6}$$

We obtain the real gauge superfields $G_+^{(2,0)}$ and $\Gamma_+^{(2,0)}$.

The minimal coupling

$$S = S_{\text{scalar}} + S_{\text{fermionic}} \tag{4.7}$$

with

$$S_{\text{scalar}} = \frac{1}{4}\int d^2x d\theta_1 d\theta_2\left(\Phi^* e^{-q_1\tilde{g}V}(\nabla_+\Phi) - (V+@)^*c^{-q_1\tilde{g}V}\Phi\right) \tag{4.7a}$$

$$S_{\text{fermionic}} = -\frac{i}{2}\int d^2x d\theta_1 d\theta_2 \Phi^* e^{-q_2\tilde{g}V} \tag{4.7b}$$

also requires the presence of a third gauge superfield V which transforms as

$$V'^{(2,0)} = V^{(2,0)} - \frac{i}{\tilde{g}}\left(\Lambda^{*(2,0)} - \Lambda^{(2,0)}\right) \tag{4.8}$$

where $V^{(2,0)}$ is real and Lorentz scalar.

Writing in components, these three superfields are read off as

$$\Gamma_+^{(2,0)}(x,\theta,\theta^*) = A_+(x) + \frac{\theta\rho(x)}{\sqrt{2}} - \frac{\theta^*\rho^*(x)}{\sqrt{2}} + \frac{\theta\theta^*}{2}B(x) \tag{4.9}$$

$$G_{+}^{(2,0)}(x,\theta,\theta^{*}) = W_{+}(x) + \frac{\theta\sigma(x)}{\sqrt{2}} - \frac{\theta^{*}\sigma^{*}(x)}{\sqrt{2}} + \frac{\theta\theta^{*}}{2}S(x) \tag{4.10}$$

$$V^{(2,0)}(x,\theta,\theta^{*}) = C_{+}(x) + \frac{\theta\lambda(x)}{\sqrt{2}} - \frac{\theta^{*}\lambda^{*}(x)}{\sqrt{2}} + \frac{\theta\theta^{*}}{2}M_{-}(x) \tag{4.11}$$

Thus, one gets the following spectrum: $\Gamma_{+}(x,\theta,\theta^{*})$ contains the $A_{+}(x)$ component of a vector, two left-handed spinors $\rho(x)$ and $\rho^{*}(x)$ and one scalar $B(x)$; similarly $G_{+}(x,\theta,\theta^{*})$ has a vector $W_{+}(x)$, spinors $\sigma(x)$ and $\sigma^{*}(x)$ and a scalar $S(x)$; $V(x,\theta,\theta^{*})$ establishes one scalar $C(x)$, two right-handed spinors $\lambda(x)$ and $\lambda^{*}(x)$ and the component M_{-}(x) of a vector.

Re-expressing the superfields Γ_{+}, G_{+} and V in terms of a θ_1, θ_2-expansion and redefining the θ,θ^{*} component fields according to

$$\rho-\rho^{*}\rightarrow i\rho,\ \ \rho+\rho^{*}\rightarrow\tilde{\rho},\ \ \sigma-\sigma^{*}\rightarrow i\sigma,\ \ \sigma+\sigma^{*}\rightarrow\tilde{\sigma},\lambda-\lambda^{*}\rightarrow i\lambda,\lambda+\lambda^{*}\rightarrow\tilde{\lambda},$$

we substitute (4.9), (4.10) and (4.11) in (4.6) and (4.8) and obtain the following gauge transformations,

$$\begin{aligned} A'_{+}(x) &= A_{+}(x) - \frac{1}{g}\partial_{+}\mathrm{Re}\ \alpha(x) \\ \rho'(x) &= \rho(x) - \frac{1}{g}\partial_{+}\mathrm{Im}\ \eta(x) \\ \tilde{\rho}'(x) &= \tilde{\rho}(x) + \frac{1}{g}\partial_{+}\mathrm{Re}\ \eta(x) \\ B'(x) &= B(x) + \frac{1}{2g}\partial_{+}\partial_{-}\mathrm{Im}\ \alpha(x) \end{aligned} \tag{4.12}$$

