Revista Brasileira de Física, Vol. 20, no 4, **1990**

Alternative off-shell k-symmetry for supermembranes

R. Amorim and J. Barcelos-Neto*
Instituto de Física, Universidade Federal do Rio de Janeiro, Caixa Postal 68528, Rio de Janeiro, 219.45, RJ, Brasil

Received August 8, 1989

Abstract The verification of the on-shell kappa-syrnmetry for supermembranes is usually done by means of a projector operator. In the off-shell case (Polyakov-like action) this verification is more subtle and that one found in literature does not make use of such operators. In this work we show that the off-shell kappa-symmetry can also be achieved by means of a projector operator.

The action for supermembranes, in its Polyakov form[1], is given by2

$$S = S_1 + S_{WZ} \tag{1}$$

where

$$S_1 = -\frac{T}{2}\int d^3\xi\,\sqrt{-g}\,\left(g^{ab}Z_a\cdot Z_b - 1\right) \tag{2}$$

and

$$S_{WZ} = -\frac{iT}{2}\int d^3\xi\,\epsilon^{abc}\,\bar\theta\Gamma_{\mu\nu}\partial_a\theta\,\left(Z^\mu_b Z^\nu_c + iZ^\mu_b\bar\theta\gamma^\nu\partial_c\theta - \frac{1}{3}\bar\theta\gamma^\mu\partial_b\theta\bar\theta\gamma^\nu\partial_c\theta\right) \tag{3}$$

is a Wess-Zumino term.

In expressions **(1)** and (2), the superspace tangent is defined as

$$Z^\mu_a = \partial_a X^\mu - i\bar\theta\gamma^\mu\partial_a\theta \tag{4}$$

where indices $a,b,$ **...** vãry from 0 to 2. γ^μ and ú are respectively D = **11** Dirac matrices and Majorana spinors, since only in this spacetime dimension supermenibranes can be consistently formuiated. The convention and notation we are

* **Bitnet address:** ift03001 **at ufrj.**

following, as well as some identities which are used throughout, are listed in the appendix.

In eq.(2), g_{ab} can be considered as an auxiliary field, since it has no dynamics. Actually, an independent variation of the action with respect to g_{ab} gives the embedding relation

$$g_{ab} = Z_a \cdot Z_b \tag{5}$$

If (5) is substituted in (2), one gets the on-shell form of the supermembrane action (Nambu-Goto-Dirac)[3]

$$S_1' = -\frac{T}{2}\int d^3\xi \left[-\det\left(Z_a \cdot Z_b\right)\right]^{1/2} \tag{6}$$

Of course, this is a classical equivalence.

The fermionic or κ-symmetry[4] is related to the following transformations

$$\delta\theta = (1+\Gamma)\kappa \tag{7}$$

$$\delta X_\mu = i\bar{\theta}\gamma_\mu\delta\theta \tag{8}$$

where $\kappa = \kappa(\xi)$ is an arbitrary Majorana spinor and $(1 \pm \Gamma)$ are projection operators, since

$$\Gamma = \frac{1}{3!\sqrt{-g}}\,\epsilon^{abc}\not{Z}_a\not{Z}_b\not{Z}_c \tag{9}$$

satisfies

$$\Gamma^2 = 1 \tag{10}$$

if (5) holds. This is the on-shell κ-symmetry, and the starting action is $S_1' + S_{WZ}$[5].

