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On the critical behavior of a self-dual Ising Model with multispin interactions

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Abstract We introduce and study the critical properties of a self-dual Ising model with multispin interactions. The model is described in terms of two- and four-spin interactions and has a global $Z(2)$ symmetry. The system is studied in Euclidean and Hamiltonian formalisms. The phase diagram is calculated using the self-duality of the model and finite-size scaling. The conformal anomaly and critica1 exponents are determined by exploring their relationship with the mass gap amplitudes predicted by conformal invariance.

1. Introduction

Ising models with multispin interactions, although less studied than those involving two-spin interactions, are known to exhibit a rich variety of critical behavior. The most studied examples in two dimensions are the eight-vertex model' and the Ashkin-Teller mode1[2] both of which may be formulated as Ising models with two- and four-spin interactions[3]. The multispin interactions, in these cases, produce a $Z(2) \otimes Z(2)$ nonlocal symmetry which can be spontaneously broken, producing a critical line with continuously varying critical exponents[3].

In this paper we introduce and study a self-dual Ising model with multispin interactions, having a simple $Z(2)$ nonlocal symmetry like the standard two-body Ising model. The phase diagram is calculated exactly exploring the self-duality of the model. Our numerical analysis shows us that the phase diagram, with positive couplings, has a line with the exponents of the standard Ising model. The model is defined explicitly in the next section. In section 3 we calculate the row-to-row transfer matrix and using the self-duality of the model the phase diagram is derived. In section 4 we take the **time-continuum** limit to define an associated quantum Hamiltonian and some interesting limiting cases are analysed.

In section 5 we exploit the consequences of conformal invariance of the infinite system to obtain the conforma1 anomaly and scaling dimensions (related to the critical exponents) in the critical line. The paper closes, in sections 6, with a conclusion and summary of our results.

2. The model

We denote the lattice points of a square lattice by a pair of integers (i,j). At each lattice point there is an Ising variable $S(i,j) = \pm 1$. The model we study in this paper is defined by the Hamiltonian

$$
\begin{aligned}
H = -\sum_{i\,\mathrm{even}} \{ & J_2 S(i,j)S(i+1,j)S(i,j+1)S(i+1,j+1) \\
& + J_1[S(i,j)S(i,j+1) + S(i+1,j)S(i+1,j+1)] \\
& + J_1[S(i,j)S(i+1,j) + S(i,j+1)S(i+1,j+1)] \\
& + J_2[S(i,j)S(i+2,j)]\}
\end{aligned} \tag{1}
$$

where the index i must assume only even values. This Hamiltonian describes an Ising model with multispin interactions and in fig. 1 we show these interactions schematically. There is a four-spin interaction between the spins on the corners of the elementary squares of the lattice (coupling J_2) and two-spin interactions between nearest neighbours (coupling J_1) and second nearest neighbours in the i-direction (coupling J_2). These coupling constants are chosen in order to ensure the self-duality of the model, as well shall see in the next section.

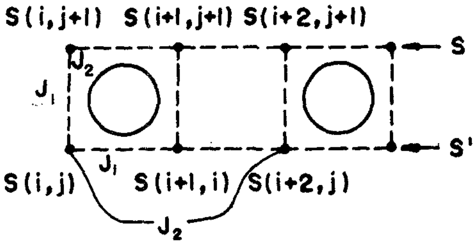

Fig. 1 - Schematic representation of the interactions in the Hamiltonian (1).