$$\begin{aligned} W'_{+}(x) &= W_{+}(x) - \frac{1}{g}\partial_{+}\mathrm{Im}\ \alpha(x) \\ \sigma'(x) &= \sigma(x) + \frac{i}{g}\partial_{+}\mathrm{Re}\ \eta(x) \\ \tilde{\sigma}'(x) &= \tilde{\sigma}(x) + \frac{1}{g}\partial_{+}\mathrm{Im}\ \eta(x) \\ S'(x) &= S(x) - \frac{1}{2g}\partial_{+}\partial_{-}\mathrm{Re}\ \alpha(x) \end{aligned} \tag{4.13}$$

$$
\begin{aligned}
C'(x) &= C(x) - \frac{2}{\tilde{g}}\text{Im } \alpha(x) \\
\lambda'(x) &= \lambda(x) + \frac{2}{\tilde{g}}\text{Re } \eta(x) \\
\tilde{\lambda}'(x) &= \tilde{\lambda}(x) + \frac{2}{\tilde{g}}\text{Im } \eta(x) \\
M'_-(x) &= M_-(x) - \frac{1}{\tilde{g}}\partial_-\text{Re } \alpha(x)
\end{aligned}
\tag{4.14}
$$

Observing the transformations for $A_+(x)$ and $M_-(x)$, one can assume $\tilde{g} = g$ and then, identify $M_-(x)$ as A–(x) in such a way that

$$
A'_+(\text{s}) = A_\pm(\text{s}) - \frac{1}{g}\partial_\pm \text{Re } \alpha(x) \tag{4.15}
$$

This means that the potential $A_\mu(x)$ has components: $A_+(x) \in \Gamma_+^{(2,0)}$ and $A_-(x)$ E $V^{(2,0)}$.

However, the relevant conclusion is that the combination between the second supersymmetry with the U(1)-symmetry leads, naturally, to the existence of a second gauge potential $W_+(x) \in G_+^{(2,0)}$. It transforms as (1.1). This property is not necessarily eliminated through a Wess-Zumino gauge. Another observation to point out is about the presence of a Lorentz scalar fields S that contains a gauge transformation.

A second observation about this extra potential field W_+ can be obtained by rewriting the scalar part (4.7a).

$$
\begin{aligned}
S_{\text{scalar}} = \frac{1}{4}\int d^2x d\theta_1 d\theta_2 \Big(&\Phi^* e^{-q_1 gV}\partial_+\Phi + \\
&- \partial_+\Phi^* e^{-q_1 gV}\Phi + 2iqg\Phi^* e^{-q_1 gV}\Gamma_+\Phi\Big)
\end{aligned}
\tag{4.16}
$$

(4.16) shows that the superfield $G_+^{(2,0)}$ decouples from the charged-matter while $\Gamma_+^{(2,0)}$ and $V^{(2,0)}$ are coupled through the charged-matter.

The appearance of the second potential, $W_+(x)$, is also better enforced if a reduction of (2,0) superfields in terms of the (1,0) superspace is accomplished. Indeed, in so doing, it is readly seen that the (2,0) superfield G_+ breaks down into

(1,0) gauge and matter superfields, as explicitly shown in Appendix A. Hence, W_+ emerges as a genuine gauge vector field with matter super-partners. Also, we shall see below, such a decomposition will prove very efficient in getting effective (2,0) gauge theories written in terms of (1,0) gauge and matter superfields.

Thus, integrating over θ_2 the scalar part (4.16) gives

$$\begin{aligned} S_{\text{scalar}} = +\frac{i}{4}\int d^2x d\theta_1 [&(D_1\Phi^*)(\partial_+\Phi) - \Phi^*(\partial_+ D_1\Phi) + \\ &- q_1 g\Phi^*\Gamma\partial_+\Phi - (D_1\partial_+\Phi^*)\Phi + (\partial_+\Phi^*)(D_1\Phi) + \\ &+ q_1 g(\partial_+\Phi^*)\Gamma\Phi + 2iq_1 g\Phi^*\chi\Phi - 2i(q_1 g)^2\Phi^*\Gamma\Gamma_+\Phi + \\ &+ 2iq_1 g(D_1\Phi^*)\Gamma_+\Phi - 2iq_1 g\Phi^*\Gamma_+(D_1\Phi)]e^{-q_1 gV} \end{aligned} \tag{4.17}$$

