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Radio frequency effect on the sheath capacitance in a low density plasma

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Abstract In this work we describe the use of the nonlinear properties of tlie sheath capacitance in a low density plasma to produce parametric amplification of RF signals in a high frequency band (H.F.). The experiment has been carried out in the Linear Mirror Device LISA of the Universidade Federal Fluminense, where a helium plasma was produced using a radio-frequency source built at UFF, with variable (10 watts to 100 watts) and frequency of 28 MHz. The experimental results shows good agreement between the theoretical model of sheat capacitance. This allows one to predict, within a limited range, the sheath capacitance variation as a function of certain plasma parameters.

1. Introduction

In plasma physics research, mainly in plasma diagnostics with resonant probes, attention is focused on the mechanisms that are important for the alternate current (AC) signals in the plasma sheath.

Aihara, Lampis and Takayama[1] and later Rapozo et al.[2] measured the dependente of the sheath resonance with the applied RF voltage. The analysis by Rosa[3] of the sheath resonance as a function of the transit time of the ions descrihed the capacitive characteristic of the sheath, with the possibility of negative admittance in the sheath resonance region.

In this work we consider the results obtained from the electrical model set up for the plasma sheath region[4], where the nonlinear characteristic of the sheath capacitance was used for the parametric amplification of radio-frequency signals.

The experimental data from which we derived the plasma admittance profile sheath resonance frequency profile versus the applied RF voltage, were obtained in a helium plasma produced by the RF source in the Linear Mirror Device LISA of the Universidade Federal Fluminense (Niterói, RJ) shown in Figure 1. The main parameters of the LISA device can be found in the work of Rapozo et al.[5]. The helium plasma is weakly ionized ($< 1\%$) and the pressure is 1.8×10^{-4} Torr. A device to detect a possible parametric amplification of the sheath was constructed as shown in Figure 2. In order to do the diagnostics, have used a circular disk Langmuir probe, oscilloscope and a linear wattmeter.

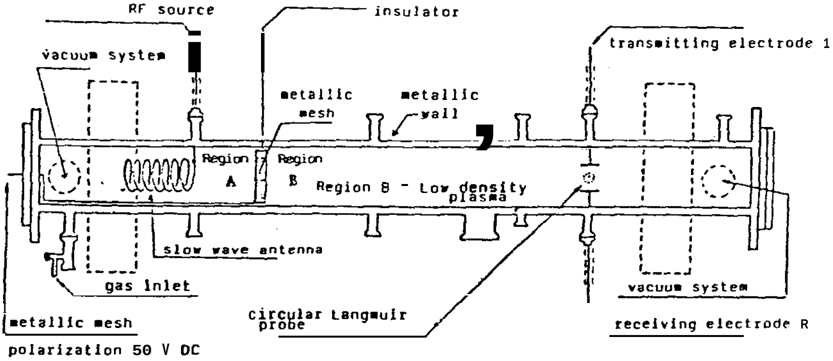

Fig. 1 - The linear device LISA.

This work is organized as follows: In Section 2 the experimental device is described. In Section 3 we present electrical model for the system of plasma sheath plus electrodes T and R, and its theoretical analysis. In Section 4 we present the experimental results and analysis. The conclusions are discussed in Section 5.

2. The experiment

The experiment was carried out on the linear mirror machine LISA' designed and constructed at the Max-Planck Institut für Plasmaphysik (Garching, West Germany)..

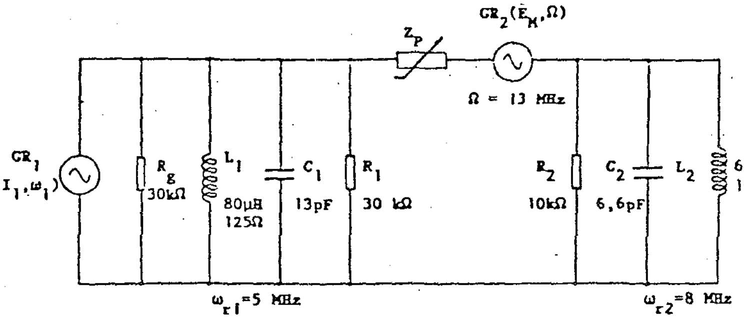

Fig. 2 - Electrical device of a parametric amplifier at two frequencies.

