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Testing the commutator method for boson mapping in a single j-shell with protons and neutrons

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Abstract The approximate boson mapping developed by Bonatsos, Klein and Li is tested in a single j-shell with protons and neutrons. Systems with up to 4 protons and 4 neutrons are considered in the diagonalization of a pairing plus quadrupole hamiltonian. The resulting energy levels are compared with the exact fermionic case. In all cases the results show a poor quality in the rotational limit, confirming what had already been seen in a single j-shell where just one kind of nucleon had been taken into account.

1. Introduction

Recently, the bosonization of fermion problems has become a widespread activity in theoretical physics. Transforming the fermion systems into boson ones is a technique used in solid state physics [1], quantum field theories [2] and lately, also in nuclear physics. The relative success of the interacting boson model (IBM) [3], proposed by Arima and Iachello one decade ago, can be microscopically justified by boson mappings. Many different boson mapping techniques [4] have been used to establish a connection between the IBM and the shell model.

Thus, the accuracy of the boson mappings needs to be tested in order to guarantee that a correct link between both above mentioned nuclear models has really been obtained. Some works on this line have already been performed. In single j-shells, some boson mappings are successful in describing nuclei in the vibrational limit but fail drastically in the rotational region [5]. In many non-degenerate j-shells, some boson mappings have also been tested [6] and for this purpose, collective bosons had to be selected. The results were considerably better than in a single j-shell mainly for rotational nuclei.

However, in the previous works, just one kind of nucleon was considered, i.e., the isospin degree of freedom was not taken into account. To improve our tests on boson mappings, we introduce here, explicitly, both nucleons in our calculations.

In the following, we consider the BKL method, which is a commutator mapping and is based on the condition that the boson operators satisfy the same commutation relations as their fermionic archetypes. It is an approximate mapping which utilizes the seniority scheme and hence, it is expected to work well near the spherical limit. We study here systems containing up to four neutrons and four protons. A pairing plus quadrupole hamiltonian is diagonalized in both the fermion and boson spaces, and the resulting energy levels are compared. We know that the pairing plus quadrupole interaction is not realistic for identical particles due to the violation of generalized seniority by its quadrupole portion. But for a single j-shell with protons and neutrons, it is the simplest and most realistic hamiltonian. In this work, the proton and neutron shells differ by parity, for instance, so that the isospin degree of freedom need not be introduced explicitly.

Calculations and Results

The hamiltonian we diagonalize here consists of a pairing and a quadrupole interaction and reads

$$H = \Omega[G_\pi A_0^\dagger(\pi) \cdot A_0(\pi) + G_\nu A_0^\dagger(\nu) \,.\, A_0(\nu)] + R_{\pi\nu} Q_\pi \cdot Q_\nu \tag{1}$$

where

$$A_0^{\dagger^F}(\rho) = \frac{1}{\sqrt{2}} \sum_m \frac{(-1)^{j-m}}{\sqrt{2j+1}} a_m^\dagger(\rho) a_{-m}^\dagger(\rho) \tag{2}$$

and

$$Q_\rho^F = \frac{1}{\sqrt{5}} \sum_{mm'} (jmj - m'|2\mu)(-1)^{j-m'} a_m^\dagger(\rho) a_{m'}(\rho) \tag{3}$$

are respectively the pair operator and the quadrupole operator for fermions where $\rho = \pi$ for protons and $\rho = v$ for neutrons. G,, G, and R, are the strengths of the interactions involved.

The boson images of the above operators given by the BKL are

$$A_0^{\dagger B}(\rho) = s^\dagger(\rho)\sqrt{r_\rho}, \tag{4}$$

and

$$Q_\rho^B = \sqrt{\frac{2}{5\Omega}} d_\mu^\dagger(\rho) s(\rho)\sqrt{r_\rho}\phi(n_v,\rho) + \sqrt{\frac{2}{5\Omega}}\sqrt{r_\rho}\phi(n_v,\rho) s^\dagger(\rho)\tilde{d}_\mu(\rho)$$
$$-\sqrt{\frac{\Omega}{10}}[d^\dagger(\rho)\otimes\tilde{d}(\rho)]_\mu^2 (r - \frac{n_s(\rho)}{\Omega})\psi(n_v,\rho) \tag{5}$$

