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Analytic stochastic regularization and gauge theories

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Abstract We prove that analytic stochastic regularization breaks gauge invariance. This is done by an explicit one loop calculation of the two, three and four point vertex functions of the gluon field in scalar chromodynamics, which turns out not to be gauge invariant. We analyse the counterterm structure, Langevin equations and the construction of composite operators in the general framework of stochastic quantization.

1. Introduction

Due to many technical aspects, non abelian gauge theories are very difficult to deal with. Nonperturbatively, there are the Gribov ambiguities which prevent a clear gauge specificationl. On the other hand, computer simulations using Monte Carlo methods have unveiled a lot about the structure of the models on a lattice[2]. In spite of this success, crucial problems still persist whenever fermions are included. Even at the perturbative leve1 the situation is not much better since, in many instances, it is not easy to find a gauge preserving regularization scheme[3]. Case examples are the supersymmetric gauge theories where most of the popular schemes fail.

With such a pletora of troubles we found very fortunate the Parisi and Wu proposal of stochastic quantization as a way to circunvemt some of the above

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problems[4]. To begin with, gauge specification is not necessary or, better saying, it is automatically incorporated, thus evading the Gribov ambiguities. Moreover, concerning gauge theories on a lattice, the introduction of a fifth variable (the Langevin time) permits a unique updating of the whole lattice data in each step, saving a huge amount of computer time.

After Parisi and Wu, some authors have proposal new regularization approaches based on the Langevin equation with a non-white noise[5,6,7]. Since, apparently, such procedures do not affect the physical space-time coordinates, it was expected that the new regularization schemes would preserve most of the symmetries. Indeed, a number of results pointed in that direction, indicating that gauge invariance holds in the regularized theory. For example, in the abelian case it was shown that, as it happens in dimensional regularization, vertex functions with at least one gauge field carrying zero momenta vanish[8]. This implies that the highest divergence in the corresponding diagrams always cancel. As a consequence there is no induced mass counterterm. This result has been substantiated by an explicit calculation of the one loop polarization tensor in scalar QED, which turned out to be transversal. A similar reasoning can be applied to the two dimensional Yang-Mills model, in which the regularized polarization tensor is also transversal[6].

The absence of mass counterterms does not preclude the induction of gauge breaking terms, containing derivatives of the gauge field. In fact, it is one of the purposes of this work to present a detailed calculation of the polarization tensor of scalar QED, using the so-called stochastic analytic regularization. Besides the usual transversal terms we found a divergent part of the form $A_\mu \partial^2 A^\mu$. Although innocuous in this abelian situation, such terms is potentially dangereous in the non-abelian case. We have confirmed this suspicion, calculating the divergence of graphs with three and four non-abelian externa1 fields. Our calculations also shows how one could overcome this problem and gives some insight on the higher order corrections.

A possible failure of current conservation in the framework of stochastic regularization has already been noticed[5]. Nevertheless, its consequences it were not clear for the gauge symmetry, since this symmetry is apparently preserved in the

Langevin equation. In this paper, we discuss the appearence of infinite terms and the possible implications for gauge theories. In section 2, we review the general rules of analytic stochastic regularization and analyse the polarization tensor for two-dimensional QED. This is done to expose the mechanism by which it is transversal, contrarily to what happens with ordinary analytic regularization of field theory. In section 3, we study gauge invariance of four-dimensional non-abelian scalar QCD, in the externa1 field approximations. The standard renormalization methods are dissected is the light of stochastic quantization, in section 4. In particular, there we discuss the construction of composite operators and the formal derivation of renormalization group equations. Finally, section 5 presents our conclusions and an overview of our work. For the reader's convenience, we have included two appendices, one to compare our calculations with those of dimensionally regularized field theory and the other containing some technical details ommited in the text.

2. Stochastic quantization and regularization

a. Feynman rules

The basic element in stochastic quantization is the Langevin equation

$$\frac{\partial\varphi}{\partial t}(x,t) = -\frac{\delta S}{\delta\varphi(x,t)} + \eta(x,t) \tag{2.1}$$

where t is a fifth time variable and x represents a four-dimensional space-time coordinate. S is the classical action and η a random field with gaussian probability, defining a Marcovian process. The two-point correlation function of the η field is given by

$$< \eta_i(x,t)\eta_j(y,t') >= 2\delta_{ij}\delta(x-y)\delta(t-t') \tag{2.2}$$

and higher point Green functions are obtained with the help of Wick's decomposition Using (2.1) and (2.2) averages can be computed through

$$< F[\varphi(t)] >_\eta = \int D\eta\, F[\varphi_\eta(t)] \exp\left[-\int_0^\infty dt dx \eta_j \eta_j\right]. \tag{2.3}$$

The above Marcovian process can be related to the field theory specified by the action S as follows: the Green functions of the quantum field are given by the stationary limit of the equal (fifth) time averages of the random field φ, namely

$$< T\phi_{i_1}(x_1)...\phi_{i_N}(x_N) >= \lim_{t\to\infty} < \varphi_{i_1}(x_1,t)...\varphi_{i_N}(x_N,t) >_\eta \tag{2.4}$$

For perturbative purposes, it is convenient to split the action in two terms; a gaussian, quadratic in the fields, and an interaction term

$$S[\varphi] = \int dx \left[\frac{1}{2}\varphi_i D_{ij}\varphi_j + V(\varphi)\right] \tag{2.5}$$

so that the Langevin equations can be rewritten as

$$\frac{\partial\varphi_i}{\partial t} + D_{ij}\varphi_j = -\frac{\partial V}{\partial\varphi_i} + \eta_i \tag{2.6}$$

where

$$D_{ij} = \delta_{ij}(-\partial^2 + m^2), \text{ for a scalar field} \tag{2.7a}$$

and

$$D_{\mu\nu} = -\partial^2\delta_{\mu\nu} + \partial_\mu\partial_\nu \text{ for a gauge field.} \tag{2.7b}$$

As noted elsewhere, due to the time derivative on the left-hand side of (2.6), the propagator

$$G_{ij} = [\partial_t + D]^{-1}_{ij} \tag{2.8}$$

exists even for a gauge theory. Indeed, we have

$$\tilde{G}_{ij}(k,t) = \delta_{ij}\theta(t)\exp[-t(k^2+m^2)] \tag{2.9}$$

for a scalar propagator and

$$\tilde{G}_{\mu\nu}(k,t) = \left\{\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)e^{-tk^2} + \frac{k_\mu k_\nu}{k^2}\right\}\theta(t) \tag{2.10}$$

for a propagator of a gauge field (the tildes denote Fourier transformation).

In (2.10), the presence of a longitudinal part is to be noted. It has been remarked that such a term does not contribute to the Green functions of gauge invariant objects[4]. However, as we shall see later, that term becomes particularly dangereous if a non invariant regularization scheme is employed.