The helium plasma is produced by a plasma RF source in region A in a stainless steel cylindrical vessel (diameter 17 cm, length 255 cm) (Figure 1). The RF source whose power can be varied from 10 watts to 100 watts has been connected to a slow wave antenna (diameter 9.7 cm, length 22 cm and pitch 1.5 mm); the forward power from the source has been measured by a linear wattmeter. The cylindrical vessel was separated into two regions, A and B, by a metallic mesh insulated from the inner wall of the cylindrical vessel (Figure 1). We use this metallic mesh for two main reasons: first, it is used to shield the region B from the RF fields, so there is no RF field in region B. Secondly, we use the metallic mesh to act as an anode for electrons that are created in region A. The electrons are accelerated by the metallich mesh, from the high electron density (region A) to the low density plasma (region B). The plasma density and the electron temperature are monitored by the radial Langmuir probe between the electrodes T and R.

An oscillator whose frequency can be varied from 190 kHz to 80 MHz and whose output voltage can be varied between 0 and 7 $V_{(RMS)}$ injects RF current into the low density plasma (region B) through the movable plane radial electrode T, which has a diameter of 3.0 cm (Figure 1). The RF current transmitted across the plasma is received by the second radial electrode R and is fed to a 50 Ω load. The output amplified voltage is measured by an oscilloscope.

In order to obtain the admittance profile we have used the applied RF voltage fixed at 3.0 $V_{(\mathrm{RMS})}$. For a fixed electrode diameter of 3.0 cm, the sheath resonance frequency profile was measured versus applied RF voltage $V_{(\mathrm{RMS})}$ and the admittance profile versus frequency. In this experiment the distance between the circular radial electrodes T and R is kept fixed at 10 cm.

Based on the results obtained by Rapozo et al.[2] for the sheath thickness profile versus the applied RF voltage V_{RMS} and the work of Rosa[3] the device shown in Figure 2 was built to demonstrate possible parametric amplification due to the capacitive characteristic of the sheath. It is a two frequency parametric amplifier, where GR_1 is the RF generator and R_g and R_1 are the internal resistance and load resistance respectively. L_1, C_1 and L_2, C_2 are tunning tanks in ω_{r_1} and ω_{r_2}, where L_1 and L_2 are the inductances and C_1 and C_2 the capacitances of the circuits respectively. R_2 is the load resistance for the RF generator GR_2 and Z_p is a parametric impedance produced by plasma in which we use the sheath capacitance as the parametric element.

3. Plasma sheath electrical model and theoretical analysis

One of the aims of this Section is to study the resonance and anti-resonance of the admittance profile described by Rapozo et al.[2]. The electrical model proposed for the system of transmitting (T) and receiving (R) electrodes and plasma sheath is shown in Figure 3, where C, represents the capacitance between the electrodes T and R and C, is the sheath capacitance[4,8] which is defined by $C_s = \epsilon A/S$ (A is the electrode surface, ϵ is the dielectric constant and S is the sheath thickness); finally L_p and R_p are the plasma inductance and resistance, respectively.

Carneiro and Rapozo[4] have shown that the resonances of the circuit, for $R_p = 0$, are

$$\begin{aligned} \omega_a^2 &= \frac{1}{LC_s} \text{for maximum admittance (sheath resonance)} \\ \omega_b^2 &= \frac{1}{LC_t} \text{for minimum admittance (plasma resonance)} \end{aligned} \tag{1}$$

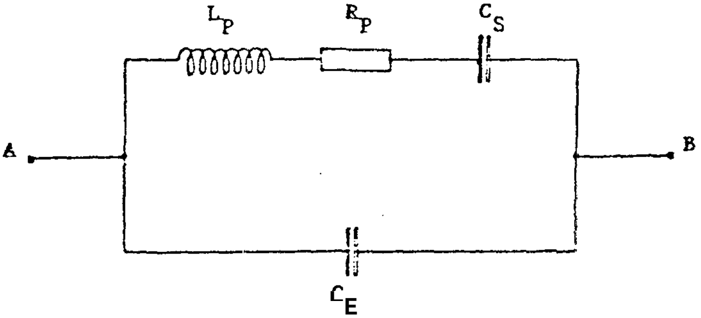

Fig. 3 – Electrical model for the concentrate elements between the electrodes T and R.