where

$$r_\rho = 1 - \frac{(n_s(\rho)+2n_v(\rho))}{\Omega}, \tag{6}$$

$n_s(\rho)$ stands for the number of ρ-bosons with L=0, $n_v(\rho)$ for the number of non (L=0) ρ- bosons and Ω is the degeneracy of the level, i.e.,$2\Omega = 2j+1$,

$$\phi(n_v,\rho) = \frac{1}{\sqrt{r_{2\rho} - \frac{1}{\Omega}}}, \tag{7}$$

$$\psi(n_v,\rho) = 10\sqrt{\frac{2}{\Omega}}\begin{Bmatrix} 2 & 2 & 2 \\ j & j & j \end{Bmatrix}\frac{1}{r_{2\rho}}, \tag{8}$$

$$r_{2\rho} = 1 - \frac{2n_v(\rho)}{\Omega} = r_\rho + \frac{n_s(\rho)}{\Omega}. \tag{9}$$

We test here a commutator method which exactly maps the pairing operator. Therefore, the pairing interaction must give exactly the same energy levels in both spaces. This really occurs and just some results for the pure pairing case will be shown for the sake of making comparisons easier.

We start by considering a two-boson system with 1ν-boson and 1π -boson. There are just two L=O and three L=2 states. In a shell model picture, if $j = 5/2$, we would be treating a ^{20}Ne nucleus. The energy levels found for the pairing plus quadrupole hamiltonian are shown in table 1.

Next we look at a three boson system with ln-boson and 2ν-bosons. There are three L=O and seven L=2 states involved. Considering again $j = 5/2$, this would be a ^{22}Ne nucleus in a shell model picture. The low-lying energy levels with L=O are shown in table 2.

Table 1 - Pairing plus quadrupole hamiltonian diagonalization ($G_\nu = -0.1$ Mev, $G_\pi = -0.13$ Mev and $R_{\pi\nu} = -1.0$ Mev) for 1π-boson (2 protons) and 1ν-boson (2 neutrons) in a j=5/2 shell.

L	BKL	Shell Model
0	-0.81	-0.81
2	-0.53	-0.55
2	-0.21	-0.27

Table 2 - Pairing plus quadrupole hamiltonian diagonalization ($G_\nu = -0.1$ Mev, $G_\pi = -0.13$ Mev and $R_{\pi\nu} = -1.0$ Mev) for 1π-boson (2 protons) and 2ν-bosons (4 neutrons) in a j=5/2 shell.

L	BKL	Shell Model
0	-0.89	-0.89
0	-0.1	-0.1

Finally, we investigate a 4-boson system with 2π-bosons and 2ν- bosons. Nine L=O states and seventeen L=2 states are involved. In most cases just L=O states will be shown due to the length of the calculation necessary to find higher L states.

We considered 3 different single j-shells, namely, $j = 5/2$ (representing ^{24}Mg), $j = 7/2$ and $j = 9/2$. Our aim is to investigate how the exact results change with the introduction of the quadrupole interaction. For this purpose we vary the strength parameter $R_{\pi\nu}$ so that the hamiltonian (1) can be diagonalised in the vibrational and rotational regions. For the vibrational region we take $R_{\pi\nu} = -0.1$ Mev and -0.3 Mev. For the rotational region we take it equal to -0.9 and -1.0 Mev. We also expect the BKL results to show better agreement in higher j-shells since the mapping converges more rapidly in this case. Unfortunately, it takes too much CPU time to run the shell model code for values of j larger than 9/2.

Considering first j=5/2, we show in figure 1 the energy levels for the pure pairing case. In figures 2,3,4 and 5 we show the deviations from the exact results when $R_{\pi\nu}$ is respectively -0.1, -0.3, -0.9 and -1.0 Mev.

The same investigations follows for $j = 7/2$ and the results are shown in figures 6,7,8,9 and 10.

Finally, for j=9/2 we also plot the spectra for the pairing case in figure 11 and forthe rotational limit, when $R_{\pi\nu} = -1.0$ Mev in figure 12.

