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A constraint analysis for an N = 1/2, D = 2 supersymmetric model

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Abstract In order to understand about the presence of different potential fields transforming under the same local symmetry group, this work gauge covariantizes an N = 1/2, D = 2 supersymmetric theory. Then, by relaxing the, so-called, conventional constraint, a second gauge-potential field naturally emerges.

1. Introduction

Supersymmetry is a rich laboratory to unveil the meaning of the gauge principle[1]. Therefore, this work intends to focus its different instructions through a very simple case: by covariantizing an U(1), N = 1/2 supersymmetry in two dimensions[2,3]. For this we are going to explore the following two identities

$$[\nabla_A, \nabla_B] = T^c_{AB}\nabla_c + F_{AB} \tag{1}$$

and

$$[\nabla_A,[\nabla_B,\nabla_c\}\} + [\nabla_B,[\nabla_c,\nabla_A\}\} + [\nabla_c,[\nabla_A,\nabla_B\}\} = 0 \tag{2}$$

Thus, considering the superspace formulation

$$Z^A \equiv (x^+, x^-; \theta) \tag{3}$$

with the following supersymmetry transformations

$$x^{+'} = x^{+}$$

$$x^{-'} = x^{-} + i\epsilon\theta \tag{4}$$

$$\theta' = \theta + \epsilon$$

one gets a global U(1) action given by

$$S = \frac{i}{2}\int d^2x d\theta[(D\Phi)^*\partial_+\Phi - (D\Phi)(\partial_+\Phi)^*] \tag{5}$$

where Φ is a complex scalar superfield defined by its component-field content according to

$$\Phi \equiv \{\phi(x), i2^{1/4}\psi(x)\} \tag{6}$$

where $\phi(x)$ is a scalar and $\psi(x)$ is a right-handed Majorana spinor. Elevating (5) to a local transformation

$$\Phi \to \Phi'(x,\theta) = e^{iq\Lambda(x,\theta)}\Phi(x,\theta) \tag{7}$$

where $\Lambda(x,0)$ is a real scalar superfield and q is the U(1)-charge corresponding to the field Φ, the gauge principle writes the following covariant derivatives

$$\nabla_+\Phi \equiv (\partial_+ + iqg\Gamma_+)\Phi \tag{8}$$

$$\nabla\Phi \equiv (D + qg\Gamma)\Phi \tag{9}$$

$$\nabla_-\Phi \equiv (\partial_- + iqg\Gamma_-)\Phi \tag{10}$$

where Γ_+, Γ and Γ_- are superfield connections transforming under U(1) symmetry as

$$\Gamma'_+ = \Gamma_+ - \frac{i}{g}\partial_+\Lambda \tag{11}$$

$$\Gamma' = \Gamma - \frac{i}{g}D\Lambda \tag{12}$$

$$\Gamma'_- = \Gamma_- - \frac{i}{g}\partial_-\Lambda \tag{13}$$

Thus, different connection superfields are generated by gauge covariantizing the space-time and supersymmetric covariant derivatives. In components, they are read as

$$\Gamma_+ = \{A_+, i2^{3/4}\rho\} \tag{14}$$

$$\Gamma = \{2^{3/4}\xi A_-\} \tag{15}$$

$$\Gamma_- = \{B_-, i2^{3/4}\chi\} \tag{16}$$

Observe that the two components of the gauge field $A_+(x)$ and $A_-(x)$ belong to different superfields. $\rho(x)$ and $\xi(x)$ are spinors, but $\xi(x)$ does not have an appropriate, dimension for being interpreted as a physical field. Thus, the gaugino will be determined through a composition, $\partial_+\xi(x) + \rho(x)$, that is gauge-invariant. The condition that the fields must be real guides spinorial fields for both possibilities of being hermitean or anti-hermitean. (8)-(10) definitions have chosen the first case. Both physics are equivalent, although do not necessarily contain the same terms. Finally, note that the space derivative ∂_- does not contribute to covariantization process. However, it influences in the constraint mechanism that the theory formulates. It is the main aspect of this work.