3. Transfer matrix and duality transformation

To analyee the model, at first we write the transfer matrix. The row-to-row transfer matrix T for the model (1) is easily derived by standard methods[4,5]. We write

$$T = T_1 \,.\, T_2 \tag{2}$$

where

$$T_1 = \prod_{i \text{even}} \exp\{\beta J_1(\sigma_i^x\sigma_{i+1}^x + \sigma_{i+1}^x\sigma_{i+2}^x) + \beta J_2\sigma_i^x\sigma_{1+2}^x\} \tag{3}$$

and

$$T_2 = C \prod_{i \text{even}} \exp(\beta\tilde{J}_1(\sigma_i^z + \sigma_{i+1}^z) + \beta\tilde{J}_2(\sigma_i^z\sigma_{i+1}^z)) \tag{4}$$

where σ_i^z and σ_i^x are spin-1/2 Pauli matrices and C is a harmless constant. The constant $\tilde{J}_1$ and $\tilde{J}_2$ appearing in (4) are related to the coupling constants J_1 and J_2 through

$$\tilde{x}_1 = \frac{1 - x_2}{1 + 2x_1 + x_2} \quad \text{and} \quad \tilde{x}_2 = \frac{1 - 2x_1 + x_2}{1 + 2x_1 + x_2} \tag{5}$$

where

$$x_1 = e^{-2\beta(J_2+J_1)} \quad , \quad \tilde{x}_1 = e^{-2\beta(\tilde{J}_2+\tilde{J}_1)} \ ,$$

$$x_2 = e^{-4\beta J_1} \qquad \text{and} \qquad \tilde{x}_2 = e^{-4\beta\tilde{J}_1} \tag{6}$$

In order to perform a dual transformation[6] in the Hamiltonian (1) we define the dual operators:

$$\tilde{\sigma}_i^z = \sigma_i^x\sigma_{i+1}^x \quad \text{and} \quad \tilde{\sigma}_i^x = \prod_{k=0}^{i-1} \sigma_{i-k}^z \ , \tag{7}$$

with $i = 1, 2, \ldots$, which obey the same algebra as the original Pauli matrices σ_i^z, σ_i^z. In terms of these new variables the transfer matrix is still given by (2), but now

$$T_1 = \prod_{i_{\text{even}}} \exp\{\beta J_1(\tilde{\sigma}_i^z + \tilde{\sigma}_{i+1}^z) + \beta J_2 \tilde{s}_i^z \tilde{\sigma}_{i+1}^z\} \tag{8}$$

$$T_2 = \prod_{i_{\text{even}}} C \exp\{\beta \tilde{J}_1(\tilde{\sigma}_i^z + \tilde{\sigma}_{i+1}^z + \tilde{\sigma}_i^z \sigma_{i+1}^z) + \beta \tilde{J}_2 \tilde{s}_{i-1}^z \tilde{\sigma}_{i+1}^z\} \tag{9}$$

Comparing this dual transfer matrix with the original one (2-4) we see that the duality transformation simply relate two different points in the parameter's space $(J_1, J_2) \to (\tilde{J}_1, \tilde{J}_2)$. The sef-dual line follows from the equalities

$$\tilde{J}_1 = J_1 \quad , \quad \tilde{J}_2 = J_2 \tag{10}$$

In terms of the variables x_1 and x_2 defined in (5, 6), this line is given by

$$x_2 = 1 - 2x_1 \tag{11}$$

This self-dual line (curve (b) in fig. 2),should coincide with the critical line in the regions of the parameter's space (J_1, J_2) or (x_1, x_2) where the transition is unique.

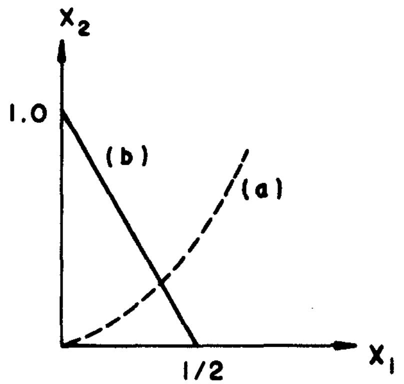

Fig. 2 - Phase diagram of the model defined by Hamiltonian (1). The parameters x_1 and x_2 are defined in (6). Curve (a) is given by $x_2 = x_1^2$ and curve (b) is the self-dual curve $x_2 = 1 - 2x_1$.