The strengths of the interactions included in the calculation of the energy levels can be read off the tables and figures captions. A 4-boson system has already been studied by Navratil and Dobes, where they test a non-hermitian Dyson mapping [7]. We considered in our work, G_π and G_ν to be the same as the ones used by them, but we vary the quadrupole strength. However, choosing $R_{\pi\nu}$ the way they

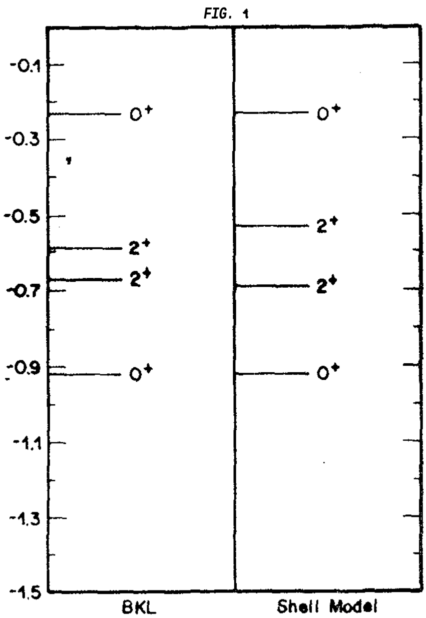

Fig. 1 – Spectra of j=5/2 (G, = -0.1 Mev, G, = -0.13 Mev) for 2π-bosons (4 protons) and 2ν-bosons (4 neutrons). 0^+ and 2^+ states are shown.

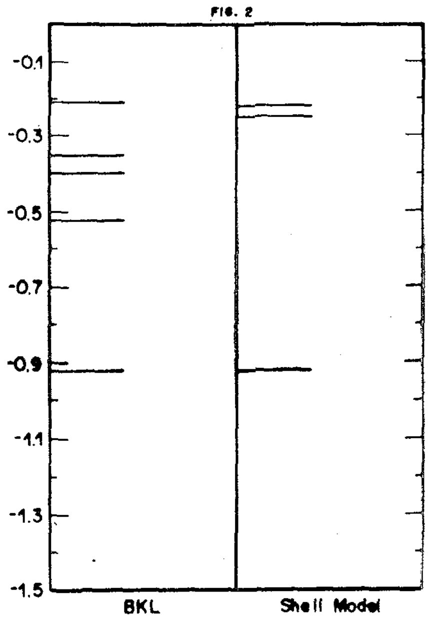

Fig. 2 – Spectra of j=5/2 (G, = -0.1 Mev, G_π = -0.13 Mev and R_π = -0.1 Mev) for 2π-bosons (4 protons) and 2ν-bosons (4 neutrons). Just 0^+ states are shown.

did, we could reproduce with the BKL, the 0^+ energy levels displayed by them in figures 1 and 2, column e. These energy levels correspond to what they call seniority boson hamiltonian.

Conclusion

BKL is a seniority mapping and, as we have already mentioned, it maps the pairing operator exactly. Thus, it is expected to work well in describing nuclei in the vibrational region. This fact is observed in all cases where the pure pairing interaction is considered. It is well known that BKL fails in describing rotational nuclei in single j-shells with just one kind of nucleon [5] and here we tested this method in a single j-shell with protons and neutrons. Various system configurations were considered in three different single j-shells. Due to computing limitations the j-shells treated in this work are not as high as in the case where just one type of nucleon was taken into account. Nevertheless, we could observe how BKL

FIG. 3

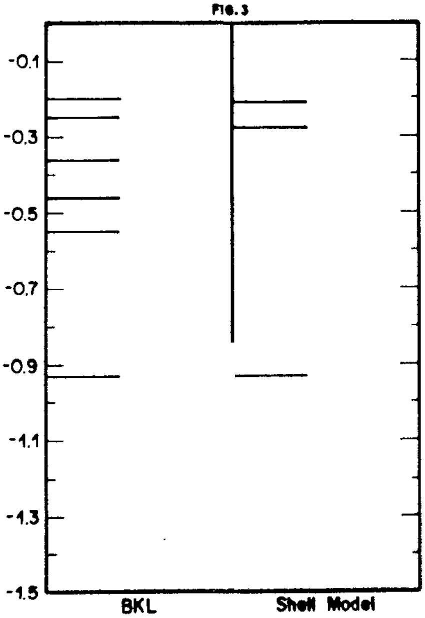

Fig. 3 – Spectra of $j=5/2$ (G, $= -0.1$ Mev, $G_\pi = -0.13$ Mev and *R,,* $= -0.3$ Mev) for 2π-boson (4 protons) and 2ν-bosons (4 neutrons). Just 0^+ states are shown.

