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A note on the Landau gauge in Yang-Mills theory

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Abstract We study the Landau limit of a class of non-linear covariant gauges which contains only one gauge parameter. The stability of the model under radiative corrections is ensured by means of a Ward identity.

1. Introduction

Some years ago G. Curci and R. Ferrari[1] proposed a massive non-abelian gauge model whose renormalization properties were controlled by means of a modified BRS[1] symmetry. A more recent investigation[2] has shown that this model can be reformulated in a simpler way if one makes use of a non-linear covariant gauge-fixing which contains only one gauge parameter and whose stability properties under radiative corrections are controlled by a massive Slavnov identity[2].

The airn of this work is to investigate the classical and quantum properties of the model in the Landau limit.

The work is organized as follows: sect. 2 covers the classical aspects of the model; sect. 3 is devoted to the quantum corrections.

2. Model

The classical action with which we start is

$$S = S_{\rm YM}(A) + \frac{m^2}{2}\int d^4x A^\alpha_\mu A^{\alpha\mu} + \int d^4x (b^\alpha \partial A^\alpha + \bar{c}^\alpha \partial^\mu (D\mu c)^\alpha) \tag{2.1}$$

where

$$S_{\mathrm{YM}}(A) = -\frac{1}{4g^2}\int d^4x F^{\alpha}_{\mu\nu}F^{\alpha\mu\nu} \tag{2.2}$$

is the gauge invariant Yang-Mills action with simple compact gauge group G, b^α the Lagrangian-multiplier, $\bar{c}^\alpha$ and c^α the Faddeev-Popov ghosts and

$$(D_\mu c)^\alpha = \partial_\mu c^\alpha + f^{abc}A^b_\mu c^c \tag{2.3}$$

the covariant derivative.

The classical action (2.1) is that which one obtains by setting to zero the gauge parameter of the model studied in reference 2, and it represents a massive Yang-Mills model quantized in a Landau gauge.

As can be easily verified, the action **(2.1)** is invariant under the following transformations

$$\begin{cases} sA^\alpha_\mu = -(D_\mu c)^\alpha \\ sc^\alpha = \frac{f^{abc}}{c^b c^c} \\ s\bar{c}^\alpha = b^\alpha \\ sb^\alpha = -m^2 c^\alpha \end{cases} \tag{2.4}$$

Notice that, due to the presence of a mass term, the transformations (2.4) are not nilpotent.

To write down the Ward identity corresponding to (2.4), we couple the non-linear variations to external sources[3]

$$S_1 = \int d^4x\Big(-\Omega^{\alpha\mu}(D_\mu c)^\alpha + L^\alpha \frac{F^{abc}}{2}c^b c^c\Big) \tag{2.5}$$

then the complete action

$$\Sigma = S + S_1 \tag{2.6}$$

satisfies the classical Ward identity

$$\varphi(\Sigma) = 0 \tag{2.7}$$

where

$$\varphi(\Sigma) = \int d^4x\Big(\frac{\delta\Sigma}{\delta\Omega^{\alpha\mu}}\frac{\delta\Sigma}{\delta A^\alpha_\mu} + \frac{\delta\Sigma}{\delta L^\alpha}\frac{\delta\Sigma}{\delta c^\alpha} + b^\alpha\frac{\delta\Sigma}{\delta\bar{c}^\alpha} - m^2 c^\alpha\frac{\delta\Sigma}{\delta b^\alpha} \tag{2.8}$$

and the equation of motion of the Lagrangian-multiplier[3]

$$\frac{\delta\Sigma}{\delta b^\alpha} = \partial^\mu A^\alpha_\mu \tag{2.9}$$

Commuting eq. (2.9) with eq. (2.7) one gets one more constraint

$$\partial^\mu \frac{\delta\Sigma}{\delta\Omega^{\alpha\mu}} + \frac{\delta\Sigma}{\delta\bar{c}^\alpha} = 0 \tag{2.10}$$

Eq. (2.10) is the equation of motion of the antighost $\bar{c}^\alpha$. The mass dimensions of the fields, A^α_μ, b^a, c^α, $\bar{c}^\alpha$, L^α, Ω^α_μ are respectively 1, 2, 0, 2, 4, 3 and the assigned ghost charges 0, 0, 1, -1, -2, -1. Equations (2.7), (2.9) and (2.10) are all the classical properties by means of we study the stability of the model under radiative corrections.

3. Stability

To study stability of the model under radiative corrections we restrict ourselves to local perturbations $\Gamma^c\,(A, c, \bar{c}, b, L, R, m^2)$ which are polynomials integrated of four-dimensional and with zero Faddeev-Popov charge and we impose to the first order in ϵ that, the perturbed action

$$(\Sigma + \epsilon\Gamma^c) \tag{3.1}$$

satisfies the Ward identity (2.7)

$$\varphi(\Sigma = \epsilon\Gamma^c) = 0 + 0(\epsilon^2) \tag{3.2}$$

i.e.:

$$\int d^4x \Big(\frac{\delta\Sigma}{\delta\Omega^{\alpha\mu}} \frac{\delta\Sigma}{\delta A^\alpha_\mu} + \frac{\delta\Sigma^c}{\delta A^\alpha_\mu} \frac{\delta\Sigma^c}{\delta\Omega^{\alpha\mu}} + \frac{\delta\Sigma}{\delta c^\alpha} \frac{\delta\Sigma^c}{\delta L^\alpha} + \frac{\delta\Sigma}{\delta L^\alpha} \frac{\delta\Sigma^c}{\delta c^\alpha} + b^\alpha \frac{\delta\Sigma^c}{\delta\bar{c}^\alpha} - m^2 c^\alpha \frac{\delta\Sigma^c}{\delta b^\alpha} \Big) = 0 \tag{3.3}$$