A study of gauge principle properties through a supersymmetric model $N = 1/2$ in $D = 2$ is the motivation of this work. In this introducing we have noted that, preceding any dynamics, there it already exist facts such as (1), (2), (8)-(13). Another intrinsic aspect that the gauge principle develops is about the possibility of supressing some physical regions through constraints. Sections 2 and 3 develop both sides of this relaxing strategy of the constraints. Finally, in the conclusion a comment about the nature of the physics of the gauge principle is made.

2. Standard case

The systematics to be followed here consists of two considerations. First, to impose constraints in the gauge covariant commutation relations and, then, to consult the Bianchi identities in order to stipulate relationships between the various superfields. From (1) and taking the torsion identically to the case with ordinary supersymmetric-covariant derivatives one gets,

$$\{\nabla,\nabla\} = -2i\nabla_- + W \tag{17}$$

Similarly the superfields field strength W_+, W_- and W_{+-} are obtained

$$[\nabla_+,\nabla] = W_+ \tag{18}$$

$$[\nabla_-,\nabla] = W_- \tag{19}$$

$$[\nabla_+,\nabla_-] = W_{+-} \tag{20}$$

Nonetheless, the determinism that symmetry generates for this model contain constraints. This means that it contains the property of imposing constraints. For this, take the following relationship

$$W = 0 \tag{21}$$

and substitute it in (17). Then, the connection Γ_- is eliminated through

$$\Gamma_- = D\Gamma \tag{22}$$

writing in components, such arranged dependence shows

$$B_- = A_-$$

$$\chi = -\partial_-\xi \tag{23}$$

A second limitation to be considered is that (18), (19) and (20) are related through the Bianchi identity. It yields,

$$W_- = 0 \tag{24}$$

$$W_{+-} = iDW_+ \tag{25}$$

Thus, there is only one independent field-strength

$$W_+ = qg(\partial_+\Gamma - iD\Gamma_+) \tag{28}$$

that in components is written as

$$W_+ \equiv \{2^{3/4} qg\lambda, qgF_{+-}\} \tag{27}$$

where

$$\lambda = \partial_+\xi + \rho \tag{28}$$

$$F_{+-} = \partial_+ A_- - \partial_- A_+ \tag{29}$$

Observe that W_+ is real, gauge invariant, accommodates the gaugino and the gauge field strength.

(21) yields the standard case[3],

$$S = S_{\text{gauge}} + S_{\text{g.f.}} + S_{\text{matter}} \tag{30}$$

where

$$S_{\text{gauge}} = -\frac{1}{2q^2g^2}\int d^2x d\theta W_+ DW_+ \tag{31}$$

$$S_{\text{g.f.}} = \frac{1}{2\alpha}\int d^2x d\theta GDG$$

with

$$G = \partial_+\Gamma + iD\Gamma_+ \tag{32}$$

$$S_{\text{matter}} = \int d^2x d\theta \Big\{ \frac{i}{2}[\nabla\Phi(\nabla_+\Phi)^* - (\nabla\Phi)^*\nabla_+\Phi] + \\ + \frac{1}{2}[\Psi^*\nabla\Psi + \Psi(\nabla\Psi)^*] + \frac{1}{2}m[\Phi^*\Psi + \Phi\Psi^*] \Big\} \tag{33}$$

considering the field contents (6) and

$$\Psi = \{2^{1/4}\beta(x), F(x)\} \tag{34}$$

$$\Lambda = \{\alpha(x), ie^{1/4}\eta(x)\} \tag{35}$$

one gets

$$\mathcal{L} = -\frac{1}{2}F_{+-}^2 - i\sqrt{2}\lambda\partial_-\lambda + \\ + \frac{1}{\alpha}\Big[\frac{1}{2}(\partial_+A_- + \partial_-A_+)^2 + i\sqrt{2}(\partial_+\xi - \rho)\partial_-(\partial_+\xi - \rho)\Big] +$$