where

$$C_t = \frac{C_s C_e}{C_s + C_e}$$

For $R_p \neq 0$, we have

$$\omega^2_{\underset{+}{1},\underset{-}{2}} = -\left[R_p^2 - \frac{L_p}{C_k}\right] \pm \sqrt{\left. k_p^2 - \frac{L_p}{C_k}\right)^2 - \frac{4L_p^2}{C_s C_e}}\ , \tag{2}$$

where the indices $\begin{pmatrix} 1 \\ + \end{pmatrix}$ denote the positive signal for ω_1 and the indices $\begin{pmatrix} 2 \\ - \end{pmatrix}$ denote the negative signal for ω_2, in front of the square root in Eq. 2.

The nonlineat characteristic of the sheath capacitance is used as a parametric factor of the impedance Z_p (Figure 2). This capacitance is modulated according to the sheath thickness variation as a function of the applied RF voltage. Considering the results obtained by Rapozo et al.[2] and the expression for the sheath thickness given by Bohm[6]

$$S = \frac{1}{3\sqrt{\pi}} \left(\frac{1}{en_0}\right)^{1/2} \frac{V_{FO}^{3/4}}{V_a^{1/4}} \tag{3}$$

where e is the electron charge, n_0 is the electron plasma density, V_{FO} is the floating potential and V, is the acceleration potential of the ion, we can see that the applied RF voltage produces a variation in V_{FO}; so, in a first approximation, it is possible to estimate the dependence on the sheath capacitance modulation which is necessary for parametric amplification.

Assuming that V_{FO} is going to vary according to the applied modulate signal ($V_{\mathrm{RF\ mod}} \cos \omega_{\mathrm{RF}} t$)', We can write $V_{FO} = V_{f_0} + V_{\mathrm{RF}}$ mod $\cos \omega_{\mathrm{RF}} t$; consequentiy, eq. (3) will be rewritten as follows

$$S = \frac{1}{3\sqrt{\pi}}\left[\frac{1}{en_0}\right]^{\frac{1}{2}}\frac{1}{V_a^{1/4}}(V_{fo} + V_{\mathrm{RFmod}}\cos\omega_{\mathrm{RF}}t)^{3/4} \qquad (4)$$

where V_{fo} is the floating potential and V_{RFmod} is the radio frequency voltage modulated. Then the sheath capacitance expression can be rewritten as follows

$$C_s = \frac{\epsilon A\beta^{-1}V_{fo}^{3/4}}{V_a^{1/4}}\left(1 + \frac{V_{\mathrm{RFmod}}}{V_{fo}}\cos\,\omega_{\mathrm{RF}}t\right)^{-3/4}, \qquad (5)$$

where

$$\beta = \frac{1}{3\sqrt{\pi}}\left(\frac{1}{en_0}\right)^{1/2}$$

If $V_{fo} >> V_{\mathrm{RFmod}}$, we have

$$C_s \simeq C_{so}\left(1 - \frac{3}{4}\frac{V_{\mathrm{RFmod}}}{V_{fo}}\cos\,\omega_{rmRF}t\right) \qquad (6)$$

that is, in a first approximation, the modulation law of C, is a necessary condition for parametric amplification. Equation (6) can be rewritten as

$$C_s \simeq C_{so}[1 - \gamma\cos\,\omega_{\mathrm{RF}}t]\,, \qquad (7)$$

where $\gamma = (3/4)(V_{\mathrm{RFmod}}/V_{fo})$ is the skin depth modulation coefficient for the classical theory of parametric amplification.

To get full functioning of the amplifier, it is necessary to follow the conditions: 1) the signal circuit formed by $L_1//C_1$ must be tuned to the frequency ω_1, that is $\omega_1 \simeq \omega_{r_1}$, where ω_{r_1} is the resonant frequency of the tank $L_1//C_1$ and ω_1 is the signal frequency to be amplified; 2) the iddler frequency ω_2 must be near the resonance of the tank $L_2//C_2$ and has to be different from ω_1, that is, $\omega_2 \simeq \omega_{r_2}$ and $\omega_{r_2} \neq \omega_{r_1}$; 3) the pumping frequency needs to obey the relation $\Omega = \omega_{r_1} + \omega_{r_2}$ [7].