In order to calculate the phase diagram of the model (1) let us consider some limiting cases. In the case where $J_2 = 0$ we recover the standard Ising model in a square lattice. Consequently from (5) and (6) the line $x_2 = x_1^2$ in the (x_1, x_2)-space is the thermodynamic path of the Ising model (see curve (a) in fig. 2). The phase transition, which is unique in this case, is given by the crossing of (11) and the curve $x_2 = x_1^2$ (see fig. 2). In the case $x_1 \to 0$ and $x_2 \to 1$ we see, from (5) and (6), that $J_1 \to 0$ and $J_2 \to \infty$. To see the physical meaning of this limit we rewrite the Hamiltonian (1) as

$$H = -J_2 \sum_{i\,\text{even}} \{S(i,j)S(i+1,j)S(i,j+1)S(i+1,j+1) + S(i,j)S(i+2,j)\} \quad (12)$$

Consider now the following one-to-one transformation of variables $\{S\} \to \{\gamma\}$

$$\begin{aligned} \gamma(1,j) &= S(1,j) \\ \gamma(2,j) &= S(2,j)S(1,j) \\ &\vdots \\ \gamma(i,j) &= S(i,j)S(i-1,j) \quad , \quad j \in Z \end{aligned} \quad (13)$$

In term of these new variables (12) is given by

$$H'(\gamma) - J_2 \sum_{i\,\text{odd}} \{\gamma(i,j)\gamma(i,j+1) + \gamma(i,j)\gamma(i+1,j)\} \quad (14)$$

Since $\gamma(i,j) = \pm 1$ the above Hamiltonian is also a standard Ising Hamiltonian, however since the coordinate i assumes only odd values the lattice is decoupled as shown in fig. 3. Therefore, in the limiting case $x_1 = 0$ we have a unique phase transition point, like the Ising model in one dimension, occuring at $T = 0$ or $x_2 = 1$. Lastly, in the limit $x_2 \to 0$, from relation (6) we have $J_1 \to \infty$ and $J_2 \to \infty$ and we cannot analyse this limit directly. However, due to the fact that the model is invariant under the nonlocal symmetry $Z(2)$, which does not have any subgroup to be broken spontaneously we expect that the self-dual line coincides with the phase transition line along the whole parameter space (x_1, x_2). It is interesting to point out here that the equations arising from the duality transformation (5) and (6) are the same as those obtained from the $Z(4)$[7] or Ashkin-Teller[2] model in the square lattice. In this latter case, however two phase transitions occur due to a partial breaking of the Z(4) symmetry[7].

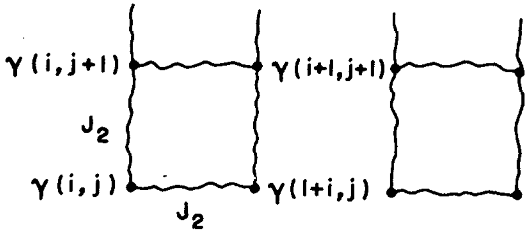

Fig. 3 - Location of interactions in the Hamiltonian (14) (wavy lines).

We want to stress that the transfer matrix has almost all elements non-zero, because it is formed by a product of operators. Consequently it is difficult to compute its spectrum, even for relatively small lattices. To circumvent this problem we will derive in the next section an equivalent quantum Hamiltonian (sum , of operators) which is expected to preserve the long-distance behavior, and whose spectrum determination is an easier task.

4. The time continuum Hamiltonian

The transfer matrix derived in the last section may be considered as a "time"-evolution operator

$$T = \exp(-\tau H) \tag{15}$$

where τ is the lattice spacing in the discrete "time"-direction and H is the associated quantum Hamiltonian. The operator H is complicated but assumes a simpler form in the so-called "time continuum" limit[5,8] ($\tau \to 0$). This limit can be performed in several ways. Here we choose the particular parametrization

$$J_1, J_2 \propto \tau \quad , \quad \tilde{J}_1 = \lambda J_1 \ \text{ and } \ \tilde{J}_2 = \lambda J_1 \tag{16}$$