$$+\frac{1}{2}[(D_-\phi)(D_+\phi)^* + (D_-\phi)^*(D_+\phi)] + gq\xi[\psi(D_+\phi)^* - \psi^* D_+\phi] +$$
$$+\frac{i\sqrt{2}}{2}A[(D_+\psi)^* - igq\sqrt{2}\rho\phi^*] + \frac{i\sqrt{2}}{2}A^*[D_+\psi + iqg\sqrt{2}\rho\phi] +$$
$$+\frac{i\sqrt{2}}{2}\beta^*[(D_\beta - i\sqrt{2}qg\xi F] + \frac{i\sqrt{2}}{2}\beta^*[(D_-\beta)^* + i\sqrt{2}qg\xi F^*] +$$
$$+\frac{1}{2}[F^*B + FB^*]\frac{1}{2}m[i\sqrt{2}(\psi^*\beta + \psi\beta^*) + \phi^*F + \phi F^*] \quad (36)$$

where

$$D_\pm = \partial_\pm + iqgA_\pm$$
$$A = \psi - i\sqrt{2}qg\xi\phi$$
$$B = F + 2qg\xi\beta \quad (37)$$

with the following transformations

$$F'_{+-} = F_{+-}$$
$$\lambda' = \lambda$$
$$A' = e^{i\alpha(x)}A$$
$$B' = e^{i\alpha(x)}B \quad (38)$$

For a more physical approach to analyze the quanta involved in (36), one can take the unitary Wess Zumino gauge.

$$\xi(x) = 0$$
$$\rho(x) = \lambda(x) \quad (39)$$

Calculating the equations of motion in terms of superfields, one gets for Γ_+,

$$\partial_- W_+ + \frac{qg}{\alpha}\partial_- G = iq^2g^2 J_- \quad (40)$$

For Γ,

$$\partial_+ DW_+ - \frac{qg}{a}\partial_+ DG = -q^2g^2 J \quad (41)$$

Where

$$J_- = -\frac{i}{2}[\Phi(\nabla\phi)^* + \Phi^*\nabla\Phi] \quad (42)$$

$$J = \frac{i}{2}[\Phi(\nabla_+\Phi)^* - \Phi^*\nabla_+\Phi] + \Psi\Psi^* \quad (43)$$

For the matter superfield Φ,

$$\frac{i}{2}\{\nabla,\nabla_+\}\Phi - \frac{1}{2}m\Psi = 0 \quad (44)$$

and for 9

$$\nabla\Psi + \frac{m}{2}\Phi = 0 \quad (45)$$

In components, one reads for A+ and *A*-

$$\partial_- F_{+-} + \frac{1}{\alpha}\partial_-(\partial_+ A_- + \partial_- A_+) = iqgj_- \quad (46)$$

$$\partial_- F_{-+} + \frac{1}{a}\partial_+(\partial_+ A_- + \partial_- A_+) = iqgj_+ \quad (47)$$

where

$$j_- = \frac{1}{2}[\phi(D_-\phi)^* - \phi^* D_-\phi] - qg\xi(\phi\psi^* + \phi^*\psi) + \frac{i\sqrt{2}}{2}(\psi^* A - \psi A^*) \quad (48)$$

$$j_+ = \frac{1}{2}[\phi(D_+\phi)^* - \phi^* D_+\phi] - i\sqrt{2}\beta\beta^8 \quad (49)$$

The dynamics for the gaugino-components ξ and ρ fields is

$$\partial_-\lambda + \frac{1}{\alpha}(\partial_+\xi - \rho) = iqg\frac{\sqrt{2}}{}[\phi^* A - \phi A^*] \quad (50)$$

$$\partial_+\partial_-\lambda - \frac{1}{al}\partial_+\partial_-(\partial_+\xi - \rho) = -i\rho g D J|_{\theta=0} \quad (51)$$

Observe that, in order to show consistency between (46) and (47) and between *(50)* and **(51)**, theory **should provide** another information. The matter-field components ϕ, |>, β and F yields, respectively, the following equations of motion,

$$(D_+D_-\phi)^* - \frac{m}{2}F^* = -gqj_\phi \quad (52)$$

$$(D_+\psi)^* + m\beta^* = -gqj_\psi \quad (53)$$

$$(D_-\beta)^* - \frac{1}{2}m\psi^* = -gqj_\beta \quad (54)$$

$$F^* + \frac{m}{2}\phi^* + 2gq\beta^*\xi = 0 \quad (55)$$

$$j_\phi = \psi^*(D_+\xi)^* + (D_+\psi)^*\xi + \psi^*\rho - \xi\partial_+\psi^* + i2\sqrt{2}gq\xi\rho\phi^* \quad (56)$$