Equation **(2.6)** can be solved iteratively, giving

$$\varphi_i(x,t) = G_{ij} * \left(- \frac{\partial V}{\partial \varphi_j}\left[G * \left(- \frac{\partial V}{\partial \varphi} + \eta\right)\right] + \eta_j\right) \tag{2.11}$$

where the asterisk is to remind that the products must be taken in the convolution sense. Using **(2.11)** and **(2.2)** the N-point Green functions of the φ field can be computed. In a given order of perturbation the following results obtain[g] :

1. Draw all topologically distinct diagrams.

2. Use a cross, $+$, to represent the contraction of two η's. Thus, a line connecting a pair of vertices can be either a crossed or an uncrossed line. The crossed lines are distributed in the graphs so that

2.1 Every loop has, at least, one crossed line.

2.2 Two external vertices can not be connected by a continuous path of lines without crosses.

2.3 Any crossed line can be connected with an external line by a path without crosses.

The number of crossed lines in a graph is

$$\text{number of } + = \text{number of loops} + \text{nurnber of externa1 lines} - 1 \tag{2.12}$$

Observe also that the vertices at the ends of an uncrossed line are naturally ordered according the values of their fifth times. On the other hand, if all lines linking a pair of vertices are crossed lines, then the amplitude for the graph decomposes into a sum corresponding to the two possible (fifth) time ordering of the vertices.

3. To the lines are associated the propagators

$$\text{Uncrossed line} \rightarrow G(x,t) (\text{as given by } (2.910))$$

$$\text{Crossed line} \rightarrow D(x-x';t,t') = \int_0^t d\tau \int dy G(x-y;t-\tau) G(x'-y;t'-\tau) \tag{2.13}$$

A look at the amplitudes constructed with the above rules shows that they, in general, diverge. It is true that graphs with more interna1 crossed lines have a better ultraviolet behaviour, but it is always possible to find graphs as divergent as those in the usual formulation of field theory. Following refs. (5,6) we introduce a non-white noise

$$< \eta(x,t)\eta(x',t') >= \delta(x-x')f_\epsilon(t-t') \tag{2.14}$$

where

$$\lim_{\epsilon\to 0} f_\epsilon(t) = 2\delta(t). \tag{2.15}$$

To be concrete, we choose a particular form for f_ϵ

$$f_\epsilon(t) = \epsilon|t|^{\epsilon-1}. \tag{2.16}$$

The meaning of the above process is the introduction of a non-Marcovian element in the process described by (2.2).

The Green functions regulated by the use of (2.14-16) are meromorphic functions of ϵ, with poles on the real axis. As in the case of analytic regularization of field theory, we could adopt different ϵ's for each η contraction[10]. Although arbitrary, this has some advantages over the use of a unique ϵ.

Using (2.14-16), the crossed propagator must be replaced by

$$D^\epsilon(x-x';t,t') = \int_0^t d\tau \int_0^{t'} d\tau' \int dy G(x-y;t-\tau)G(x'-y;t'-\tau')f_\epsilon(\tau-\tau') \tag{2.17}$$

which for a scalar field gives

$$\tilde{D}^\epsilon(p) = \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{e^{-i\omega(t_1-t_2)}}{(p^2+m^2)^2+\omega^2}\tilde{f}_\epsilon(\omega) \tag{2.18}$$

In the special case defined by (2.16), we obtain

$$\tilde{f}_\epsilon(\omega) = \epsilon\Gamma(\epsilon)|\omega|^{-\epsilon}\sin\left[\frac{\pi}{2}(1-\epsilon)\right] = |\omega|^{-\epsilon}\hat{f}_\epsilon \tag{2.19}$$

such that

$$\begin{aligned}\tilde{D}^\epsilon(p) =& \frac{\hat{f}_\epsilon}{(p^2+m^2)^{1+\epsilon}}\Big[e^{-(p^2+m^2)|t_1-t_2|} \\ &- \epsilon\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\frac{\ln\omega}{1+\omega^2}e^{-i\omega(p^2+m^2)(t_1-t_2)} + O(\epsilon^2)\Big]\end{aligned} \tag{2.20}$$

Note the similarity of the first term on the right-hand side of this equation with the usual analytically regularized Feynman propagator. Since, as mentioned before, the divergences of the regularized amplitudes appear as simple poles, the second term will only contribute to their finite parts. For the same reason the amplitudes containing the $O(\epsilon^2)$ term vanish as the regulator is removed.

b. A sample calculation

To illustrate the use of the regularization method introduced before, we will calculate the lowest order contributions to the polarization tensor of two-dimensional scalar QED. The photon polarization tensor, $\Pi_{\mu\nu}$ is a convenient object to focus our attention, as it must be transversal if gauge symmetry holds. This is also a good test on the advantages of the new regularization method because, as the reader probably knows, the usual analytic regularization method of field theory induces a mass counterterms, breaking gauge invariance.

The model is described by the Lagrangian density

$$L = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \varphi^* D^\mu D_\mu \varphi + m^2 \varphi^* \varphi \tag{2.21}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A$, and $D_\mu = \partial_\mu + ieA + \mu$ is the covariant derivative. The Langevin equations governing the evolution of the fields A,, φ and φ^* are[8,11]

$$\begin{aligned} \dot{A}_\mu &= -\frac{\delta S}{\delta A_\mu} + \eta_\mu = \partial_\rho F^\rho{}_\mu - ie\varphi^* D_\mu \varphi + \eta_\mu \\ \dot{\varphi} &= -\frac{\delta S}{\delta \varphi^*} + \eta = D^2 \varphi - m^2 \varphi + \eta \\ \dot{\varphi}^* &= -\frac{\delta S}{\delta \varphi} + \eta^* = D^2 \varphi^* - m^2 \varphi^* + \eta^* \end{aligned} \tag{2.22}$$

with the random fields η_μ, η and η^* satisfying

$$\begin{aligned} &< \eta_\mu(x,t) \eta_\nu(x',t') > = \delta_{\mu\nu} f_\epsilon(t-t') \delta(x-x') \\ &< \eta(x,t) \eta^*(x',t') > = f_\epsilon(t-t') \delta(x-x') \\ &< \eta(x,t) \eta(x',t') > = 0 \end{aligned} \tag{2.23}$$

Solving iteratively these equations, we found, in lowest order of perturbation, the graphs shown in fig. I. Note that there is one graph contributing to fig. I.a, four graphs contributing to fig. I.b - they correspond to different graphs with the same topology having two crossed lines, one externa1 and the other internal - and two graphs for the fig. I.c. To get a better idea of the details of the calculation, we divide it in two parts. In the first part, we calculate the amplitudes for the graphs, neglecting the contributions of the second term in *(2.20)* to the crossed propagators. Also, for simplicity, in computing correlation functions, we suppose that the fifth times of the fields are equal and very large. We, then, integrate over the fifth times of the internal vertices and keep only the dominant terms (i.e., only those surviving in the infinite fifth time limit). In appendix A, for the reader's convenience, we have included more details of the calculation. For the graph I.a we get