Under these conditions, we have only the frequency ω_1 in the signal circuit, and only the iddler frequency ω_2 in the auxiliar circuit.

If we consider that the amplitudes $E_1(\omega_1)$ and $E_2(\omega_2)$ are smaller than the amplitude $E_n(\Omega)$ of the pumping signal, we can neglect the non-linearity of the sheath capacitance C, in relation to $E_n(\Omega)$ and we replace it by a linear parametric amplificatiori in relation to Ω [7].

In this case, we can show that in the resonance regime, that is $\omega_1 = \omega_{r_1}$ and $\omega_2 = \omega_{r_2}$, respectively, we have a conductance $G_e(\omega_1)$ seen by the signal[7] given by

$$G_e(\omega_1) = -\left(\frac{\Delta C_s}{2}\right)^2 \omega_1\omega_2 R_2 \tag{8}$$

and the amplification gain K_p will be

$$K_p = \frac{1}{\left(1 + \frac{G_e}{2G_1}\right)^2} \tag{9}$$

where $G_1 = 1/R_1$.

When the signal of frequency ω_1 deviates from the frequency ω_{r_1} and, otherwise, the frequency ω_2 deviates from ω_{r_2}, the absolute value $Z_2(\omega_2)$ decreases; consequently the power gain decreases.

4. Experimental results and analysis

In this work the main goal is to study parametric amplification using the nonlinear properties of the sheath capacitance in a low density helium plasma which was created in the metallic cylindrical vessel of the Linear Mirror Device LISA.

The average electron density, measured by means of a circular Langmuir probe L (diameter = **0.2** cm) between the radial electrodes (transmiting electrode T and receiving electrode R) is $\bar{n} = 1.1. \times 10^6$ cm^{-3} and the electron temperature is $T_e = 6.2$ eV.

Figure 4 shows the admittance profile versus frequency for an applied RF voltage fixed in 3.0 V_{RMS}. The admittance characteristic shows a sheath resonance frequency $f_R \approx 8.5$ MHz and an average electron plasma frequency, $\bar{f}_{Pe} \cong 10.5$ MHz, coupled by the expression given by Rapozo et al.[2]

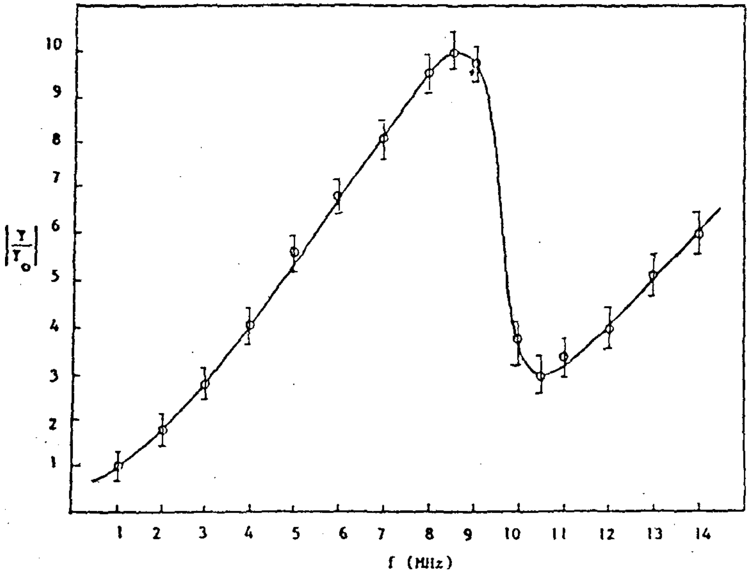

Fig. 4 – Admittance profile versus frequency for an applied RF voltage fixed in 3.0 $V_{(RMS)}$.

$$f_R = \bar{f}_{Pe}\sqrt{\frac{2S}{d}} \tag{10}$$

where f_R is the resonance sheath frequency and $\bar{f}_{Pe}$ is the average electron plasma frequency given by

$$\bar{f}_{Pe} = 10^4\sqrt{\bar{n}} \qquad (Hz, \mathrm{cm}^{-3})\ ; \tag{11}$$

The distance d between the electrodes T and R is fixed at 10 cm. The average electron plasma density $\bar{n}$ is given by $\bar{n} = n_0(1 - 2S_0/d)/(1 - 2S_0/d)/(1 - 2S/d)$, where n_0 and $\bar{n}$ are the electron plasma density without and with RF injection, respectively, and S_0 and S are the thickness of the sheath without and with the applied RF voltage[2].