From the equations (2.9), (2.10) one gets

$$\frac{\delta\Sigma^c}{\delta b^\alpha} = 0 \tag{3.4}$$

and

$$\left(\partial^\mu \frac{\delta\Sigma^c}{\delta\Omega^{\alpha\mu}} + \frac{\delta\Sigma^c}{\delta\bar{c}^\alpha}\right) = 0 \tag{3.5}$$

Equations (3.4), (3.5) imply that Γ^c is b-independent and that $\bar{c}^\alpha$ and Ω^α_μ enter only through the combination

$$\gamma^\alpha_\mu = \Omega^\alpha_\mu + \partial_\mu \bar{c}^\alpha \tag{3.6}$$

i.e.:

$$\Gamma^c = \Gamma^c(A, c, L, \gamma, m^2) \tag{3.7}$$

Introducing the action $\tilde{\Sigma} = \tilde{\Sigma}(A, c, L, \gamma, m^2)$ defined by

$$\Sigma = \tilde{\Sigma} + \int d^4x b^\alpha \partial a^\alpha \tag{3.8}$$

$$\begin{aligned}\tilde{\Sigma} = S_{\mathrm{YM}}(A) &+ \int d^4x \frac{m^2}{2} A^\alpha_\mu A^{\alpha\mu} \\ &+ \int d^4x \Big(-\gamma^{\alpha\mu}(D_\mu c)^\alpha + L^\alpha \frac{f^{abc}}{2} c^b c^c \Big)\end{aligned} \tag{3.9}$$

The equation (3.3) becomes

$$B\hat{\Sigma}\Gamma^c = 0 \tag{3.10}$$

where

$$\begin{aligned}B\hat{\Sigma} = \int d^4x \Big(&\frac{\delta\hat{\Sigma}}{\delta\gamma^\alpha_\mu}\frac{\delta}{\delta A^{\alpha\mu}} + \frac{\delta\hat{\Sigma}}{\delta A^\alpha_\mu}\frac{\delta}{\delta\gamma^{\alpha\mu}} \\ &+ \frac{\delta\hat{\Sigma}}{\delta c^\alpha}\frac{\delta}{\delta L^\alpha} + \frac{\delta\hat{\Sigma}}{\delta L^\alpha}\frac{\delta}{\delta c^\alpha}\Big)\end{aligned} \tag{3.11}$$

The most general form for Γ^α compatible with the above constraints and the assigned quantum numbers reads:

$$\begin{aligned}\Gamma^\alpha = \hat{\Gamma}^c(A) &+ \int d^4x \beta \frac{m^2}{2} A^\alpha_\mu A^{\alpha\mu} \\ &+ \int d^4x (\sigma\gamma^{\alpha\mu}\partial_\mu c^\alpha + \omega^{abc}\gamma^{\alpha\mu}A^b_\mu c^\alpha) \\ &+ \int d^4x l^{abc} L^\alpha \frac{c^b c^c}{2}\end{aligned} \tag{3.12}$$

where β and a are arbitrary parameters, ω^{abc} and l^{abc} arbitrary tensor of the gauge group G and $\hat{\Gamma}^c(A)$ is a local functional which depends only on **A.** The condition (3.10) gives

$$\begin{cases} \beta = \sigma \\ \omega^{abc} = -l^{abc} = -\xi f^{abc} \end{cases} \tag{3.13}$$

and

$$\hat{\Gamma}^c(A) = -\frac{\rho}{4g^2}\int d^4x F^{\alpha}_{\mu\nu}F^{\alpha\mu\nu} - \left(\frac{\xi-\sigma}{2g^2}\right)\int d^4x F^{\alpha}_{\mu\nu}(F^{\alpha\mu\nu} + f^{abc}A^{b\mu}A^{c\nu} \tag{3.14}$$

where ρ, ξ are arbitrary parameters. Equations (3.13) and (3.14) show that the perturbation Γ^c contains three arbitrary parameters ξ, σ, p which can be reabsorbed by redefining the fields, the coupling constant g and the mass parameter m^2 of the classical action C :

$$\Sigma(A, b, c, ?, L, R, m^2, g) + \epsilon\Gamma^c = \Sigma(A^0, b^0, c^0, \bar{c}^0, L^0, \Omega^0, m_0^2, g_0) + O(\epsilon^2) \tag{3.15}$$

with

$$A^0 = Z_A A$$

$$b^0 = Z_A^{-1} b$$

$$\Omega^0 = Z_A^{-1}\Omega$$

$$m_0^2 = Z_A^{-1} Z_c^{-1} m^2$$

$$g_0^2 = Z_g g^2$$

$$c^0 = Z_c c$$

$$\bar{c}^0 = Z_A^{-1}\bar{c}$$

$$L^0 = Z_c^{-1} L$$

where

$$\begin{cases} Z_g = 1 - \epsilon p \\ Z_A = 1 + \epsilon(\sigma + \xi) \\ Z_c = 1 + \epsilon\xi \end{cases} \tag{3.17}$$

Thus the model described by the action (2.1) will, in the absence of gauge anomalies, be multiplicative by renormaliiable.

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References

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Resumo

Estuda-se o limite de Landau de uma classe de calibres não-covariantes os quais contém apenas um parâmetro de gauge. A estabilidade do modelo a respeito de correções radiativas é assegurada através da Identidade de Ward.