FIG. 4

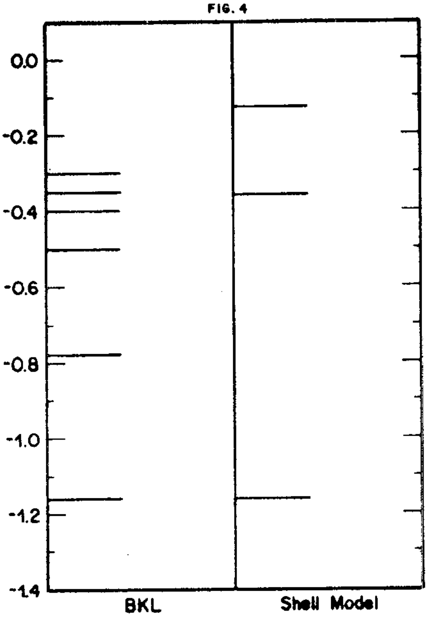

Fig. 4 – Spectra of $j=5/2$ ($G_\nu = -0.1$ Mev, $G_\pi = -0.13$ Mev and *R,,* $= -0.9$ Mev) for 2π-boson (4 protons) and 2ν-bosons (4 neutrons). Just 0^+ states are shown.

deteriorates when we move from the vibrational to the rotational region, where the first energy level is almost always exactly reproduced but higher states are poorly obtained.

In conclusion, we remark that although we expected that the inclusion of protons and neutrons would improve the BKL results near the rotational limit, this improvement was not markedly notable. Thus, we continue stating that the BKL is a good tool in justifying the IBM just near the spherical limit. So far, the only test of BKL which showed reasonable results near the rotational region was the one where non-degenerate j-shells were considered [6].

At this point, we shall mention that there is at least one more test of boson mappings we find worth doing. The most sophisticated nucleus we could try to investigate would be a nucleus containing two non-degenerate j-shells with protons ($j = 13/2$ and $j = 17/2$ for example, to mimic the 50-82 major shell) and also

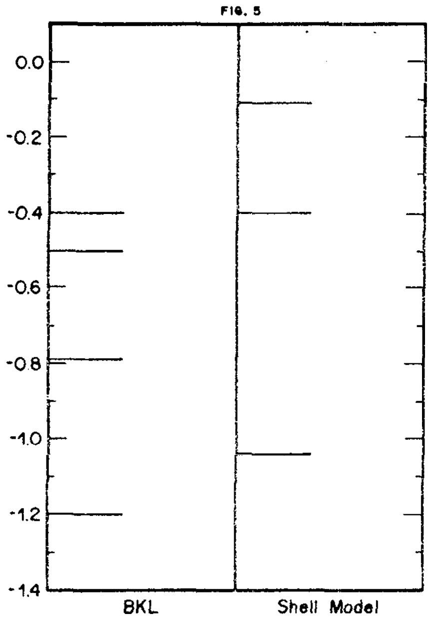

Fig. 5 - Spectra of j=5/2 (G, = −0.1 Mev, G_π = −0.13 Mev and R_ν = −1.0 Mev) for 2π-bosons (4 protons) and 2ν-bosons (4 neutrons). Just 0^+ states are shown.

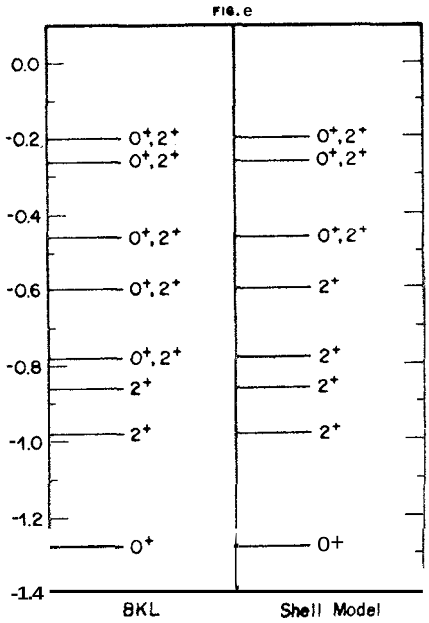

Fig. 6 - Spectra of j=7/2 (G_ν = −0.1 Mev, G_π = −0.13 Mev) for 2π-bosons (4 protons) and 2ν-bosons (4 neutrons). 0^+ and 2^+ states are shown.

two non-degenerate j-shells with neutrons ($j = 17/2$ and $j = 25/2$, to mimic the **82-126** major shell). So far, we have already learnt many of the necessary steps towards accomplishing this hard task. The selection of collective proton and neutron bosons will be necessary and a shell model code which includes particles in two different shells for protons and two different shells for neutrons would have to be written. This idea is already under consideration.