The amplification caused by the parametric effect due to the nonlinear characteristic of the sheath capacitance resulted in a maximum power gain $K_p(\omega_1)$ of 5.7 as indicated in Figure 5 which also shows that $K_p(\omega_1)$ is asymmetric at the resonance. The absence of symmetry can be explained by Figure **6,** which is the superposition of the $K_p(\omega_1)$, $|Z_p/Z_0|$ and $|Z_1/Z_0|$ profiles versus normaliied frequency f_1, where $|Z_p/Z_0|$ represents the plasma-electrode impendance and $|Z_1/Z_0|$ represents the signal circuit impedance, respectively.

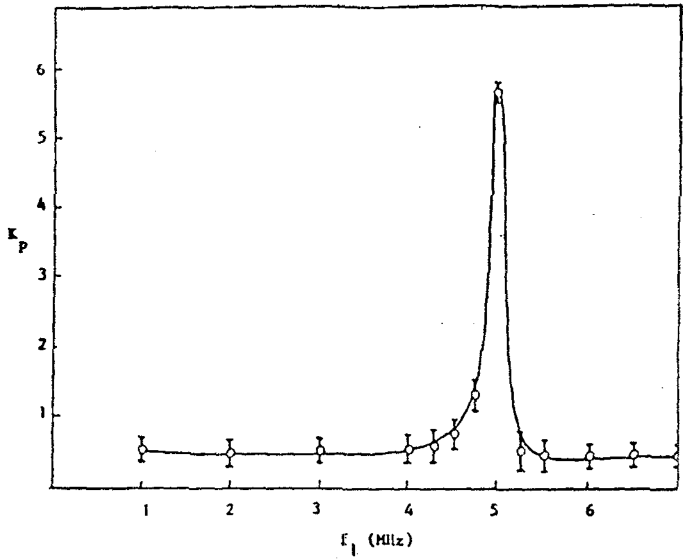

Fig. 5 - Power gain profile K_p versus input signal frequency.

The classical analysis of the two frequency parametric amplifier gives us for the circuit of Figure 2 a negative conductance[8] $G_e^* = -(\Delta C/2)^2\omega_1\omega_2 Z_2^*(\omega_2)$, when Z_p is changed by a nonlinear capacitance. ΔC represents the variation of this capacitance due to the modulation introduced by the RF generator GR_2, ω_1 is the frequency of the signal that is amplified and ω_2 is the iddler frequency. The power gain of the electrical circuit is given by[8] $K_p = 1/(1+G_e/2G_1)^2$, where $G_1 = 1/R_1$ is the conductance of the signal circuit at resonance $(\omega_1 = \omega_{r_1})$ of the tank L_1, C_1.

In the two-frequency parametric amplifier which uses a nonlinear capacitance, a weak variation in the frequency ω_1 about the maximum K_p shows a symmet-

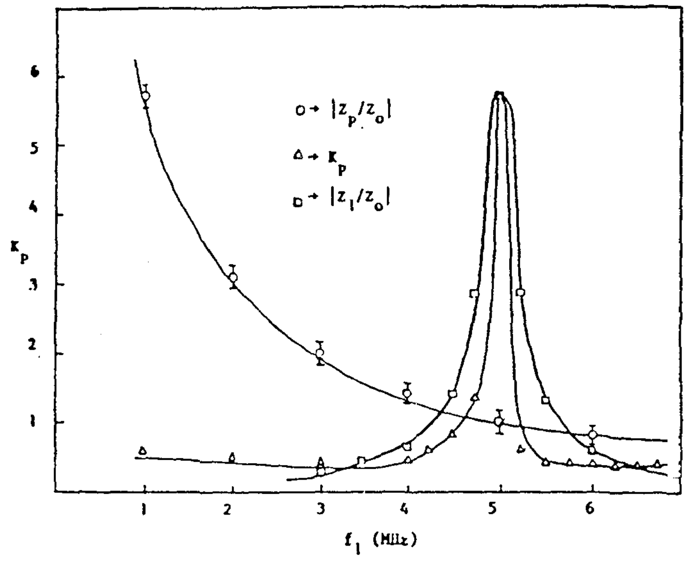

Fig. 6 - Overposition between $K_p(\omega_1)$, $|Z_p/Z_0|$ and $|Z_1/Z_0|$ profiles versus frequency f_1 normalized.

rical profile for the gain. In this work this is not observed because Z_p cannot be considered as a single parametric capacitance.