Now SUSY(1,0) is manifested due to the fact that the action (4.17) written in the superspace (1,0) is in terms of (1,0) superfields. Then, observe that the existence of a SUSY(2,0) is not explicit anymore. It was converted into superfields (1,0) on the superspace (1,0). Therefore, in order to characterize a (2,0) SUSY, it becomes necessary to impose relations that connect the (1,0) superfields $\Phi^{(1,0)}$, $\Gamma_+^{(1,0)}$, $\chi^{(1,0)}$, $V^{(1,0)}$, and $\Gamma^{(1,0)}$. The latter are all defined in Appendix A.

From the second supersymmetry, given by

$$\delta_{\epsilon_2}\Phi^{(2,0)} = i\epsilon_2 Q_2\Phi^{(2,0)}, \tag{4.18}$$

one gets

$$\delta_{\text{effective}}\Phi^{(1,0)} = -i\epsilon_2 D_1\Phi^{(1,0)}, \tag{4.19}$$

where (4.19) shows how the second supersymmetry acts upon the scalar superfield (1,0): the generator is the covariant derivative of the first SUSY and it takes the superfield into a translation.

Similarly one concludes that

$$\delta_{\text{eff}}\Gamma_+^{(1,0)} = i\epsilon_2\chi^{(1,0)} \tag{4.20}$$

$$\delta_{\text{eff}}\chi^{(1,0)} = -\epsilon_2\partial_-\Gamma_+^{(1,0)} \tag{4.21}$$

and that

$$\delta_{\text{eff}}V^{(1,0)} = i\epsilon_2\Gamma^{(1,0)} \tag{4.22}$$

$$\delta_{\text{eff}}\Gamma^{(1,0)} = -\epsilon_2\partial_- V^{(1,0)} \tag{4.23}$$

A different aspect from the second supersymmetry appears in (4.20)-(4.21). It envolves supersymmetric rotations. This means that it transforms fermions into bosons.

Similarly the spinorial part (4.7b) is rewritten in the (1,0) superspace as

$$S = +\frac{1}{2}\int d^2x d\theta_1[\Psi^* e^{-q_2gV} D_1\Psi + \\ + (D_1\Psi^*)e^{-q_2gV}\Psi + q_2g\Psi^* e^{-q_2gV}\Gamma\Psi] \tag{4.24}$$

By definition

$$\delta_{\epsilon_2}\Psi^{(2,0)} = i\epsilon_2 Q_2\Psi^{(2,0)} \tag{4.25}$$

one gets

$$\delta_{\text{eff}}\Psi^{(1,0)} = -i\epsilon_2 D_1\Psi^{(1,0)} \tag{4.26}$$

Substituting (4.19)-(4.23) and (4.26) in (4.17) and (4.24), we have

$$\delta_{\text{eff}} S_{\text{scalar}} = \frac{i}{4}\int d^2x d\theta_1\{e^{-q_1gV}\epsilon_2[+q_1g\Phi^*(\partial_- V)(\partial_+\Phi) + \\ - q_1g(\partial_+\Phi^*)(\partial_- V)\Phi - 2i(q_1g)^2\Phi^*(\partial_- V)\Gamma_+\phi + \\ - \phi^*\partial_+\partial_-\phi - (\partial_-\phi^*)(\partial_+\phi) + (\partial_+\phi^*)(\partial_-\phi) + \\ + (\partial_+\partial_-\phi^*)\phi + 2iq_1g\phi^*(\partial_-\Gamma_+)\phi \\ + 2iq_1g(\partial_-\phi^*)\Gamma_+\phi + 2iq_1g\phi^*\Gamma_+(\partial_-\phi)]\} = 0 \tag{4.27}$$

and

$$\delta_{\text{eff}} S_F = -\frac{1}{2}\int d^2x d\theta_1\epsilon\{e^{-q_2gV}[\Psi^*\partial_-\Psi + (\partial_-\Psi^*)\Psi + \\ - q_2g\Psi^*(\partial_- V)\Psi]\} = 0 \tag{4.28}$$

(4.27) and (4.28) verify that the inclusion of (2,0) degrees of freedom is consistent.