which describes the model aroiind its self-dual line $(J_1 = \tilde{J}_1, J_2 = \tilde{J}_2)$. The parameter λ plays the role of temperature and the quantum Hamiltonian is given by

$$\begin{aligned} H(\lambda, J_1, J_2) = & -\sum_i^{L/2} \{J_1(\sigma^z_{2i-1}\sigma^z_{2i} + \sigma^z_{2i}\sigma^z_{2i+1} \\ & + J_2\sigma^z_{2i-1}\sigma^z_{2i+1}\} - \lambda \sum \{J_1(\sigma^z_{2i-1} + \sigma^z_{2i}) + J_2\sigma^z_{2i-1}\sigma^z_{2i}\} \end{aligned} \tag{17}$$

Hereafter we assume that the lattice size L is an even number. The parametrization (16) preserves the property of self-duality of the model since, by using the dual operators (17), we obtain

$$H(\lambda, J_1, J_z) = \lambda H(1/\lambda, J_1, J_z) \tag{18}$$

The self-dual curve is given by $\lambda = 1$ and we now consider some limiting cases. In the limit $J_1 = 0$, in the same way as in the Euclidean version of the model (see sec. 2) we obtain the quantum Hamiltonian associated to the standard Ising model[5,8]. In the limit $J_1 = 0$ the situation is not so simple and some additional calculations should be done in order to understand this limit. We write initially the quantum Hamiltonian H_I associated to the Ising model, on a chain of length

$$H_I = -\sum_{i=1}^{L} (\sigma_i^z \sigma_{i+1}^z + \lambda \sigma_i^x) \tag{19}$$

We assume in (19) the boundary conditions

$$\sigma_{L+1}^z = p\sigma_1^z \quad , \quad \sigma_{L+1}^x = p\sigma_1^x \tag{20}$$

where p $= 1$ for periodic, p $= -1$ for antiperiodic and p $= 0$ for free boundary conditions. We now define the following new variables

$$\mu_i^x = \sigma_i^x \sigma_{i+1}^x \quad , \quad \mu_i^z = \sigma_i^z \quad i = 1, 2, \ldots, L-1$$

$$\mu_L^x = p\sigma_L^x \sigma_1^x \quad , \quad \mu_L^z = \sigma_L^z \tag{21}$$

In terms of these new variables the Hamiltonian (19) takes a simple form

$$H_I(\mu) = -\sum_{i=1}^{N} (\mu_i^z + \lambda \mu_i^x) \tag{22}$$

However, the algebra of these new variables is more complicated. From their definitions (21) they satisfy:

$$[\mu_i^x, \mu_j^x] = [\mu_i^z, \mu_j^z] = 0 \tag{23}$$

for all pairs $(i,j) \in \{1,2,...,L\}$ and

$$[\mu_1^x, \mu_i^z] = 0 \tag{24}$$

unless $i = j$, $i = j + 1$, or $(i,j) = (1,L)$, in which cases

$$\{\mu_i^x, \mu_j^z\} = 0 \tag{25}$$

Nevertheless, in the case of toroidal boundary conditions ($p = \pm 1$) imposed, the variables (μ_i^z, μ_i^x, $i = 1,2,...,L$) are not independent but should satisfy the constraint

$$\prod_{i=1}^{L} \mu_i^x = \sigma_1^x\sigma_2^x\sigma_2^x\sigma_3^x \ldots \sigma_{N-1}^x\sigma_N^x p\sigma_N^x\sigma_1^x = p \tag{26}$$

In the case of free ends this constraint does not exist and the variables are independent. It is important to observe that the Ising Hamiltonian (19) commutes with the parity operator

$$\hat{p} = \prod_{i=1}^{L} \mu_i^z = \sigma_1^z\sigma_2^z \ldots \sigma_L^z \tag{27}$$