The lack of symmetry in $K_p(\omega_1)$ can be seen, if we add the plasma conjugate complex impedance $Z_p^*(\omega_1)$, in the $Z_2^*(\omega_2)$ term (between the electrodes). In this case[9], $G_e^* = -(\Delta C/2)^2\omega_1\omega_2[Z_2(\omega_2) + Z_p^*(\omega_1)]$, where $\omega_1 = 2\pi f_1$.

For resonance, $\omega_1 = \omega_{r_1}$ and $\omega_2 = \omega_{r_2}$ where ω_{r_1} and ω_{r_2} are the resonance frequencies for the tank circuit $L_1//C_1$ and $L_2//C_2$, respectively. In this situation $|Z_2^*(\omega_2)| = R_2$ and $G_e = -(\text{Ç}/2)^2\omega_1\omega_2[R_2 + |Z_p^*(\omega_1)|]$.

If the plasma impedance is complex, $Z_p^*(\omega_1) = 0$ implies that $K_p(\omega_1)$ will be symmetric close to the resonance of $Z_1^*(\omega_1)$. But for $Z_p^*(\omega_1) \neq 0$, the absence of the symmetry of $K_p(\omega_1)$, close to the resonance, causes an increase proportional to the modulus of $Z_p^*(\omega_1)$.

We can see that at frequencies below or near w„„ the modulus of $Z_1(\omega_1)$ is increasing and the same occurs with the modulus of $Z_2(\omega_2)$ but $Z_p^*(\omega_1)$ is decreasing, so that the slope of $K_p(\omega_1)$ is reduced. Therefore, for frequencies close to and higher than ω_{r_1}, where the moduli of $Z_2^*(\omega_2)$ and $Z_p^*(\omega_1)$ are decreasing, the magnitudes of G_e and consequently the value of $K_p(\omega_1)$ decrease quickly.

The Figure 7 shows the evidence of the influence of $Z_p^*(\omega_1)$ on the power gain $K_p(\omega_1)$ of the system, where the frequency ω_1 and the pumping frequency Ω were kept constant at resonance. The profile shown in Figure 7 was obtained by variation of the tunning filter across C_2.

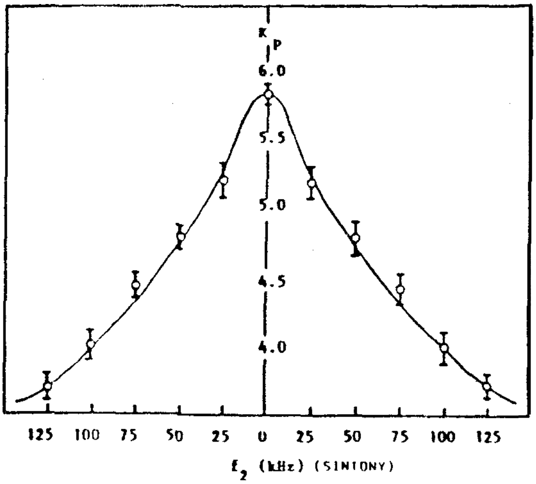

Fig. 7 - Variation of the power gain profile $K_p(\omega_{r_1})$ versus $\Delta\omega_{r_2}$.

The pumping frequency Ω was fixed and the tunning of the tank L_1, C_1 changed by the variation of C_1 was shown in Figure 8.

The symmetry of K_p occurred in both cases about the maximum value; this occurs due to the constancy of the plasma impedance (ω_1 = constant).

The negative conductance G_e responsible for the amplification of the signal can be also understood via an average kinetic energy analysis of the ion crossing the sheath in the direction toward the receiving electrode R in presence of the applied RF signal. Figure 9 shows the qualitative behaviour of the spatial potential between the electrodes T and R suggested by Rapozo et al.[2].