Thus, the (1,0) superspace also contains a second supersymmetry, differently from the approach of section 2. Tlie difference is that here such a second supersymmetry appears effectively, i.e., in a non-manifested way. Writing in components the presence of both supersymmetries for matter terms, it yields,

$$S_{\text{scalar}} = +\frac{i}{4}\int d^2x\{-iq_1g\lambda e^{-q_1gC}[-\psi^*\partial_+\phi+$$
$$-\phi^*\partial_+\psi - q_1g\phi^*\tilde{\lambda}\partial_+\phi + (\partial_+\psi^*)\psi+$$
$$+(\partial_+\phi^*)\psi + q_1g(\partial_+\phi^*)\tilde{\lambda}\phi - 2iq_1g\phi^*\tilde{\rho}\phi+$$
$$-2i(q_1g)^2\phi^*\tilde{\lambda}A_+\phi + 2iq_1g\psi^*A_+\phi + 2iq_1g\phi^*A_+\psi]+$$
$$+e^{-q_1gC}[-i(\partial_-\phi^*)(\partial_+\phi) + 2\psi^*\partial_+\psi + i\phi^*(\partial_+\partial_-\phi)+$$
$$+q_1g\psi^*\tilde{\lambda}\partial_+\phi - 2q_1g\phi^*A_-(\partial_+\phi) + q_1g\phi^*\tilde{\lambda}\partial_+\psi+$$
$$+i(\partial_-\partial_+\phi^*)\phi - 2(\partial_+\psi^*)\psi - i(\partial_+\phi^*)(\partial_-\phi)+$$
$$+2iq_1g\psi^*\tilde{\rho}\psi - q_1g(\partial_+\psi^*)\tilde{\lambda}\phi + 2q_1g(\partial_+\phi^*)A_-\phi+$$
$$-q_1g(\partial_+\phi^*)\tilde{\lambda}\psi + 2iq_1g\psi^*\tilde{\rho}\phi + 4iq_1g\phi^*B\phi+$$
$$+2i(q_1g)^2\psi^*\tilde{\lambda}A_+\phi - 4i(q_1g)^2\phi^*A_+A_-\phi - 2(q_1g)^2\phi^*\tilde{\lambda}\rho\phi+$$
$$+2i(q_1g)^2\psi^*\tilde{\lambda}A_+\psi - 2q_1g(\partial_-\phi^*)A_+\phi + 2q_1g\psi^*\rho\phi - 4iq_1g\psi^*A_+\psi+$$
$$-2q_1g\phi^*\rho\psi + 2q_1g\phi^*A_+(\partial_-\phi)]\} \tag{4.29}$$

and

$$S_F = +\frac{1}{2}\int d^2x\{-iq_2g\lambda e^{-q_2gC}[\beta^*F + F^*\beta + q_2g\beta^*\tilde{\lambda}\beta]+$$
$$+e^{-q_2gC}[2F^*F + i\beta^*\partial_-\beta - i(\partial_-\beta^*)\beta+$$
$$+g_2gF^*\tilde{\lambda}\beta - 2q_2g\beta^*A_-\beta + q_2g\beta^*\tilde{\lambda}F]\} \tag{4.30}$$

Taking g = 0, (4.29) and (4.30) verify (3.15).

Thus, from the above verifications for matter fields, one gets confident to study the implications of such a second supersymmetry.

5. A Version for a (2,0) QED

Gauge covariant derivatives are defined so as to undergo the transformations

$$\nabla \to \nabla' = e^{iq\Lambda}\nabla e^{-iq\Lambda} \tag{5.1}$$

Then, by virtue of the constraint (4.3) on A, a possible representation for the gauge-covariant spinorial derivatives is:

$$\nabla \equiv D \tag{5.2}$$

$$\nabla^* \equiv e^{+qgV^{(2,0)}} D^* e^{-qgV^{(2,0)}}, \tag{5.3}$$

which yields the following field-strengths

$$F_+ = [\nabla, \nabla_+] \tag{5.4}$$

$$\tilde{F}_+ = [\nabla^*, \nabla_+] \tag{5.5}$$

once the superspace torsion components are kept the same as in (3.6). Evaluating (5.4) and (5.5), one gets:

$$F_+ = iqgD(\Gamma_+^{(2,0)} + iG_+^{(2,0)}), \tag{5.6}$$

whose component-field expansion reads:

$$F_+ = iqg\sqrt{2}(\rho + ia) + \theta^* iqg[2B - 2i\partial_- A_+ + 2iS + 2d-W+] \tag{5.7}$$