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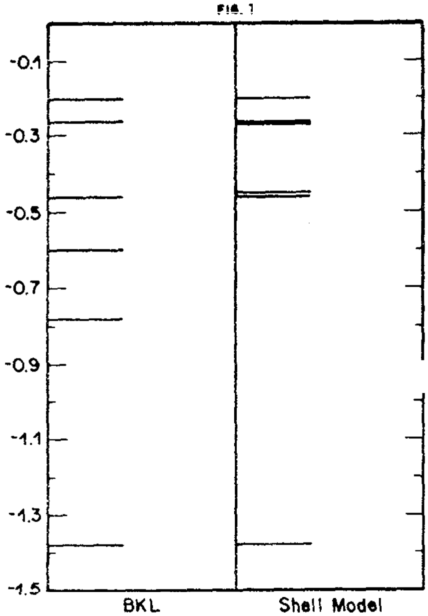


Fig. 7 – Spectra of j=7/2 ($G_\nu = -0.1$ Mev, $G_\pi = -0.13$ Mev and $R_{\pi\nu} = -0.1$ Mev) for 2π-bosons (4 protons) and 2ν-bosons (4 neutrons). Just 0^+ states are shown.

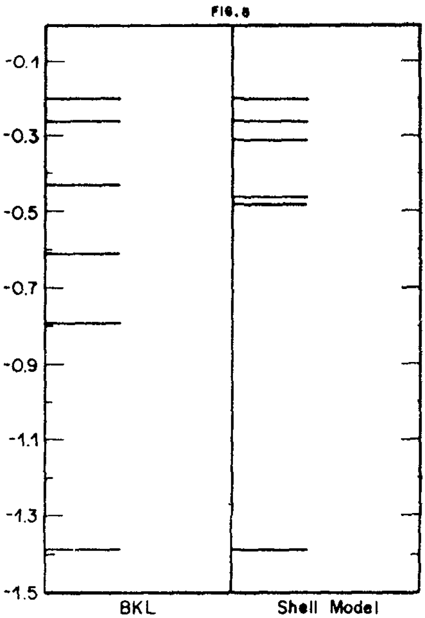


Fig. 8 – Spectra of j=7/2 (G, $= -0.1$ Mev, G, $= -0.13$ Mev and R„ $= -0.3$ Mev) for 2π-bosons (4 protons) and 2ν-bosons (4 neutrons). Just 0^+ states are shown.

FIG. 9

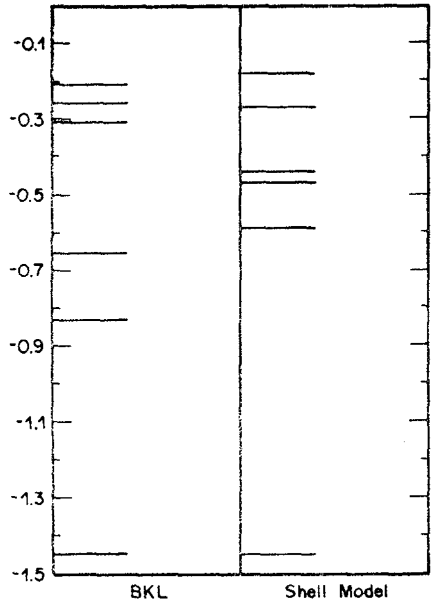


Fig. 9 – Spectra of j=7/2 ($G_\nu = -0.1$ Mev, $G_\pi = -0.13$ Mev and $R_{\pi\nu} = -0.9$ Mev) for 2π-bosons (4 protons) and 2ν-bosons (4 neutrons). Just 0^+ states are shown.