On the other hand, in the case $J_1 = 0$ the Hamiltonian (17), for $J_2 = 1$, with the boundary condition (20), is given by

$$H' = H(\lambda,0,1) = -\sum_{i}^{L/2}(\sigma_{2i-1}^x\sigma_{2i+1}^x + \lambda\sigma_{2i-1}^z\sigma_{2i}^z) \tag{28}$$

and defining the following variables

$$\varsigma_i^x = \sigma_{2i-1}^x\sigma_{2+1}^x \quad , \quad \varsigma_i^z = \sigma_{2i-1}^z\sigma_{2i}^z \quad ; \quad i = 1,\ldots,L/2-1$$

$$\varsigma_{L/2}^x = p\sigma_{L-1}^x\sigma_1^x \qquad \varsigma_{L/2}^z = p\sigma_{L-1}^z\sigma_L^z \tag{29}$$

it takes the simple form

$$H'(\varsigma) = -\sum_{i=1}^{N/2}(\varsigma_{2i+1}^x + \lambda\varsigma_{2i+1}^z) \tag{30}$$

From (29) we easily verify that the variables $\{\varsigma_i^x, \varsigma_i^z\}$ obey the same algebra as the variables $\{\mu_i^z . \mu_i^z\}$ defined in (21) and also they satisfy the same constraint

(26). Comparing *(22)* and *(30)* we thus conclude that the Hamiltonian *(28)* or *(30)* is equivalent to that of an Ising quantum chain with $L/2$ sites. This can also be seen directly in fig. 4 where we show schematically the interactions in *(28)*. It is interesting to note that the Hamiltonian *(28)* has a $Z(2)$ local gauge symmetry, because it commutes with any operator σ_i^z, with i an even number, i.e.

$$[H', \sigma_{2i}^z] = 0 \qquad i = 1, \ldots, L/2 \tag{31}$$

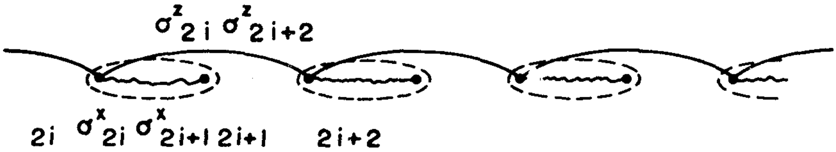

Fig. 4 - Schematic representation of the interactions on the quantum chain (28). The dashed lines encloses the elementary variables.

Consequently, in the σ^z-basis we can separate the Hilbert space associated with the Hamiltonian *(28)* into $2^{L/2}$ block disjoint sectors labeled by the eigenvalues (± 1) of the spin operators σ_{2i}^z $(i = 1, 2, \ldots, L/2)$. Each sector can still the separated into two other sectors corresponding to the eigenvalues $(p = \pm 1)$ of the parity operator

$$\hat{P} = \prod_{i=1}^{L/2} \sigma_{2i-1}^z \tag{32}$$

because $[H, \hat{P}] = 0$.

To a given choice of gauge, which corresponds to fixing the variables

$$\sigma_{2i}^z = s_i \quad i = 1, 2, \ldots, L/2, \quad s_i = \pm 1 \tag{33}$$

we can make the following canonical transformation

$$\sigma_i'^z = s_i \sigma_{2i-1}^z \quad \sigma_i'^x = \sigma_{2i-1}^x \quad ; \quad i = 1, 2, \ldots, L/2 \tag{34}$$

which gives

$$H' = -\sum_{i=2}^{L/2} (\sigma_i'^x \sigma_i'^x + \lambda \sigma_i'^z) \tag{35}$$

Consequently the $2^{L/2}$ disjoint sectors of the Hilbert space are degenerate and have the spectrum identical to that of the Ising Hamiltonian on a chain of $L/2$ sites.

5. Numerical Results

In this section we present our numerical analysis for the model (1). Our calculation will be done by using finite-size scaling and exploring the predictions of conformal invariance for the finite-size behavior of critical statistical systems. Instead of using the transfer matrix (2-4) we use the quantum Hamiltonian (17) with periodic boundaries for our spectral analysis. For a recent review of numerical methods of spectral calculations see ref. 10.