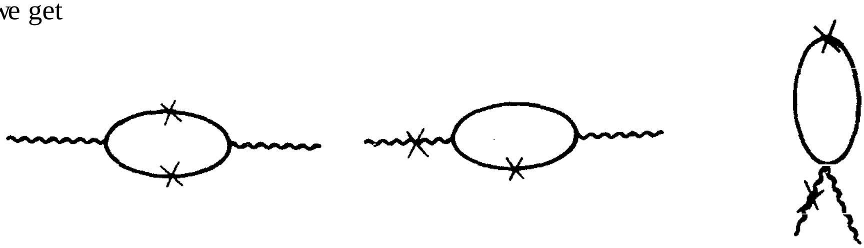

Fig. I - One-loop graph contributing to the photon self-energy in scalar electrodynamics.

$$I.a = 2\hat{f}_\epsilon^2 \int_{\tau_2>\tau_1} d\tau_1 d\tau_2 e^{-p^2(t-\tau_1)} e^{-p^2(t-\tau_2)} \times$$
$$\int \frac{d^2k}{(2\pi)^2} \frac{(2k+p)_\mu (2k+p)_\nu}{[k^2+m^2]^{1+\epsilon}[(k+p)^2+m^2]^{1+\epsilon}} e^{-[(k+p)^2+k^2+2m^2]|\tau_2-\tau_1|} \quad (2.24)$$

As explained in the appendix, the factor 2 on the right hand side of this equation comes from the two possible orderings of the internal times ($\tau_1 > \tau_2$ or $ri < \tau_2$). Integrating in τ_1 and τ_2, we get

$$I.a = \frac{\hat{f}_\epsilon^2}{p^2} \int \frac{d^2k}{(2\pi)^2} \Big[\frac{1}{(k^2+m^2)^{1+\epsilon}} \times$$
$$\frac{(2k+p)_\mu (2k+p)_\nu}{[k^2+m^2]^{1+\epsilon}[(k+p)^2 + k^2 + p^2 + 2m^2]} \Big] \quad (2.25)$$

This expression is very difficult (even for $\epsilon = 0$) to evaluate. Although in two discussions we could still obtain a closed form for it, we find it more instructive to employ a different procedure which has the advantage of being generalizable to four dimensions. The basic observation is that $\Pi_{\mu\nu}$ is analytic in m fir m big enough (equivalently, for small p). Then, $\Pi_{\mu\nu}$ can be expanded in powers of m^{-1} (or, equivalently, in powers of the external momenta) and the transversality property of $\Pi_{\mu\nu}$ will be correct only if it is satisfied in each order of the expansion. In the forthcoming calculation, we will analyse the terms of the above mentioned expansion, up to the first one to be finite when the regularization is removed. With this approximation we have

$$I.a = \frac{\hat{f}_\epsilon^2}{p^2}\int \frac{d^2k}{(2\pi)^2}\frac{2k_\mu k_\nu}{(k^2+m^2)^{3+2\epsilon}} = \frac{\delta_{\mu\nu}}{8\pi p^2 m^2} + O(\epsilon) \tag{2.26}$$

For the graph of fig.(I.b), the calculation is also straightforward, but a little bit more extense. From appendix A, we get

$$I.b = \frac{\delta_{\mu\nu}}{2\pi(p^2)^2}\left(\frac{1}{\epsilon} - 1\right) - \frac{1}{12\pi m^2 (p^2)^2}\left(\frac{5}{2}\delta_{\mu\nu}p^2 - p_\mu p_\nu\right) \tag{2.27}$$

Finally, the graph of fig. (I.c) gives

$$I.c = \frac{-2\hat{f}_\epsilon^2\delta_{\mu\nu}}{(p^2)^{2+\epsilon}}\int \frac{d^2k}{(2\pi)^2}\frac{1}{(k^2+m^2)^{1+\epsilon}} = -\frac{\delta_{\mu\nu}}{2\pi\epsilon(p^2)^2}. \tag{2.28}$$

where the extra factor of two comes from the two graphs of fig. (I.c).

Adding the contributions (2.26-28), we note that the divergent pieces exactly cancel, leaving the result

$$\frac{1}{12\pi p^2 m^2}\left(-\delta_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right) - \frac{\delta_{\mu\nu}}{2\pi(p^2)^2} \tag{2.29}$$

which is, evidently, non transversal. This expression should be compared with the one employing the usual analytical regularization of field theory. In that case, the regularised integrand is obtained by replacing the free propagator (p^2+m^2) by $(p^2+m^2)^\lambda$. With this substitution, we get a polarization tensor differing from (2.29) just by a term which vanishes after a judicious choice of the participating lambdas. Thus, up to this point, there is no great advantage in using stochastic analytic regularization, instead of the more usual one. However, we still have to compute the corrections coming from the neglected terms in (2.20). These terms are very important because, as we shall see right now, they will make the final result gauge invariant. Firstly, notice that there is no correction coming from the graph of Fig. I.a, since it is finite without the regularization. The contribution of the remaining graphs are not difficult to evaluate (see appendix A) and we get the following additional terms

$$\frac{\delta_{\mu\nu}}{\pi(p^2)^2} \quad \text{from Fig. I.b}$$

$$\frac{-\delta_{\mu\nu}}{2\pi(p^2)^2} \quad \text{from Fig. I.c}$$

Adding all these contributians, we get a miraculous cancellation of the non-transversal parts, leaving the net result

$$\frac{1}{12\pi m^2}\left(\frac{p_\mu p_\nu}{p^2} - \delta_{\mu\nu}\right) \tag{2.30}$$

The cancellation of the non-transversal terms is a consequence of a general theorem proven in ref. 7, asserting the non existence of mass corrections to the photon field. However, that theorem does not preclude the induction of non gauge-invariant terms, containing derivatives of the A_μ field.

3. Breaking of gauge invariance in the non abelian case

The calculations of previous section indicate that analytic stochastic regularization can be a very efficient tool for the study of properties of gauge theories, at least the abelian ones. Unfortunately, this result, as we will show now, does not extend to the non abelian situation. Our discussion will be based in the non abelian version of (2.21)

$$L = \mathrm{Tr}\left[\frac{1}{4}\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu} - \varphi^* \mathrm{D}^\mu \mathrm{D}_\mu \varphi + \mathrm{m}^2 \varphi^* \varphi\right] \tag{3.1}$$

where now

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ie[A_\mu, A_\nu] \tag{3.1a}$$

$$D_\mu = \partial_\mu + ieA_\mu \quad , \quad A_\mu = A_\mu^\alpha \lambda^a \tag{3.16}$$

and the λ^a are the generators of the algebra of the gauge groups.

The Langevin equations are given by (2.22) with the replacements (3.1a,b). They can be formally iterated, producing terms that can be represented by stochastic disgrams, as before. If dimensional regularization is employed, one finds that they distribute themselves into classes of gauge-invariant amplitudes. In particular, the set consisting of the diagrarns of figures I till IV is gauge-invariant and, in the large-t limit, generates a counterterm

$$Z_1 F^{\tau}_{\mu\nu} F^{\tau\mu\nu} \tag{3.2}$$

where $F^{\tau}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ieZ_2[A_\mu, A_\nu]$.