$$j_\psi = [i\sqrt{2}\xi(D_+\phi)^* + \frac{1}{gq}(\partial_+\xi - \rho)\psi^*] \qquad (57)$$

$$j_\rho = i\sqrt{2}F^*\xi \qquad (58)$$

A next information that the theory provides is the Noether current conservation. For a supersymmetric theory

$$\mathcal{L} = \int d\theta \mathcal{P}(\Phi_i \ , \partial_\pm \phi_i \ , D\Phi_i \ , \ \partial_\pm \Phi_i) \qquad (59)$$

there are two independent invariances in the superspace that depend on the parameters ϵ and $\Lambda(x,\theta)$. They are

$$\delta_{\text{su.sy.}}\mathcal{P} = -\epsilon 0 \mathcal{P} \qquad (60)$$

and

$$\delta_{\text{gauge}}\mathcal{P} = 0 \qquad (61)$$

substituting (60) in (30) one gets a continuity equation in superspace that expresses the invariance under supersymmetry

$$\partial_+\mathcal{G}_- + D\mathcal{G} = S$$

with

$$\mathcal{G}_- = \frac{i}{2}[(Q\Phi^*)\nabla\Phi - (Q\Phi)(\nabla\Phi)^*] - \frac{1}{qg}(Q\Gamma)DW_+$$

$$\mathcal{G} = \frac{i}{2}[Q\Phi^*)\nabla_+\Phi + (Q\Phi)(\nabla_+\Phi)^*] + \frac{1}{2}[(Q\Psi)\Psi^* + (Q\Psi^*)\Psi] + \\ + \frac{i}{qg}(Q\Gamma_+)DW_+ - \frac{1}{2q^2g^2}W_+QW_+$$

$$S = Q\Big\{\frac{i}{2}[\nabla\phi(\nabla_+\phi)^* - (\nabla\phi)^*\nabla_+\phi] + \frac{i}{2}[\Psi^*\nabla\Psi + \Psi(\nabla\Psi)^*] + \\ \frac{1}{2}m(\Phi^*\Psi + \Phi\Psi^*) - \frac{1}{2q^2g^2}W_+DW_+\Big\} \qquad (62)$$

Using (61) in (30) one gets the following relationship from gauge invariance

$$iq\Lambda[\partial_+J_- + DJ] = 0 \qquad (63)$$

(63) coincides for $\Lambda(x,6)$ being global or local. Considering the global condition

$$D\Lambda = \partial_+\Lambda = 0$$

it yields,

$$\partial_+(DJ_-) - ia_-J|_{\theta=0} = a, J^\mu_G = 0 \quad (64)$$

where J_G r $(iDJ_-, J)|_{\theta=0}$. In components,

$$\partial_+ j_- + \partial_- j_+ = 0 \quad (65)$$

Now the information has appeared that the equations of motion for the potential fields were expecting. (64) shows that although the structure of the field strength F_{+-} does not propitiate a conservation law, there is a combination between (46) and (47) such that there appears a conserved current $J_G(x)$, whose componets are coupled to the potential fields $A_+(x), A_-(x)$, respectively. Similarly, the spinorial equations (50) and (51) work consistently with (64) by showing the presence of only one equation of motion for the photino. These aspects show that the gauge principle offers a closure relation for the field-dynamics. It is important to note that, though the parameter $\Lambda(x,\theta)$ contains two parameters $\alpha(x)$ and $\eta(x)$, the symmetry of the theory does not work as a $U(1)$ x $U(1)$ mechanism with two conservation laws. (64) prints out the presence of just one conserved current as in ordinary QED. The other current, the one coupled to the photino, contains just a gauge invariant behaviour, as shown in (50). Again, such results carry consistency due to the fact that while the potential fields suffer gauge transformations, the gaugino is properly an invariant.