FIG. 10

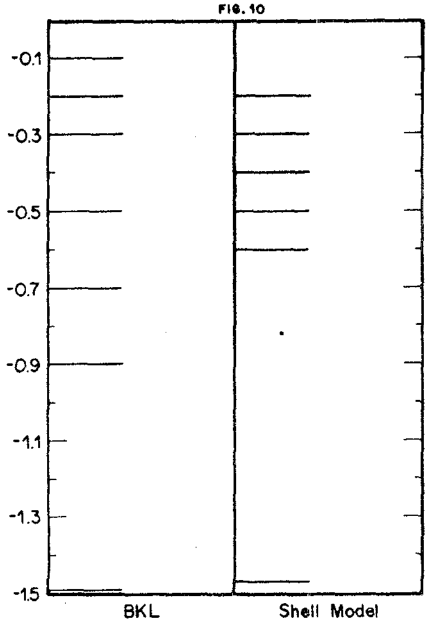
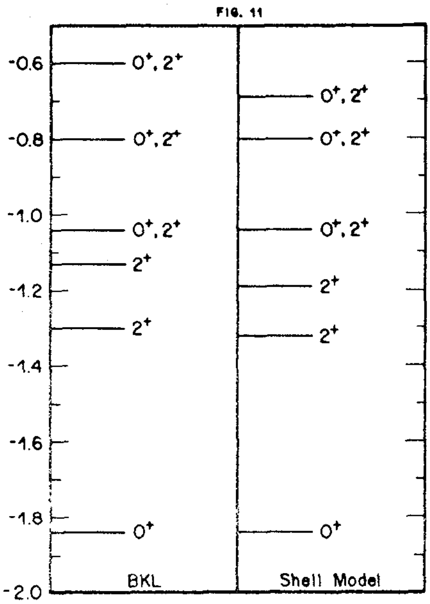


Fig. 10 – Spectra of j=7/2 ($G_\nu = -0.1$ Mev, $G_\pi = -0.13$ Mev and $R_{\pi\nu} = -1.0$ Mev) for 2π-bosons (4 protons) and 2ν-bosons (4 neutrons). Just 0^+ states are shown.



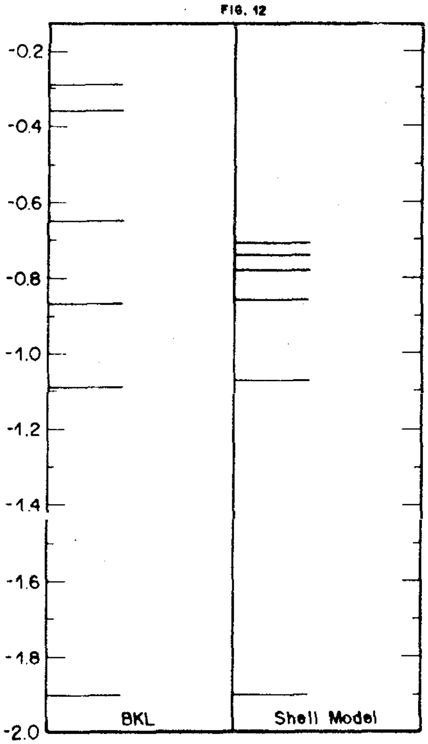


Fig. 11 – Spectra of j=9/2 (G, $= -0.1$ Mev and $G_\pi = -0.13$ Mev) for 2π-bosons (4 protons) and 2ν-bosons (4 neutrons). 0^+ and 2^+ states are shown.

Fig. 12 – Spectra of j=9/2 (G, $= -0.1$ Mev, $G_\pi = -0.13$ Mev and R, $= -1.0$ Mev) for 2π-bosons (4 protons) and 2ν-bosons (4 neutrons). Just 0^+ states are shown.

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Resumo

O mapeamento de bósons desenvolvido por Bonatsos, Klein e Li é testado em uma camada única j com prótons e nêutrons. Consideram-se sistemas com até 4 prótons e 4 nêutrons na diagonalização de uma hamiltoniana de emparelhamento mais quadrupolo. Os níveis de energia resultantes são comparados com os níveis obtidos através do modêlo de camadas. Em todos os casos, os resultados mostram uma má qualidade no limite rotacional, confirmando o que já tinha sido visto em uma camada única com um só tipo de nucleon.