However, had we used the analytic stochastic regularization method, the induced counterterm would be

$$Z_1 F^{\tau}_{\mu\nu} F^{\tau\ \mu\nu} + Z_3 A_\mu \partial^2 A^\mu \tag{3.3}$$

which is not gauge-invariant. To verify (3.3), we consider explicitly the contribution of each relevant diagram. This calculation is rather simplified by noting that, since we are interested in divergent pieces, the $O(\epsilon)$ terms in (2.20) can be freely disregarded. To be systematic, we divide the calculation into three parts, according to the number of external gluon lines:

1. Graphs with two external gluon lines. As in the two-dimensional example, there are three diagrams, shown in fig. I. Differently from two dimensions the diagram I.a is now logarithmically divergent. Actually, this diagram is the responsible for the breaking of gauge-invariance, as it will become clear shortly. Analogously to (2.26), we have

$$\begin{aligned} Fig.I.a = \frac{\hat{f}^2_\epsilon}{p^2}\int \frac{d^4k}{(2\pi)^4}\Big[\frac{1}{(k^2+m^2)^{1+\epsilon}} \times \\ \frac{(2k+p)_\mu (2k+p)_\nu}{[(k+p)^2+m^2)]^{1+\epsilon}[(k+p)^2+k^2+p^2+2m^2]}\Big] = \\ = \frac{\hat{f}^2_\epsilon}{p^2}\int \frac{d^4k}{(2\pi)^4}\frac{2k_\nu k_\nu}{(k^2+m^2)^{3+2\epsilon}} = \frac{\delta_{\mu\nu}}{64\pi^2 p^2 \epsilon} + \text{finite term} \end{aligned} \tag{3.4}$$

We would like to call the reader's attention to the exponent 2ϵ in the denominator of the last line of the above expression. The factor 2 comes in because there are two crossed lines in the graph. This implies that (3.4) is only half of the corresponding result for the dimensionally regulated amplitude. The disastrous consequences of this is that (3.4) will not "match" the contributions from the other graphs.

The computation of the graphs of Fig. I.b and I.c is also very simple and gives (up to finite terms).

$$Fig.I.b = \frac{2}{(p^2)^2}\int \frac{d^4k}{(2\pi)^4}\frac{(2k+p)_\mu(2k+p)_\nu}{(k^2+m^2)^{1+\epsilon}[p^2+k^2+(p+k)^2+2m^2]} =$$
$$\frac{1}{(4\pi)^2}\left[-\frac{2m^2\delta_{\mu\nu}}{(p^2)^2}-\frac{5\delta_{\mu\nu}}{6p^2}+\frac{p_\mu p_\nu}{3(p^2)^2}\right]\frac{1}{\epsilon} \tag{3.5}$$

$$Fig.I.c = -\frac{2\delta_{\mu\nu}}{(p^2)^2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2+m^2)^{1+\epsilon}} = \frac{2m^2\delta_{\mu\nu}}{(4\pi)^2(p^2)^2\epsilon} \tag{3.6}$$

2. Now come the graphs with three external gluon lines. These diagrams decompose into two sets of topologically similars graphs. In the first set, fig. II.a, there are 18 (linearly) divergent diagrams. However, the divergent part of these diagrams cancel in groups of two. This is to be expected, since each graph generates a counterterm of the type $\mathrm{Tr}(\partial^\mu A_\mu)A_\nu A^\nu$, which violates charge conjugation. The other set of topologically similar diagrams with 3 external gluon lines contains 18 diagrams. To illustrate the calculation, we write the amplitude for the graph of fig. II.b

$$Fig.II.b = -e^3p_1^2\frac{\mathrm{Tr}[A^\mu A^\nu A^\rho]}{(p_1^2+p_2^2+p_3^2)}\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2+m^2)^{1+\epsilon}}$$
$$\frac{(2k+p_1)_\mu(2k+2p_1-p_2)_\nu(2k-p_3)_\rho}{[p_3^2+k^2+(k-p_3)^2+2m^2][p_2^2+p_3^2+(k+p_1)^2+k^2+2m^2]} \tag{3.7}$$

where the external field propagators have been eliminated. Expanding at $p_i=0$, we find the result

$$-\frac{e^3p_1^2\,\mathrm{Tr}(A^\mu A^\nu A^\rho)}{p_1^2+p_2^2+p_3^2}[(p_1^\mu\delta^{\nu\rho}-p_3^\rho\delta^{\mu\nu}+(p_1-p_3)^\nu\delta^{\mu\rho}]\frac{1}{(4\pi)^2\epsilon}. \tag{3.8}$$

This result can be used to get the amplitudes for the other graph, just changing the momenta and Lorentz indices adequately. Adding all these contributions, we get a final result for graphs with three external gluon lines.

$$-\frac{2e^3}{(4\pi)^2\epsilon}\mathrm{Tr}[(\partial_\mu A_\nu-\partial_\nu A_\mu)A^\mu A^\nu]. \tag{3.9}$$

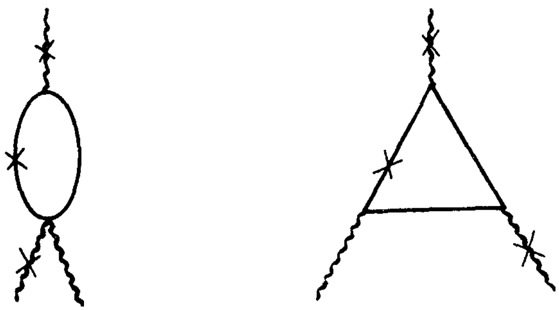

Fig. II – One-loop contribution to the gluon three-point vertex function. Graphs with two internal lines do not induce counterterms because of charge conjugation.

3. Graphs with four external gluon lines. These diagrams can be grouped into three sets of topologically similar graphs. The first set is constituted by graphs with two internal lines. There are 48 such diagrams. They can further be divided into six subsets of 8 diagrams each. In each subset, the indices and momenta of the external lines are fixed. A typical graph, fig. III.a, let us say, has an analytic expression like (for the rest of this section the propagators of the external fields have been eliminated)

$$\frac{2e^4 p_1^2 \, \mathrm{Tr}[(\mathrm{A}_\mu \mathrm{A}^\mu)^2]}{(\mathrm{p}_1^2 + \mathrm{p}_2^2 + \mathrm{p}_3^2 + \mathrm{p}_4^2)}$$

$$\int \frac{d^4 k}{(2\pi)^2} \frac{1}{(k^2 + m^2)^{1+\epsilon}(p_2^2 + p_3^2 + k^2 + (k + p_2 + p_3)^2 + 2m^2)} =$$

$$= \frac{2e^4 p_1^2 \, \mathrm{Tr}[(\mathrm{A}_\mu \mathrm{A}^\mu)^2]}{(2\pi)^2 \epsilon (\mathrm{p}_1^2 + \mathrm{p}_2^2 + \mathrm{p}_3^2 + \mathrm{p}_4^2)} + \text{finite terms} \tag{3.10}$$