In the off-shell case, eq. (10) does not hold with Γ given by (9). Even in this case, the κ-symmetry still holds[6], since the metric tensor transfornis as[7]

$$\begin{aligned}\delta(\sqrt{-g}\,g^{ab}) &= 2i\,g^{c(a}\epsilon^{b)de}\,\bar{\kappa}(1+\Gamma)\not{Z}_d\not{Z}_e\,\partial_c\theta \\ &+ \frac{2i}{3\sqrt{-g}}\,\epsilon^{cd(a}\epsilon^{b)ef}\bar{\kappa}\not{Z}^l\,\partial_l\theta(Z_c\cdot Z_e Z_d\cdot Z_f + g_{df}Z_c\cdot Z_e + g_{ce}g_{df})\end{aligned} \tag{11}$$

Our purpose in this letter is to show the off-shell κ-symmetry in an alternative way. First we construct another gamma matrix, denoted by Γ', in such a way that $(\Gamma')^2 = 1$, independently of (5). One can see that

$$\Gamma' = \frac{1}{3!\sqrt{-h}}\,\epsilon^{abc}\not{Z}_a\not{Z}_b\not{Z}_c \tag{12}$$

where

$$h = \det Z_a \,.\, Z_b \tag{13}$$

satisfies this requirement. We thus consider that θ has, instead of (7), the following transformation

$$\delta\theta = (1 + \Gamma')\kappa \tag{14}$$

So, the κ-symrnetry will be verified if we can show that the variation of action (1) is proportional to $(1 - \Gamma')\delta\theta$.

From (4) and (8), with an arbitrary variation 68, we see that

$$\delta Z_a^\mu = 2i\partial_a\bar{\theta}\gamma^\mu\delta\theta \tag{15}$$

With this result and using the identity

$$\epsilon^{abc}\left[\Gamma_{\mu\nu}\psi_a(\bar{\psi}_b\gamma^\mu\psi_c) + \gamma^\mu\psi_a(\bar{\psi}_b\Gamma_{\mu\nu}\psi_c)\right] = 0 \tag{16}$$

which holds only for D = 11, in the case of Majorana spinors, we obtain, after a long algebraic calculation

$$\delta L_{WZ} = iT\,\epsilon^{abc}\partial_a\bar{\theta}\not{Z}_b\not{Z}_c\,\delta\theta \tag{17}$$

modulo exact differentials. We assume that frontier terms can be eliminated with a proper choice of κ [6].

Considering (15) and (17) we obtain that the total variation of the Lagrangian corresponding to action S is

$$\delta L = \frac{T}{2}\,(g_{ab} - Z_a\cdot Z_b)\,\delta(\sqrt{-g}\,g^{ab}) - 2iT\sqrt{-g}\,g^{ab}\partial_a\bar{\theta}\not{Z}_b\,\delta\theta + iT\,\epsilon^{abc}\partial_a\bar{\theta}\not{Z}_b\not{Z}_c\,68 \tag{18}$$

Incidentally, we notice that on-shell, i.e., when (5) is used, we simply have

$$\begin{aligned}\delta L &= -\,2iT\sqrt{-g}\,g^{ab}\partial_a\bar{\theta}\not{Z}_b\,\delta\theta + iT\,\epsilon^{abc}\partial_a\bar{\theta}\not{Z}_b\not{Z}_c\,\delta\theta\\ &= -\,2iT\sqrt{-g}\;g^{ab}\partial_a\bar{\theta}\not{Z}_b\,(1-\Gamma)\delta\theta\end{aligned} \tag{19}$$

which vanishes for 68 given by (7).

To obtain the off-shell case, we consider the following general transformation

$$\delta(\sqrt{-g}g^{ab}) = A\,g^{c(a}\epsilon^{b)de}\partial_c\bar{\theta}\not{Z}_d\not{Z}_e\,\delta\theta + B\epsilon^{cd(a}\epsilon^{b)ef}\;\partial_c\bar{\theta}\not{Z}_d\not{Z}_e\not{Z}_f\,\delta\theta + X^{ab} \tag{20}$$

where A, B and X^{ab} are respectively two scalars and a symmetric parameter space tensor which have to be calculated.