Similarly,

$$\tilde{F}_+ = iqgD^*(\Gamma_+^{(2,0)} + iG_+^{(2,0)} + i\partial_+ V^{(2,0)}), \tag{5.8}$$

whose 6-expansion is given by

$$\begin{aligned} \tilde{F}_+ = &-iqg\sqrt{2}(\rho^* + i\sigma - i\partial_+\lambda^*) + \\ &- \theta iqg[2B + 2iS - 2\partial_- W_+ + 2\partial_+\partial_- C + 2i(\partial_- A_+ - \partial_+ A_-)] \end{aligned} \tag{5.9}$$

Nevertheless, in order to verify that from the introduction of the second supersymmetry, realized by (3.1) and (3.21, a second massless quantum, W+, naturally emerges, a consistency requirement should be imposed: to reproduce the Maxwell equations as a boundary conditions. With this in mind, a non-interacting W_+-quantum appears from the following superaction:

$$S = -\frac{1}{2(qg)^2} \int d^2x d\theta d\theta^* \tilde{F}_+^* \tilde{F}_+, \tag{5.10}$$

or

$$S = -\frac{1}{8(qg)^2}\int d^2x[(D^*\tilde{F}_+^*)(D\tilde{F}_+) + (DD^*\tilde{F}_+^*)\tilde{F}_+]| \qquad (5.11)$$

From the expansions (5.8) and (5.9) and eq. (5.11), it can be automatically checked that the usual Maxwell action $F_{\mu\nu}F^{\mu\nu}$ is generated along with its supersymmetric partners.

Just to end up this section, we would like to mention that a more constrained (2,0) gauge theory, with a different realization for the gauge-covariant superspace derivatives, has been systematically formulated in the work by Brooks, Gates and Muhamrnad[8].

6. Conclusion

This work has been motivated by the observation that the local gauge principle offers enough room for the generation of distinct intermediate quanta. To substantiate this context, whereby the gauge principle allows for more than a single gauge potential, one has chosen to work with (2,0) supersymmetry. It has the advantage of including two generators in a simple formulation. Thus, besides transforming fermions and bosons amongst themselves, the presence of a second supersymmetry offers another retransformation. This means that the number of involved fields is expected to increase and so, the possibility of finding out a second potential field transforming as (1.1) can be realized.

To implement a gauge theory with this second supersymmetry, it is necessary to consider the superspace with a second Grassmannian spinor coordinate. However, as a first attempt, we have applied relationships (2.5) and (2.6) to generate a gauge theory. Then, a possible gauge potential superfield, H, would appear as transforming in a same multiplet with the usual Γ,

$$\delta_{II}\begin{pmatrix}\Gamma\\ H\end{pmatrix} = M\xi D\begin{pmatrix}\Gamma\\ H\end{pmatrix} \qquad (6.1)$$

where M is a real 2 x 2 matrix. Nevertheless, (6.1) was unsuccessful to organize and accomodate the degrees of freedom of a supersymmetric gauge theory. This situation favoured the introduction of a second Grassmann coordinate, θ_2 [7]. Then, a gauge theory was organized and, as consequence, the second potential field $W_+(x)$

transforming through (4.13), naturaly showed up. It appears as a component of the following (2,0)-superfield:

$$G_+(x,\theta,\theta^*) = W_+(x) + \frac{\theta\sigma(x)}{\sqrt{2}} - \frac{\theta^*\sigma^*(x)}{\sqrt{2}} + \frac{\theta\theta^*}{2}S(x). \qquad (6.2)$$

However, its meaning is better systematized through a Clebsch-Gordon decomposition of the (2,0) supersymmetry. This means that superfields as the ones in (A.3) add to the usual $\Gamma_+^{(1,0)}$ a (1,0) matter multiplet. Finally, one gets the following properties for $W_+(x)$: it is massless, propagates through (5.11), and does not couple to matter.