In order to present our numerical results let us state our notations for the eigenenergies. As we clearly see, Hamiltonian (17) commutes with the parity operator (27) and consequently its Hilbert space is block separated into two disjoint sectors according to the eigenvalue $P = \pm 1$ of this parity operator. We denote by $E_n^{\{P\}}(L,p)$ the n-excited state ($n = 0,1,2,...$) with momentum p (in units of $2\pi/L$) in the sector of parity $P(\pm 1)$ of the L sites chain. The ground-state is $E_0^{(1)}(L,0)$.

The phase diagram of (17) with $J_1, J_2 \leq 0$ was calculated using standard finite-size scaling. If we denote by $G_L(\lambda) = E_0^1(L,0) - E_0^{-1}(L,0)$ the mass gap of the size-L chain, the critical coupling λ_c, for a given pair of values (J_1, J_2) is found by extrapolating the sequence of values λ for which successive ratios of $G_L(\lambda)$ and $G_{L-2}(\lambda)$ exactly scale[g], i.e., the values of λ for which

$$R(\lambda) = LG_L(\lambda)/(L-2)G_{L-2}(\lambda') = 1 \tag{36}$$

Our results reveal that for $J_1, J_2 > 0$ the Hamiltonian (17) has a unique phase transition located at the self-dual point $\lambda_c = 1$.

Statistical mechanical systems at criticality are believed to be conformally invariant[12]. In two dimensions, this assumption is particularly significant (for a review see ref. 12). Specifically, Cardy[12,13] has derived a set of remarkable relations between the eigenspectrum of the transfer matrix on a strip of finite width and anomalous dimensions of the operator algebra describing the critical behavior of the infinite system. These results can be transcribed to the quantum Hamiltonian formalism and we will use them to determine the universality class of the critical behavior of Hamiltonian (17).

The pertinent results for our purposes are as follows[13]. Corresponding to each primary operator ϕ, in the operator algebra of the infinite system, there exists a set of eigenstates of the quantum Hamiltonian on a periodic chain of L sites with energies, given, at $\lambda =$ A„ by

$$E_{N,N'} = E_0 + (2\pi/L)\varsigma(x_\phi + \mathrm{N} + \mathrm{N'}) + o(L^{-1}), \quad \mathrm{N}, \mathrm{N'} = 0,1,2... \tag{37}$$

as L $\rightarrow \infty$, where x_ϕ is the anomalous dimension of ϕ. In addition to these predictions conformal invariance also predicts[14] that the ground-state energy E_0 of H, at $\lambda =$ A, and with periodic boundary conditions, should behave as

$$E_0/L = e_\infty - \pi c\varsigma/6L^2 + o(L^{-2}) \tag{38}$$

as L $\rightarrow \infty$, where e_ϕ is the infinite-lattice value, ς is model dependent and c is the central charge or conformal anomaly of the appropriate conformal class of the transition in the bulk system. Hence finite-lattice calculations can, in principle, allow c to be direclty estimated. In order to do this we have to estimate the constant ς, which can be done, from (37), by comparng energy levels belonging to the same conformal tower of a given primary operator. Assuming that like in the Ising model the two first levels in the sector P = –1 belong to the same conformal tower, the constant ς is evaluated from the sequence

$$Z_L = E_0^{(-1)}(L,1) - E_0^{(-1)}(L,0) = 2\pi\varsigma/L + o(L^{-1}) \tag{39}$$