By using gamma-matrix algebra and some identities listed in the appendix, we arrive at

$$\begin{aligned}
\delta L = & -2iT\sqrt{-g}\, g^{ab}\partial_a\bar\theta \not{Z}_b\,\delta\theta - 3AT\sqrt{-h}\, g^{ab}\partial_a\bar\theta\not{Z}_b\,\Gamma'\delta\theta \\
& + iT\,\epsilon^{abc}\partial_a\bar\theta\not{Z}_b\not{Z}_c\,\delta\theta - 3BT\sqrt{-h}\,\epsilon^{abc}\partial_a\bar\theta\not{Z}_b\not{Z}_c\,\Gamma'\delta\theta + AT\epsilon^{abc}\partial_a\bar\theta\not{Z}_b\not{Z}_c\,\delta\theta \\
& - \frac{1}{2}AT\,(2g^{be}\epsilon^{acd} + g^{ab}\epsilon^{edc})\partial_c\bar\theta\not{Z}_d\not{Z}_e\, Z_a\,.\, Z_b\,\delta\theta \\
& - BTg\, g^{ce}(g^{af}g^{bd} - g^{ab}g^{df})\partial_c\bar\theta\not{Z}_d\not{Z}_e\not{Z}_f\, Z_a\cdot Z_b\, 68 \\
& + 2BTg\,(g^{ac}g^{bd} - g^{ab}g^{cd})\partial_c\bar\theta\not{Z}_d\, Z_a\cdot Z_b\,\delta\theta \\
& + \frac{1}{2}T\,(g_{ab} - Z_a\cdot Z_b)X^{ab}
\end{aligned} \tag{21}$$

If we choose

$$\begin{aligned}
X^{ab} = & -A(g^{e(a}\epsilon^{b)cd} + 2g^{ab}\epsilon^{edc})\partial_c\bar\theta\not{Z}_d\not{Z}_e\,\delta\theta \\
& - Bg\, g^{ce}(g^{f(a}g^{b)d} - 2g^{ab}g^{df})\partial_c\bar\theta\not{Z}_d\not{Z}_e\not{Z}_f\,\delta\theta \\
& + 2Bg\,(3g^{c(a}g^{b)d} - 4g^{ab}g^{cd})\partial_c\bar\theta\not{Z}_d\,\delta\theta
\end{aligned} \tag{22}$$

several terms cancel and we yet

$$\begin{aligned}
\delta L = & -2T(i\sqrt{-g} + 3Bg)\, g^{ab}\partial_a\bar\theta\not{Z}_b\,\delta\theta - 3AT\sqrt{-h}\, g^{ab}\partial_a\bar\theta\not{Z}_b\,\Gamma'\delta\theta \\
& + T(i + \frac{3}{2}A)\epsilon^{abc}\partial_a\bar\theta\not{Z}_b\not{Z}_c\,\delta\theta - 3BT\sqrt{-h}\,\epsilon^{abc}\partial_a\bar\theta\not{Z}_b\not{Z}_c\,\Gamma'\delta\theta
\end{aligned} \tag{23}$$

which will be proportional to $(1 - \Gamma')\delta\theta$ if

$$\begin{aligned}
3A\sqrt{-h} &= -2i\sqrt{-g} - 6Bg \\
3B\sqrt{-h} &= i + \frac{3}{2}A
\end{aligned} \tag{24}$$

whose solution is

$$\begin{aligned}
A &= -\frac{2i}{3}\frac{\sqrt{-g}}{\sqrt{-g} + \sqrt{-h}} \\
B &= \frac{i}{3}\frac{1}{\sqrt{-g} + \sqrt{-h}}
\end{aligned} \tag{25}$$

Combining (23) and (25), we finally obtain

$$\delta L = iT\frac{}{\sqrt{-g}+\sqrt{-h}}(\epsilon^{abc}\partial_a\bar{\theta}\not{Z}_b\not{Z}_c - 2\sqrt{-g}\,g^{ab}\partial_a\bar{\theta}\not{Z}_b)(1-\Gamma')\delta\theta \qquad (26)$$

which has the desired form and vanishes given by (14).