Note that a quadrilinear vertex gives a factor of 2 or 1, depending on whether there are one or two crossed gluon lines ending at its end. Adding these 48 diagrams, we obtain

$$\frac{3e^4}{4\pi^2 \epsilon} \mathrm{Tr}(\mathrm{A}_\mu \mathrm{A}^\mu)^2 \tag{3.11}$$

The next group of diagrams consists of triangle graphs, having three internal lines. There are 144 diagrams of this type and, as in the previous case, they can

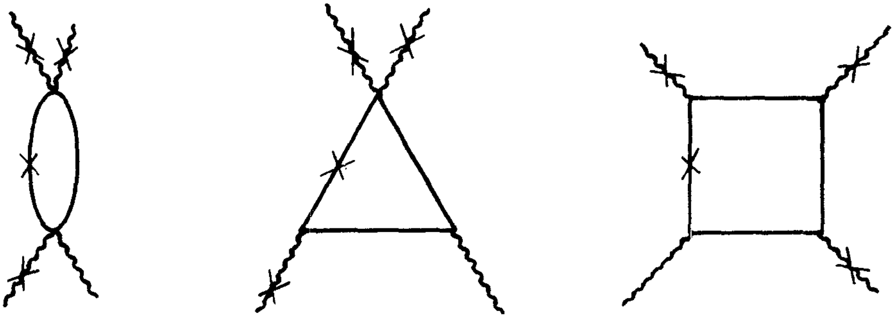

Fig. III – Typical graphs contribution to the gluon four-point vertex function.

be grouped into 12 groups of twelve diagrams, so that in each subset the labels of the external lines are fixed. A typical diagram, fig. III.b, has the expression

$$\frac{2e^4(p_1)^2\ \mathrm{Tr}[A_{\mu_1}A^{\mu_2}(A^\mu A_\mu)]}{(p_1^2+p_2^2+p_3^2+p_4^2)}\int\frac{d^4k}{(2\pi)^4}\left[\frac{1}{(k^2+m^2)^{1+\epsilon}}\right.$$

$$\left.\frac{(2k+p_1)_{\mu_1}(2k+2p_1+p_2)_{\mu_2}}{[k^2+(k+p_1+p_2)^2+p_4^2+p_3^2+2m^2][(k+p_1)^2+k^2+p_2^2+p_3^2+p_4^2+2m^2]}\right]$$

$$=\frac{e^4(p_1)^2\ \mathrm{Tr}(A_\mu A^\mu)^2}{(p_1^2+p_2^2+p_3^2+p_4^2)2(4\pi)^2\epsilon}+\text{finite terms}\ . \tag{3.12}$$

The amplitudes for the other diagrams can be obtained similarly. The final result for the sum of the 144 diagrams is

$$\frac{-6e^4\ \mathrm{Tr}[(A_\mu A^\mu)^2]}{4\pi^2\epsilon} \tag{3.13}$$

Let us, finally, consider the box diagrams shown in fig. III.c. There are 96 diagrarns which can be separated into six groups of **16** diagrams, with pf fixing the labels of the external legs. As an example, we write the expression for the graph III.c.

$$\frac{e^4(p_1)^2\ \mathrm{Tr}[A_{\mu_1}...A^{\mu_4}]}{(p_1^2+p_2^2+p_3^2+p_4^2)}\int\frac{d^4k}{(2\pi)^2}\frac{(2k+p_1)_{\mu_1}}{(k^2+m^2)^{1+\epsilon}[k^2+(k-p_4)^2+p_4^2+2m^2]}\times$$

$$\frac{(2k+2p_1+p_2)_{\mu_2}(2k+2p_1+p_2+p_3)_{\mu_3}(2k-p_4)_{\mu_4}}{[k^2+(k+p_1+p_2)^2+p_4^2+p_3^2+2m^2][k^2+(k+p_1)^2+p_2^2+p_3^2+p_4^2+2m^2]}=$$

$$= \frac{p_1^2 e^4}{12(4\pi)^2\epsilon} \frac{\mathrm{Tr}\left\{3(A_\mu A^\mu)^2 + \frac{1}{4}[A_{\mu_1}, A^{\mu_2}]^2\right\}}{(p_1^2 + p_2^2 + p_3^2 + p_4^2)} \tag{3.14}$$

The other diagrams can be computed similarly. However, we must not forget that some diagrams have additional combinatoric factors due to, possibly, different time orderings for the vertices of the diagrams. With these caveats, the remaining diagrams are calculated, giving

$$\frac{3e^4}{4\pi^2\epsilon}\mathrm{Tr}(A_\mu A^\mu)^2 + \frac{e^4}{8\pi^2\epsilon}[A_\mu, A_\nu]^2. \tag{3.15}$$

Collecting, now, the results (3.4, 5, 6, 9, 11, 13, 15) we obtain the counterterm for the graphs considered in the form (3.3), with

$$\begin{aligned} Z_1 &= \frac{1}{(4\pi)^2 12\epsilon} \\ Z_3 &= -\frac{1}{(4\pi)^2 4\epsilon}. \end{aligned} \tag{3.16}$$

From the above computation, it is clear why our result is not gauge-invariant. This is so because diagram I.b has two internal crossed lines to be regulated and this produces an overall factor of $1/2$. The solution to this problem is very simple and consists in using $\epsilon/2$ for each line of the graph I.b. In higher orders, an analogous prescription can lead to the correct result. Nevertheless our computation puts serious doubts on the consistency of the stochastic analytic regulator employed as a nonperturbative regularization scheme.

In the usual (i.e. non stochastic) formulation of gauge theories, we have to add a gauge fixing term (and possibly Faddeev-Popov ghost) to the Lagrangian density. In a Lorentz gauge, this term is given by $\frac{1}{2\alpha}(\partial_\mu A^\mu)^2$ and, in general, the Green functions are a dependent. Observables (including the S matrix), on the other hand, must be a independent. If a non-gauge regularization scheme is employed, then the a-independence can be achieved only at the expense of adding gauge-dependent couterterms to the original Lagrangian. In the abelian situation, the putative a-dependence of the Green functions comes from graphs pictorially represented in fig. IV. The contributions of this type of diagrams vanish on shell only if the gauge field is coupled to a conserved current*

In the framework of stochastic quantization, the above problem can be particularly dangereous, since the longitudinal part of the A, field has a piece proportional to the fifht time. Therefore, it is mandatory that the gauge field be coupled to a conserved current.

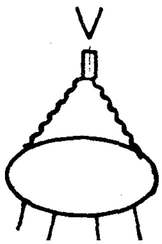

Fig. IV - The non vanishing of this graph makea the Green functions gauge dependent. The special vertex V corresponds to the insertion of the operator $(\partial_\mu A^\mu)^2$.