3. Natural action

The most immediate action taken from the gauge principle is the one for which no constraints are used. Practical experiences with the reality constraint, as in four dimensions, have been showing that degrees of freedom of the fields can be eliminated from the theory by imposing suitable constraints on the superfields. This shows that the same symmetry is realized in different layers depending on the constraints' nature. Thus this section intends to explore this fact by digging down the su.sy. region for N $= 1/2$, d $= 2$ until one finds out a symmetry layer with no constraint[4].

Relaxing (21), we have

$$W = 2qg(D\Gamma - \Gamma_-) \tag{66}$$

The field-strength superfields W_- and W_{+-} are fixed from (2) as

$$W_- = \frac{i}{2}(DW) \tag{67}$$

and

$$W_{+-} = -i\Big(\frac{1}{2}\partial_- W_- - DW_+\Big) \tag{68}$$

Thus (66)-(68) shows the existence of three independent potential superfields: Γ_+, Γ and Γ_-. Reading off the gauge invariant W components, one gets

$$A'_\pm(x) = A_\pm(x) - \frac{1}{g}\partial_\pm \alpha(x) \tag{69}$$

$$B'_-(x) = A_-(x) - \frac{1}{g}\partial_- \alpha(x) \tag{70}$$

$$\xi' = \xi + \frac{1}{g\sqrt{2}}\eta \;\; ; \;\; \rho' = \rho - \frac{1}{g\sqrt{2}}\partial_+ \eta \tag{71}$$

$$\chi' = \chi - \frac{1}{g\sqrt{2}}\partial_- \eta \tag{72}$$

where (70) informs about the existence of a second gauge potential in theory.

A next stage is to show that (69) and (70) do not represent a version of a same field obtained from a linear combination. Thus, it becomes necessary to study the quanta and interaction between such fields. The following bilinear terms are added to (31)

$$S_1 = \frac{\lambda_1}{(2qg)^2}\int d^2x d\theta W_+(\partial_+ W) \tag{73}$$

$$S_2 = \frac{\lambda_1}{(2qg)^2}\int d^2x d\theta\theta (DW_+)(\partial_+ W) \tag{74}$$

$$S_3 = \frac{\lambda_1}{(2qg)^2}\int d^2x d\theta\theta (\partial_+ W)^2 \tag{75}$$

where λ_1, λ_2 are free coefficients. Expressing the kinetic part in momentum space, one gets the following set of propagators

$$F.T. \begin{bmatrix} < A_+A_+ > & < A_+A_- > & < A_+B_- > \\ < A_-A_+ > & < A_-A_- > & < A_-B_- > \\ < B_-A_+ > & < B_-A_- > & < B_-B_- > \end{bmatrix}$$

$$= \frac{1}{4\Delta} \begin{bmatrix} \frac{p}{k_+^2} & \frac{m}{k_+k_-} & \frac{n}{k_+k_-} \\ \frac{m}{k_+k_-} & \frac{p}{k_-^2} & \frac{\ell}{k_-^2} \\ \frac{n}{k_+k_-} & \frac{\ell}{k_-^2} & \frac{p+q}{k_-^2} \end{bmatrix} \tag{76}$$

Where

$$\Delta = \lambda_1^2 + 2\lambda_2$$

$$p = 2\lambda_2(1-\alpha) + \alpha\lambda_1^2$$

$$m = -2\lambda_2(1+\alpha) - \alpha\lambda_1^2$$

$$n = -2\lambda_2(1+\alpha) - \lambda_1(2+\alpha\lambda_1)$$

$$\ell = 2\lambda_2(1-\alpha) + \lambda_1(2-\alpha\lambda_1)$$

$$q = -4\Big[1 - \lambda_1(1-\alpha) + \frac{1}{2}\alpha\lambda_1^2\Big] \tag{77}$$

(76) is not to be diagonalized, otherwise it would loose its local interpretation. This is due to the fact that its eigenvalues would be determined in terms of non-polynomial functions of the momentum. Nevertheless, (76) is enough to inform about the theory spectrum. From the first line, one reads that there is a probability related to the residue p to create a quantum of $A+$. Similarly, the A– and B_+ quanta are determined with probabilities related to p and $p+q$. In order to obtain the mass eigenvalue, that is expressed from the square of a quadrimomentum operator, it is necessary to read off the pole of a corresponding two-point Green function. Thus, observing (76), we note that each line contain at least one term whose denominator is in the Lorentz manifest form k_+k_-. This yields that the quanta associated with $A+$, A–, B are massless. Note that a massive term might be obtained from (69) and (70), but it would violate Lorentz covariance.