Using the ground-state energy of two lattice sizes L and L' the relations (38) and (39) give us a finite-size sequence

$$c = \frac{12[E_o^{(1)}(L,0)/L - E_0^{(1)}(L',0)/L']}{Z_L/L - Z'_L/L'} \tag{40}$$

which in the bulk limit L, L' $\rightarrow \infty$ will give us the conformal anomaly c. In table 1 we show two of these sequences together with the extrapolated results. Our calculations indicate that the Hamiltonian *(17)* is governed by an Ising conformal field theory having $c = 1/2$ for all values of $J_1, J_2 \leq 0$. Consequently we should expect, like the standard Ising chain[12], the existence of energy and magnetic operators with dimensions $x_\epsilon = 1$ and $x_m = 1/8$, respectively. In order to calculate these conformal dimensions we used the relations *(37)*. Because of the $Z(2)$ symmetry of the model the structure of eigenenergies is the same as that of the Ising model. The eigenenergies in the sector with parity $P = +1$ are associated to the conformal towers of the identity $(x_I = 0)$ and to the energy operators, those in the sector with parity $P = -1$ to the conformal tower of the magnetic operator. The dimensions $x_i^P(p)$ associated to the energy level $E_i^P(L,P)$ are calculated from the sequence

$$(E(L,P) - E(L,0))/Z_L \rightarrow x_i^P(p) \tag{41}$$

In table *2* we show some of these sequences together with the extrapolated results. The dimensions of the energy and magnetic operators are given by $x_\epsilon = x_1^1(0)$ and $x_m = x_0^{-1}(0)$ and our extrapolated results give us the Ising values $x_\epsilon = 1$ and $x_m = 1/8 = 0.125$. We have also computed some other sequence related to secondary operators of the identity, energy and magnetic conformal blocks.

Table *1* - Finite-size sequences $C_{L,L+2}$ for the conformal anomaly, together with the extrapolated results.

$J_1 = 0 \quad J_2 = 1$		$J_1 = 1 \quad J_2 = 0.5$	
L	$c_{L,L+2}$	L	$c_{L,L+2}$
6	0.717714	6	1.380742
8	0.614784	8	0.775024
10	0.572067	10	0.639855
12	0.549803	12	0.586490
14	0.536604	14	0.559316
Extr.	0.5001	Extr.	0.5005

Table 2 - Finite-size sequences (42) corresponding to the scaling dimensions x_ϵ and x_m of the energy and magnetic operators of the Hamiltonian (17). The extrapolated results are also presented.

$J_1 = 0$ $J_2 = 1$			$J_1 = 0$ $J_2 = 1$		
L	x_ϵ	x_m	L	x_ϵ	x_m
6	1.154700	0.154700	6	1.012715	0.159405
8	1.082392	0.140652	8	0.997646	0.142708
10	1.051462	0.134729	10	0.996557	0.135893
12	1.035276	0.131652	12	0.997116	0.132407
14	1.025716	0.129842	12	0.997131	0.130374
16	1.019591	0.128685	16		
Extr.	1.000	0.125	Extr.	1.005	0.125

6. Conclusion and Summary

In this paper we have introduced and studied a particular multispin Ising model in the Euclidean (1) and Hamiltonian (17) formalisms. In both formulations the model has the useful property of self-duality. Using this property and analysing some limiting cases of the model the phase diagram is derived.

Exploring the consequence of conformal invariance we calculate, using finite-size calculations in the Hamiltonian formalism, the conformal anomaly and dimensions of operators governing the critical fluctuations. Our results show that, for ferromagnetic couplings, the model is in the Ising universality class, having a conformal anomaly $c = 1$ and energy and magnetic operators with dimensions $x_\epsilon = 1$ and $x_m = 1/8$, respectivel.

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Resumo

Introduzimos e estudamos as propriedades críticas de um modelo de Ising **auto-dual** com interações de **multispins.** O modelo é descrito em termos de interações de dois e quatro corpos possuindo uma simetria global $Z(2)$. O sistema é estudado no formalismo Euclideano e Hamiltoniano. O diagrama de fases é calculado usando a auto-dualidade do modelo e transformações de escala para sistemas finitos. A anomalia conforme e os expoentes críticos são determinados explorando-se **as** suas **relações** com as lacunas de massas, preditas pela invariância conforme.