4. Renormalization

In this section, we shall discuss some peculiarities of the renormalization in the formalism of stochastic quantization. In particular, we will show that the counterterms necessary to render the field theory finite can also be interpreted as counterterms in the stochastic theory, i.e., at finite fifth time. In spite of that, there may be differences (finite renormalizations), since the degree of divergence depends not only on the topology, but also on the number of internal crossed lines. For simplicity, but without loss of generality, we choose the ϕ^4 model and scalar electrodynamics to base our considerations.

First of all, we notice that for finite non-zero times there will be a strong convergence factor, provided by the exponential in the stochastic propagators. As a specific example, consider the case of the photon self-energy in scalar electrodynamics

$$\int \frac{d^4k}{(2\pi)^4} \frac{\exp\{-\tau[(k+p)^2+k^2+2m^2]\}\exp\{-(t-\tau)p^2\}}{k^2+m^2 \qquad\qquad\qquad p^2}, \tag{4.1}$$

* **For a rigorous discussion of gauge-invariance in QED, see ref. 12**

which is finite, insofar as τ is non vanishing. If we now integrate over the fictitious time, then a divergence shows up. Indeed, we have

$$\int_0^t e^{-t\tau} dx = \frac{1-e^{-tx}}{x} \tag{4.2}$$

and the first term gives origin to a (logarithmic) divergent integral. To exhibit this divergence in a more natural way, it is convenient, also, to Fourier transform with respect to the fifth time. Our notational convention is that the Fourier transform, $\tilde{f}(\omega)$, of f (t) is given by

$$\tilde{f}(\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i\omega t} f(t) \tag{4.3}$$

and we observe that w has dimension two in mass units.

With the above definition, the Fourier transform of the propagators are

$$Uncrossed: \ G(p,\omega) = \frac{1}{p^2+m^2+i\omega} \tag{4.4}$$

$$Crossed: \ D(p,\omega) = \frac{1}{p^2+m^2+i\omega}\frac{1}{p^2+m^2-i\omega} \tag{4.5}$$

As an illustration, in the following we will be specific to the four-dimensional Φ^4 theory. Using (4.4,5) and noting, also, that each loop contributes with **6** to the power counting, we get the degree of superficial divergence of a graph

$$\delta(\gamma) = 6m - 2n - 2x \tag{4.6}$$

where

$$m = \#\ of\ loops$$

$$n = \#\ of\ internal\ lines$$

$$X = \#\ of\ internal\ crossed\ lines$$

Using, now, the relations

$$X = m + N_{unc} - 1 \tag{4.6a}$$

$$m = n - V + 1 \tag{4.6b}$$

and

$$4V = 2n + N \tag{4.6c}$$

with N, N_{unc}, and V denoting the numbers of external lines, uncrossed external lines and vertices, we obtain

$$\delta(\gamma) = 6 - N_c - 3N_{unc} \tag{4.7}$$

where N_c is the number of external crossed lines. We see that graphs with $N_{unc} > 2$ are superficially convergent. Let us now examine each case of possible divergence:

1. $N_{unc} = 2$, $N_c = 0$. These are graphs contributing to the two point function and having no external crossed lines (see fig. V). It is easily verified that such divergences can be absorbed in a multiplicative renormalization of the random field.

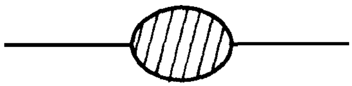

Fig. V – Logarithmic divergent contnbution to the renormalization of the random field η.

2. $N_{unc} = 1$, $N_c = 3$. This is the usual logarithmic divergence of graphs with four external lines (see example in fig. VI). It corresponds to the usual four point vertex renormalization.

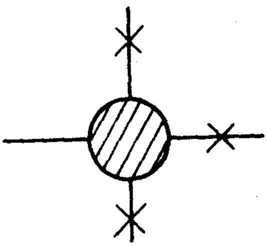

Fig. VI – Lowest order graph contnbution to the charge renormalization in the framework of stochastic quantization.

3. $N_{unc} = 1$, $N_c = 1$. This is the usual quadratic divergence of self-energy graphs (see fig. VII). It can be eliminated through a mass and wave function renormalization.

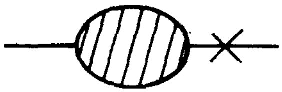

Fig. VII – **This graph contains part of the usual mass and wave function renormalization.**

In the stationary limit of field theory, all the externa1 fifth-times are made equal. This corresponds to joining all external lines of each contributing graph in a new vertex, V_∞, and then integrating over the w variables of the additional loops. One can verify that no divergences arise in this process. Actually, a simple calculation, similar to the one above, gives

$$\delta(\gamma) = 4 - N_c - 3N_{unc} - 3W \tag{4.8}$$

where W denotes the number of lines meeting at V, Since W > 1, then, necessarily, $\delta(\gamma) < 0$. These conclusions agree with those of ref. (13), where a similar power counting was done directly in the space of the fifth-time coordinate.

From the above discussion, we conclude that the renormalization problem is essentially the same as in the usual formulation of the theory. The divergent parts can be removed by reparametrizing the original model. This can be implemented by adopting a convenient subtraction scheme. Without commiting ourselves to any particular scheme, we want to add some remarks on the properties of the resulting theory. The first issue eoncerns the derivation of the field equations. If, for instance, we consider the bilinear $N[\Phi(-\partial^2 + m^2)9]$, where the symbol N indicates a normal product prescription, then, a basic step in the derivation is the amputation of the line associated to the operator $(-\partial^2 + m^2)\Phi$. This is a trivial task since, in momentum space, $(-a^2 + m^2)$ is just the inverse of the propagator and one can always arrange things so that it also happens in the regulated theory. So, it is possible to define bilinear normal products formally obeying the classical Euler-Lagrange equations. However, this result does not imply the absence of anomalies in Ward identities, since the Green function of the current operator have an independent definition and, in principle, their divergences are unrelated to the above rnentioned normal products. Indeed, Green functions of the object $\Phi^* \bar{\partial}^\mu \Phi$, regulated with the use of equation (2.16), do not satisfy current conservations because the regularized propagator is not the inverse of $(-a^2 + m^2)$. This observation is in complete accordance with the results of section 2.