Thus, (70) and (76) show that $B_-(x)$ does not belong to the class of compensating fields. However it is still necessary to analyse its physical consistency. First, the inclusion of a $B_-(x)$ field without a partner $B_+(x)$ does not break the manifest Lorentz covariance. However, the propagator is able to reproduce a well-defined mass term. Second, aboiit the presence of ghosts: considering that residues p and p + q depend on parameters $\lambda_1, \lambda_2, a,$ the theory is provided with enough constraints to build up probabilities with same sign. Thus, a healthy quantum for $B_-(x)$ exists.

Now, we should understand the presence of this non-covariant $B_-(x)$ field through its interactions. Selection rules brought in by Lorentz weight, dimensional analysis and renormalizability are guiding aspects for possible interacting terms. As an example to show how the interaction terms are selected, we are going to study the case involving the field-strengths W_+ and W. The general expression is

$$\frac{1}{g^s}\int d\theta W^m W_+^n D^p \partial_+^q \partial_-^r \tag{77}$$

Then, considering the above conditions, respectively, this yields

$$\frac{1}{2} + m - \frac{n}{2} + \frac{p}{2} - q + r = 0 \tag{78}$$

$$-s + \frac{1}{2} + m + \frac{3n}{2} + \frac{p}{2} + q + r = 2 \tag{79}$$

$$m + n \geq s \tag{80}$$

and including the interaction condition.

$$m + n \geq 3 \tag{81}$$

one gets only one possibility. It is the non-abelian term

$$\frac{1}{g^4}\int d^2x\ d\theta\ \mathrm{Tr}(WW_+^3) \tag{82}$$

Similarly, through explicity breaking the abelian supersymmetric case, one obtains

$$\frac{\lambda_3}{g^4}\int d^2x d\theta\theta W^2 W_+(\partial_+ W_+) \tag{83}$$

Matter couplings involving Γ_- are also obtained by breaking supersymmetry,

$$\lambda_4 \int d^2x d\theta\theta (\nabla_+\Phi)^*(\nabla_-\Phi)(\Phi^*\Phi)^n + h.c. \tag{84}$$

$$\lambda_5 \int d^2x d\theta\theta \Psi^* (\nabla_- \Psi)(\Phi^* \Phi)^r + h.c. \tag{85}$$

Finally, there is a massive term

$$S_{\text{mass}} = \int d^2x d\theta (m\Phi^* \Psi + m\Phi \Psi^*) \tag{86}$$

The new equations of motion for this, considered as the natural region, are:
For Φ,

$$\frac{i}{2}\{\nabla, \nabla_+\}\Phi - \frac{1}{2}m\Psi + \frac{\lambda_4\theta}{2}\{\nabla_+[(\nabla_-\Phi)(\Phi\Phi^*)^n] + \\
+ \nabla_-[(\nabla_+\Phi)(\Phi\Phi^*)^n] - \frac{n}{2}[(\nabla_+\Phi)^*\nabla_-\Phi + (\nabla_+\Phi)(\nabla_-\Phi)^*](\Phi\Phi^*)^{n-1}\Phi\} + \\
- \frac{i\lambda_5 r}{4}\theta\{[\Psi^*\nabla_-\Psi + \Psi(\nabla_-\Psi)^*](\Phi\Phi^*)^{r-1}\Phi\} = 0 \tag{87}$$

For Ψ,

$$\nabla\Psi + \frac{1}{2}m\Phi + \frac{i\theta\lambda_5}{2}\{\nabla_-[\Psi(\Phi\Phi^*)^r] + (\nabla_-\Psi)(\Phi\Phi^*)^r\} = 0 \tag{88}$$

For Γ_+,

$$\partial_- W_+ - \frac{i\lambda_1}{4}(\partial_+ DW - i\theta\partial_-\partial_+ W) + \\
- \frac{1}{4(2qg)^2} D[\theta W^2 \partial_+ W_+ + \theta\partial_+(W^2 W_+)] = iq^2 g^2 J'_- \tag{89}$$