Thus, three different proofs have already been collected from supersymmetry to identify the presence of a second potential field embedded in the context of gauge invariance. They can be traced back to two possible origins that differ by the relaxation of constraints usually imposed on the field-strength superfields. The first aspect was obtained through the cases $N = \frac{1}{2} - D = 2$, $N = 1 - D = 3$ and, also, $N = 1 - D = 2$ follows the same pattern[6]. Then, by means of certain constraints, a second potential field'has emerged. In the first case such a field is able, at the non-abelian level, to interact without breaking supersymmetry. Making, if one wishes, a convenient soft-breaking supersymmetry in the second case, one gets a scalar electrodynamics for this second potential field. Increasing the scale of possibilities introduced by supersymmetry, this work has tried to bypass such a stage of constraints. So, through a gauge theory with a second supersymmetry, the existence of such a second potential field can be explicitly identified. To illustrate the second origin, no constraints were necessary to be relaxed. Concluding, (2,0) supersymmetry is rich enough to establish the thesis of a second potential without touching the constraints of that superspace. On the other hand, combining this second supersymmetry and relaxing the constraints probably will generate a third potential field.

Just to conclude, we would like to point out that (1.1)can also be supported by arguments based on the fibre bundle approach to gauge theories[g]. This is presently under investigation and it shall be reported elsewhere.

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Appendix A

A (2,0) Clebsch-Gordon Decomposition

A table parametrizing the superfields (2,0) in terms of the superfields (1,0) is catalogued below.

For the matter fields,

$$\Phi^{(2,0)} = \Phi^{(1,0)} - i\theta_2 D_1 \Phi^{(1,0)}$$

$$\Phi^{*(2,0)} = \Phi^{*(1,0)} - i\theta_2 D_1 \Phi^{*(1,0)} \qquad (A.1)$$

$$\Psi^{(2,0)} = \Psi^{(1,0)} - i\theta_2 D_1 \psi^{(1,0)}$$

$$\Psi^{*(2,0)} = \Psi^{*(1,0)} - i\theta_2 D_1 \psi^{*(1,0)} \qquad (A.2)$$

For the potential fields,

$$\begin{aligned} \Gamma_+^{(2,0)} &= \Gamma_+^{(1,0)} + i\theta_2 \chi^{(1,0)} \\ G_+^{(2,0)} &= G_+^{(1,0)} + i\theta_2 \Omega^{(1,0)} \\ V_+^{(2,0)} &= V_+^{(1,0)} + i\theta_2 \Gamma^{(1,0)} \end{aligned} \qquad (A.3)$$

where the expansion in components for the superfields (1,0) is given as

$$\begin{aligned}
\Phi^{(1,0)} &= \phi(x) + \theta_1\psi(x) \\
\Psi^{(1,0)} &= \beta(x) + \theta_1 F(x) \\
\Gamma_+^{(1,0)} &= A_+(x) + i\theta_1\rho(x) \\
\chi^{(1,0)} &= \tilde{\rho}(x) + 2\theta_1 B(x) \\
G_+^{(1,0)} &= W_+(x) + i\theta_1\sigma(x) \\
\Omega^{(1,0)} &= \tilde{\sigma}(x) + 2\theta_1 S(x) \\
V^{(1,0)} &= C(x) + i\theta_1\lambda(x) \\
\Gamma^{(1,0)} &= \tilde{\lambda}(x) + 2\theta_1 A_-(x)
\end{aligned} \tag{A.4}$$

By means of Majorana-Weyl spinors θ_1 and θ_2, the covariant derivatives are:

$$\begin{aligned}
D_1 &= \frac{\partial}{\partial\theta_1} - i\theta_1\partial_- \\
D_2 &= \frac{\partial}{\partial\theta_2} - i\theta_2\partial_-
\end{aligned} \tag{A.5}$$

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Resumo

Uma teoria de gauge exibindo duas supersimetrias é formulada e discutida. Para descrevê-la, uma coordenada Grassmanniana adicional, de natureza espinorial, é acrescentada ao superespaço $N = 1/2$. O modelo em supercampos resultante, quando colocado em termos dos campos componentes, revela a existência de dois campos vetoriais transformando-se sob um único grupo de gauge $U(1)$.