Another remark concerns the behaviour of the Green functions under renormalization group transformations at finite fifth-time. Since, as we have seen before, the elimination of all divergences can be accomplished by the usual wave, mass and charge renormalization (the η field renormalization is, as we saw before, part of the wave function renormalization), then, one should expect that the Green functions $G(x_1, x_2, .., x_N)$ of the stochastic field would obey the renormalization group equation

$$\left[\mu^2 \frac{\partial}{\partial \mu^2} + \beta \frac{\partial}{\partial g} + N\gamma\right] G^{(N)} = 0 \tag{4.9}$$

with the same β and 7 as in the íimiting field theory. Equation (4.9) can be proved as follows: we introduce differential vertex operators[14] (DVO), corresponding to the different field monomials of the lagrangian of the model. Thus, in Φ^4 theory we consider (to be more careful, we could dimensionally regularize our amplitudes)

$$\begin{aligned}
\Delta_1 &= \frac{1}{2} \int d^4x \Phi^2 \\
\Delta_2 &= \frac{1}{2} \int d^4x \partial^\mu \Phi \partial_\mu \Phi \\
\Delta_3 &= \frac{1}{24} \int d^4x \Phi^4
\end{aligned} \tag{4.10}$$

which are defined by the same Feynman rules specifying the Green functions. Thus, Δ_1 means the insertion of a mass vertex in the graphs contributing to the Green functions. It is also convenient to introduce a DVO, Δ_4, which counts the number of crossed lines in a graph. It is given by

$$\Delta_4 = \int d^4x \eta^2 \tag{4.11}$$

As will be clear shortly, the latter is not independent of those in (4.9). Using these DVO's the action becomes formally

$$S[\Phi] = (1+b)\Delta_2 + (m^2 - a)\Delta_1 + (c-g)\Delta_3 + d\Delta_4 \tag{4.12}$$

where a, b, c and d are counterterms. We adopt intermediate renormalization so that the propagator has a pole at $p^2 = -m^2$ but its residue and the other renormalization conditions are dependent on the value of a new parameter μ^2, a different normalization point.

The derivatives $\partial/\partial g$ and ∂/μ^2 have simple expressions in terms of the DVO's (4.10)

$$\frac{\partial}{\partial g}G^{(N)} = \Big[-\frac{\partial a}{\partial g}\Delta_1 + \frac{\partial b}{\partial g}\Delta_2 + \frac{\partial(c-g)}{\partial g}\Delta_3 + \frac{\partial d}{\partial g}\Delta_4\Big]G^{(N)} \tag{4.13a}$$

$$\frac{\partial}{\partial \mu^2}G^{(N)} = \Big[-\frac{\partial a}{\partial \mu^2}\Delta_1 + \frac{\partial b}{\partial \mu^2}\Delta_2 + \frac{\partial c}{\partial \mu^2}\Delta_3 + \frac{\partial d}{\partial \mu^2}\Delta_4\Big]G^{(N)}. \tag{4.13b}$$

There is also a counting identity, which is nothing but the integrated equation of motion

$$-NG^{(N)} = [(1+b)\Delta_2 + (m^2-a)\Delta_1 + (c-g)\Delta_3 + d\Delta_4]G(N). \tag{4.14}$$

Beside the above identities, typical of the usual formulation of field theory, we have another equation which is a consequence of Δ_4 being an operation counting the number of crossed lines. Explicitly, from (4.6a-c),

$$X = \frac{n}{2} - \frac{N}{4} + N_{unc}. \tag{4.15}$$

Using now (4.13-15), we can easily establish the renormalization group equation (4.9). We replace (4.13-15) into (4.9) and equate to zero the coefficient of each DVO. Two of these equations (namely those associated to the coefficients of Δ_1 and Δ_2) can be used to fix β and γ. The remaining one is, then, shown to be identically satisfied by virtue of our on-shell mass renormalization.

5. Conclusions

In this paper we have discussed the use of analytic stochastic regularization in field theory. For abelian gauge theories there is some clear advantage in the use of such a method. Indeed, in lower dimensions (< 4) the polarization tensor turns out to be transversal and gauge-invariance is not broken. In four dimensions, because of quadratic divergences, there is the induction of a non gauge-invariant counterterm of the form $A_\mu \partial^2 A^\mu$. Technicaliy, this happens because the number of internal crossed lines (which are the ones affected by the regularization) varies from graph to graph. This causes unbalances in the usual weights of each individual graph, so that non-invariant pieces do not cancel anymore. These observations are, of course, related to the fact that the gauge field is not coupled to a conserved current, as would be mandatory in a gauge preserving scheme. Nevertheless, this induction of a non gauge-invariant counterterm is not really dangerous in the abelian case, since it can be traded for a renormalization of the gauge-fixing term.

The situation is more complicated in the non-abelian case. It is not possible to absorve the counterterm $A_\mu \partial^2 A^\mu$ into a change of the gauge-fixing term. The problem is that the renormalization of the gluon polarization tensor is now dependent on the renormalization of the three and four point proper vertex functions of the gluon field. At the perturbative level, we could remedy this disease by attributing different epsilons to each crossed propagator[10]. This is not in the original spirit of the method, but can produce gauge-invariant amplitudes, if the epsilons are chosen adequately. Non-perturbatively however, there is not, up to the present, any prescription which guarantees gauge-invariant results.

To define composite operators, care must be taken, since there are at least two "natural", but inequivalent, ways to introduce normal products. The first possibility, which we have adopted in the derivation of the renormalization group equations, is to treat the formal product in the same way as an ordinary Lagrangian vertex. Another possibility, inequivalent to this one, is obtained by taking the product of fields at different points and, then, letting the points coincide. The difference between these two objects can be traced back to the flow of the fifth time. In the first case the fifth time flows into the special vertex (i.e., its time variable is higher than those of any vertex nearby), whereas in the latter case the flow goes continuously through it.

Appendix A

In this appendix we will show details of the calculation of the graph I.b in two dimensions. The calculation of the other graphs in both two and four dimensions is completely analogous and should not present any additional difficulties. Using the η field two-point function (2.14,16), it is easy to see that the crossed propagator is given by (2.21). As the divergences of the Feynman integrals appear as simple poles, the term of order of ϵ in (3.21) will contribute only in those integrals which becomes divergent as ϵ tends to zero.

Since our primary interest is in the stationary limit of quantum field theory, then the external fifth times in fig. I.b are equal and very large. Moreover, because of the rules of stochastic quantization, the internal times satisfy

$$\tau_1 < \tau_2 < t_2 = t_1 = t \to \infty. \tag{A.1}$$

Therefore, we have

$$\begin{aligned}
&Fig.I.b = \\
&\int d\tau_1 d\tau_2 D_c(p;t_1,\tau_1) \int \frac{d^2k}{(2\pi)^2} D_c(k+p;\tau_1,\tau_2) D(k;\tau_1,\tau_2) D(p;t_2,\tau_2) \\
&= \hat{f}_\epsilon^2 \int_{-\infty}^{t} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_1 \int \frac{d^2k}{(2\pi)^2} \frac{(2k+p)_\mu (2k+p)_\nu e^{-(k^2+m^2)(\tau_2-\tau_1)-p^2(t_2-\tau_2)}}{(p^2)^{1-\epsilon}[(k+p)^2+m^2]^{1+\epsilon}} \\
&\times \int_{-\infty}^{\infty} \frac{dx_1}{\pi} \frac{(1-\epsilon \ln|x_1|)}{1+x_1^2} e^{-ix_1 p^2 (t_1-\tau_1)} \\
&\times \int_{-\infty}^{\infty} \frac{dx_2}{\pi} \frac{(1-\epsilon \ln|x_2|)}{1+x_2^2} e^{-ix_2[(k+p)^2+m^2](\tau_1-\tau_2)}
\end{aligned} \tag{A.1}$$