For Γ,

$$\partial_+ DW_+ + \frac{i\theta}{(2qg)^2} D[WW_+\partial_+ W] - \partial_+\Big[\frac{\lambda_1}{4}\partial_+ W - \frac{i\theta}{4(2qg)^2}W^2\partial_+ W_+\Big] + \\
- \partial_+ D\Big[\frac{\lambda_1\theta}{4}\partial_+ W + \frac{1}{2}\lambda_1 W_+ + \frac{\lambda_1\theta}{2}DW_+ + \lambda_2\theta\partial_+ W\Big] + \\
+ \partial_+\partial_+\Big[\frac{i\theta}{4(2qg)^2}W^2 W_+\Big] = -q^2 g^2 J' \tag{90}$$

For Γ_-,

$$\partial_+\Big[-\frac{\lambda_1}{2}W_+ - \frac{\lambda_1\theta}{2}DW_+ - \lambda_2\theta\partial_+ W\Big] + \frac{i\theta}{(2qg)^2}WW_+\partial_+ W_+ = iq^2 g^2 J'_+ \tag{91}$$

where

$$J'_- = J_- + \frac{\lambda_4\theta}{2}[\Phi(\nabla_-\Phi)^* - \Phi^*\nabla_-\Phi](\Phi\Phi^*)^n \tag{92}$$

$$J'_+ = \frac{\lambda_4\theta}{2}[@(V+@)^* - @^*V+@](@@^{+*}; i\lambda_4\theta(\Psi^*\Psi)(\Phi\Phi^*) \tag{93}$$

$$J' = J \tag{94}$$

Finally, the Noether theorem gives the following conserved current

$$\partial_+ J'_- + \partial_- J'_+ + DJ = 0 \tag{95}$$

4. Conclusion

The main effort in this work has been an attempt at understanding some of the instructions that the gauge principle displays. Whenever combined with supersymmetry, their discussion becomes more enlighthening as the latter touches clearly the organization of the degrees of freedom of theory.

Here, we have adopted to work with a sypersymmetry of the (1,0) - type in order to illustrate that the gauge principle does not compel us to necessarily fix the number of vector potentials appearing in a given gauge model. We have explicitly constructed an (1,0) - supersymmetric Abelian gauge theory characterized by the preserve of two gauge potential transforming similarly, under a common U(1) group. This has been achieved upon the relaxation of typical superspace constraint imposed on the algebra of gauge-covariant derivatives: the so-called conventional constraint.

In our study, we have been able to identify three *independent* gauge connection superfields that accomodate two independent component-field gauge potentials, $A_\mu(x)$ and $B_\mu(x)$. Though in terms of light-cone coordinates only the component $B_-(x)$ of $B_\mu(x)$ appears in the superspace expansion for the superfield $\Gamma_-(x_i\theta)$, the model can be shown to exhibit manifest Lorentz covariance and the propagators $< A_\mu\ A_\nu >$, $< B_\mu\ B_\nu >$ and $< A_\mu\ B_\nu >$ for the vectors can be written down which display the right structure of poles[5].

The question of explicit soft breakings of (1,0) - supersymmetry has also been addressed to and possible interactions for the extra potential, $B_-(x)$, have been found out and written down based on general grounds. It is noterworthy to stress that such interactions respect supersymmetry for the non-Abelian case: the need for a soft breaking is a peculiarity of the U(1) version of the (1,0) gauge model.

The next step of our investigation concerns the study of more general (p, q) - supersyrnmetric gauge models, once we have found a less constrained (1,0) gauge theory. The possibility of extending the results found in this work to the more interesting case of (2,0) and (4,0) supersymmetries is now under consideration and we shall soon report on the results elsewhere.

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Resumo

Para entender-se sobre a existência de diferentes potenciais de calibre transformando-se sob um mesmo grupo local de simetria, este trabalho estuda a teoria supersimétrica N $= 1/2$, $D = 2$. Então, relaxando-se o assim chamado vínculo convencional, um segundo potencial de calibre emerge naturalmente.