If we collect all the terms, we obtain that the variation of the metric tensor is given by

$$\begin{aligned}\delta(\sqrt{-g}g^{ab}) = &-\frac{2i}{3}\frac{\sqrt{-g}}{\sqrt{-g}+\sqrt{-h}}\Big\{\sqrt{-g}(3g^{c(a}g^{b)d} - 4g^{ab}g^{cd})\partial_c\bar{\theta}\not{Z}_d \\ &+ (g^{c(a}\epsilon^{b)de} + g^{e(a}\epsilon^{b)dc} + g^{ab}\epsilon^{cde})\partial_c\bar{\theta}\not{Z}_d\not{Z}_e \\ &+ [\sqrt{-g}(g^{ab}g^{df} - \frac{1}{2}g^{f(a}g^{b)d})g^{ce} - \frac{1}{2\sqrt{-g}}\epsilon^{cd(a}\epsilon^{b)ef}]\partial_c\bar{\theta}\not{Z}_d\not{Z}_e\not{Z}_f\Big\}\delta\theta\end{aligned} \qquad (27)$$

In conclusion, we have discussed an alternative way of obtaining the off-shell κ-symmetry for supermembranes by means of a projector operator. We observe that the transformation for the metric field given by (27) has a different expression of the corresponding one presented in ref. 7. This is an acceptable result since the initial transformations for the theta field are not the same in both cases.

Appendix

We present here the convention and some identities which we have been used through this paper.

$\eta_{\mu\nu} = \text{diag.}(-1, 1, \cdots 1)$

Signature of $g_{ab} = (-, +, +)$

$\epsilon_{012} = 1$

$\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}, \qquad [\gamma_\mu, \gamma_\nu] = \frac{1}{2}\Gamma_{\mu\nu}$

$g^{ab}\epsilon^{cde} = \frac{1}{3}g^{a[c}\epsilon^{de]b}$

$\epsilon^{abc}\epsilon^{def} = g\,(g^{a[d}g^{|b|e}g^{f]c} - g^{a[d}g^{|b|f}g^{e]c})$

Acknowledgment

This work is supported in part by Conselho Nacional de Desenvolvimento Científico e Tecnológico - CNPq (Brazilian Research Agency).

References

1. A.M. Polyakov, Phys. Lett. **103B**, 207 (1981); P.S. Tucker, J. Phys. **A10**, L155 (1977).
2. A. Sugamoto, Nucl. Phys. **B215**, 381 (1983).
3. Y. Nambu, *Lectures* at the Copenhagen *Summer* Symposium, 1970; T. Goto, Prog. Theor. Phys. 46 1560 (1971); P.A.M. Dirac, Proc. R. Soc. London **268A**, 57 (1962); 270A 354 (1962).
4. W. Siegel, Phys. Lett. 128B (1983) 397; Class. Quantum Grav. 2, L95 (1985).
5. For a general review, see M. Ruiz-Altaba, Supermembranes, Preprint CERN-TH 5048/88 and references therein.
6. J. Hughes, J. Liu and J. Polchinski, Phys. Lett. 180B 370 (1986); E. Bershoeff, E. Sezgin and P.K. Townsend, Phys. Lett. **189B**, 75 (1987); A. Achúcarro, J.M. Evans, P.K. Townsend and D.L. Wiltshire, Phys. Lett. **198B**, 441 (1987).
7. E. Bergshoeff, E. Sezgin and P.K. Townsend, Ann. Phys. (NY) 185, 330 (1988).

Resumo

A verificação da simetria kapa "on-shell" para supermembrana é geralmente feita por meio de um operador de projeção. No caso "off-shell" (ação tipo Polyakov), esta verificação é mais delicada e a que é encontrada na literatura não faz uso de tais operadores. Neste trabalho, mostramos que a simetria kapa "off-shell" pode também ser obtida por meio de um operador de projeção.