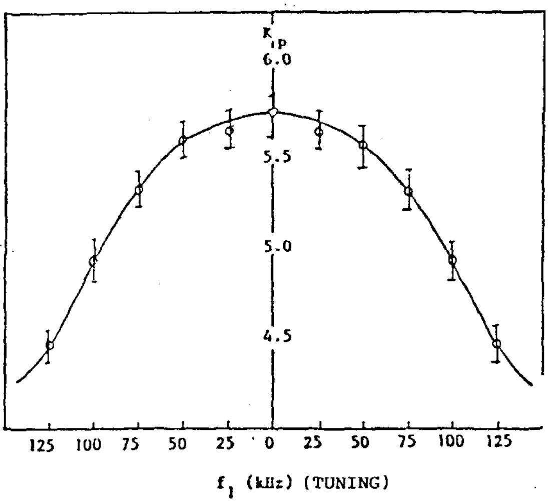

Fig. 8 - Variation of $K_p(\omega_{r_1})$ versus $\Delta\omega_{r_1}$ *by* the variation of the sintony of the circuit tank $L_1//C_1$.

5. CONCLUSIONS

The results we have obtained show again the capacitive nature of the sheath with a parametric amplification of RF signals about 5 MHz. The pronounced drop of the amplification factor near or above the resonance frequency of the signal circuit shows the necessity of verifying the nature of the gain when the signal and its tuning circuit are corresponding to the sheath resonance frequency. An amplification obtained under these conditions would probably eliminate the dependence of the negative conductance on the impendance Z_p of the plasma, in order to enable one to study the variation of the sheath capacitance, which is fundamental for the amplification factor K_p.

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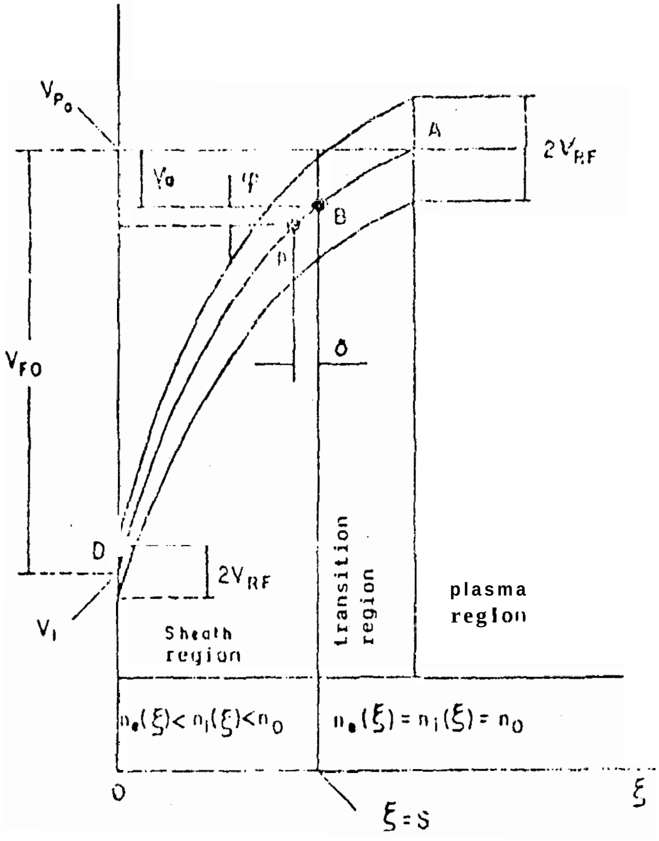

Fig. 9 - Qualitative behaviour of the spatial potential between the electrodes T and R.

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Resumo

Neste trabalho utilizamos a propriedade não linear da capacitância da sheath de um plasma de baixa densidade, para gerar uma amplificação paramétrica de sinais de RF na banda HF. O experimento foi realizado na máquina linear LISA da Universidade Federal Flurninense (Niterói, RJ). O plasma foi gerado usando-se uma fonte de RF, construída na UFF, com uma potência que pode ser ajustada entre 10 e 100 W, cuja freqüência de operação é 28 MHz. Os resultados experimentais mostraram uma boa concordância com o modelo teórico proposto da capacitância da sheath, o que nos permite prever, dentro de um certo limite, a variação da capacitância da sheath como uma função de certos parâmetros do plasma.