Integrating in τ_1 and τ_2 and incorporating also the contributions of the other three diagrams gives

$$Fig.I.b = A + B$$

$$A = \frac{2\hat{f}_\epsilon^2}{(p^2)^2} \int \frac{d^2k}{(2\pi)^2} \frac{(2k+p)_\mu (2k+p)_\nu}{(k^2+m^2)^{1+\epsilon}[(k+p)^2+k^2+p^2+2m^2]} \tag{A.2}$$

$$B = \frac{4\hat{f}_\epsilon^2}{(p^2)^2} \int_{-\infty}^{\infty} \frac{dx_1}{\pi} \frac{-\epsilon \ln|x_1|}{(1+x_1^2)(1+ix_1)} \times$$

$$\int \frac{d^2k}{(2\pi)^2} \frac{(2k+p)_\mu (2k+p)_\nu}{[(k+p)^2+m^2]^{1+\epsilon}(ix_1p^2+(k+p)^2+k^2+2m^2)} +$$

$$+ \frac{2\hat{f}_\epsilon^2}{(p^2)^2} \int_{-\infty}^{\infty} \frac{dx_2}{\pi} \frac{-\epsilon \ln|x_2|}{(1+x_2^2)} \times$$

$$\int \frac{d^2k}{(2\pi)^2} \frac{(2k+p)_\mu (2k+p)_\nu}{((k+p)^2+m^2]^{1+\epsilon}(p^2+k^Z+m^2+ix_2((k+p^2)+m^2)]}. \qquad (A.3)$$

We now rescale the loop momentum, $k \to mk$, and expand the integrand in powers of p/m. Keeping again only the contributions surviving in the $\epsilon \to 0$, we note that the two terms in **(A.3)** give identical results. We get

$$A = \frac{\delta_{\mu\nu}}{2\pi(p^2)^2}\left(\frac{1}{\epsilon}-1\right) - \frac{5\delta_{\mu\nu}}{24\pi p^2 m^2} + \frac{p_\mu p_\nu}{12\pi (p^2)^2 m^2}. \qquad (A.4)$$

For the B term, the only divergence comes from p = 0, in which case the integral over x can be performed

$$\int_{-\infty}^{\infty} \frac{dx}{\pi} \frac{\ln|x|}{(1+x^2)(1+ix)} = -\frac{1}{2}. \qquad (A.5)$$

Using also

$$\int \frac{d^2k}{(2\pi)^2} \frac{4k_\mu k_\nu}{(k^2+m^2)^{2+\epsilon}} = \frac{\delta_{\mu\nu}}{2\pi(p^2)^2\epsilon} + \text{finite terms} \qquad (A.6)$$

we are left with

$$B = \frac{\delta_{\mu\nu}}{\pi(p^2)^2}. \qquad (A.7)$$

Observe that the contributions of the term of order of ϵ are necessarily finite. In four dimensions there is no such cancellation and the computation is simpler since it is not necessary to laok at the finite contributions.

A very important point in the whole calculation is the possibility of more than one contribution arising from the same graph, due to different time ordering for the interna1 vertices. In general, time flows from outside to inside a graph. However, if there is no uncrossed line linking a pair of adjacent vertices, then all possible orderings must be taken into account.

Appendix B

Using dimensional regularization, we will reproduce here the results for the induced counterterms associated to graphs with two, three and four external gluon fields in scalar chromodynamics. These results are ($D = 4 - \frac{\epsilon}{2}$).

1. Graphs with two external (amputated) gluon lines. The graphs are similar to those shown in **fig.** I (but, of course, all propagators are the usual ones of field theory). They give

$$e^2 \int \frac{d^D k}{(2\pi)^D} \frac{-2\delta_{\mu\nu}}{(k^2 - m^2)} +$$
$$e^2 \int \frac{d^D k}{(2\pi)^D} \frac{(2k+p)_\mu (2k+p)_\nu}{(k^2+m^2)[(k+p)^2+m^2]} =$$
$$= \frac{e^2}{3\epsilon}(-\delta_{\mu\nu}p^2 + p_\mu p_\nu). \qquad (B.1)$$

2. Graphs with three external gluon lines. Only the triangle graphs, similar to those of **fig.** II.b, contribute. The divergente part is

$$\frac{-2e^3}{(4\pi)^2\epsilon}\mathrm{Tr}[(\partial_\mu \mathrm{A}_\nu - \partial_\nu \mathrm{A}_\mu)\mathrm{A}^\mu \mathrm{A}^\nu]. \qquad (B.2)$$

3. Graphs with four external lines. The calculation is straightforward and the final result is

$$\frac{e^4}{8\pi^2\epsilon}[A_\mu, A_\nu]^2 \qquad (B.3)$$

which coincides with the one obtained by the use of analytic stochastic regularization.

We see that the breaking of gauge-invariance in the analytic stochastic regularization method is entirely due to the graphs with two external gluon lines.

References

1. V. N. Gribov, Nucl. Phys., **B139, 1 (1978).**
2. K. Wilson, ***Proceedings of Cargese Summer School,*** **1979,** Plenum Press.
3. E. Abdalla and M. C. B. Abdalla, Phys. Lett. **B160, 101** (1985); Nucl. Phys. **B266, 423 (1986).**
4. G. Parisi and Yongshi Wu, Scientia Sinica 24, 463 **(1981).**

5. J. D. Breit, S. Gupta and A. Zaks, Nucl. Phys. **B233, 61** (1984).
6. J. Alfaro, Nucl. Phys. **B253,** 464 (1985).
7. Z. Bern and M. B. Halpern, Phys. Rev. D33, 1134 (1986). D. Zwanziger, Nucl. Phys. **B192,** 483 (1981).
8. Zvi Bern, Nucl. Phys. **B251,** 633 (1985).
9. W. Grimus and H. Huffel, Z. Phys. **C18,** 129 (1983). H. Huffel and P. V. Landshoff, Nucl. Phys. **B260,** 545 (1985).
10. E. Speer, "Generalízed Feynman Amplitudes", Ann. Math. Studies 62. Princeton Univ. Press, 1969.
11. E. Gozzi, Phys. Rev. **D31,** 1349 (1985).
12. J. H. Lowenstein and B. Schroer, Phys. Rev. **D6,** 1553 (1972).
13. J. Alfaro, R. Jengo and N. Parga, Phys. Rev. Lett. 54, 369 (1985).
14. J. H. Lowenstein, BPHZ Renormalization, in *Renormalization* Theory G. Velo and A. S. Wightman eds., D. Reidel Pub. Comp., 1975.

Resumo

Provamos que regularização analítica estocástica quebra invariância de gauge. A prova é obtida através de um cálculo explícito, em um loop, das funções de dois três e quatro pontos do campo do gluon na cromodinâmica escalar, que se mostra não ser invariante de gauge. Analisamos a estrutura de contratermos, as equações de Langevin e a construção de operadores compostos no contexto geral de quantização estocástica.