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Potential representations for the delta-nucleon interaction

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Abstract We obtain rank-one separable potentials with Yamaguchi form factors to describe the AN isospin one $^5S_2$ and $^5P_3$ phases which were obtained from the analysis of πd elastic scattering observables. For both waves the separable potentials are attractive, but not strong enough to form bound states, and present range parameters of the order of three times the pion mass. We also obtain configuration space potentials of simple (Yukawa and Woods-Saxon) forms, suggested by NN phenomenology, which reproduce the same phases. Again we obtain potentials which are mainly attractive, without bound states, and with ranges corresponding to about three pion masses. For the $^5P_3$ case we have a repulsive core of about 0.7 fm range. We also analyse the AN $^3S_1$ state, whose phases show rapid change with the energy, implying on a repulsive core with a large radius of about 1.5-2 fm.

1. Introduction

Several processes[1,2] have recently exhibited the effects of a direct AN interaction of short range. These phenomena indicate that not all manifestations of the AN dynamics are consequence of the $\Delta N\pi$ vertex, and that the delta may show the behavior of a particle, similar to the nucleon, before it splits itself into N, **s.** The study of these properties of the A's is of great importance for nuclear physics, as the deltas are created inside nuclei in almost every process in which sufficient excitation energy is made available.

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The system most easily accessible to the study of the direct *AN* interaction is the πd system, where the basic calculations are reliable, at least in principle, thanks to the Faddeev equations which solve the three-body scattering problem, and thanks to the good knowledge of the deuteron wavefunction, whose loose structure helps to control off-shell extrapolations and relativistic corrections.

Although equivalent in principle, the $\pi d \to \pi d$ amplitudes obtained by different authors present important differences in their values, mainly due to existing arbitrariness in the input quantities, such as in the choice of models for the energy-shell extrapolation of the intervening two-body amplitudes. In spite of their expect'ed confiability, these theoretical calculations show marked discrepancies with respect to experiments, which are universal for different authors: the calculated differential cross- sections are always too high at intermediate and large angles, and the angular dependence of the vector analysing power iT_{11} does not present the same remarked structure as shown by the experimental data.

We have previously made an effort[3–6] to explain the discrepancies between theoretical and experimental results for πd elastic scattering observables through the influence of an additional direct *AN* interaction in the intermediate state , according to the skeleton diagram of fig 1.

In our model *AN* interaction in the J, L, S states contributes to the πd partial wave helicity amplitude $T^{\pi d \to \pi d;J}_{\lambda',\lambda}$ (λ' and X are respectively the final and initial deuteron helicities) in the form

$$
\begin{aligned}
T^{\pi d\to\pi d;J}_{\lambda'\lambda}(s) = {} & \frac{1}{2J+1}\,\frac{4}{3}\,g^2_{\Delta N\pi}\,\frac{|q_\pi|^3}{m_\Delta m_N\sqrt{s}} \sum_{S,L;S',L'} M^{S,L;S',L';J}_{\Delta N\to\Delta N}(s) \\
& \times\left\{C\begin{pmatrix} S & L & J \\ \lambda & 0 & \lambda \end{pmatrix} C\begin{pmatrix} 1 & 1 & S \\ \lambda & 0 & \lambda \end{pmatrix} K^{(S)}F_L(s)\right\} \\
& \times C\begin{pmatrix} S' & L' & J \\ \lambda' & 0 & \lambda' \end{pmatrix} C\begin{pmatrix} 1 & 1 & S' \\ \lambda' & 0 & \lambda' \end{pmatrix} K^{(S')}F_{L'}(s)\Big\}
\end{aligned}
\qquad (1)
$$

where $K^{(S)} = 1$ for $S = 2$ and $K = 1/\sqrt{3}$ for $S = 1$, and $M^{SL;S'L';J}_{\Delta N\to\Delta N}(s)$ is the AN scattering amplitude. The complex functions $F_L(s)$, which represent the results of the evaluation of the triangular structures in fig 1, present very strong energy dependence,with peaks in both real and imaginary parts located in the region just above the AN threshold, corresponding to pion kinetic energies coincident with the range of available experimental data. The overall values of the vertex functions $F_L(s)$ decrease rapidly as L increases, and we thus expect that the most important contributions come from low L values[3-5].

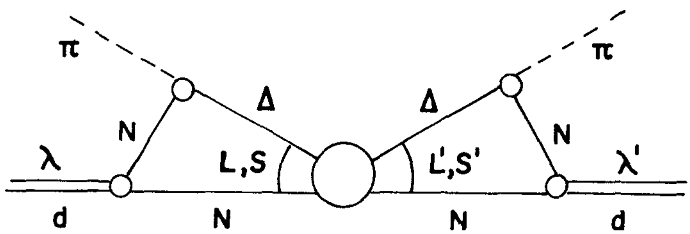

Fig. 1 - Skeleton diagram for the contribution of the ΔN interaction in the intermediate state of pion-deuteron elastic scattering.

The AN matrix elements entering into eq.(1) are unknown quantities. We have extracted their values from a comparison between the experimental values of the elastic *nd* observables (total and differential cross-sections,and vector analysing power) and the theoretical amplitudes, these formed by adding eq (1) to a background set of Faddeev amplitudes. This was done through a best fitting procedure. Due to the intricate participation of the four independent complex helicity amplitudes given by eq.(1) in the expressions for the *nd* observables, the freedom in the choice of parameters is, for most partial waves, of no use for an improvement of the fittings. Using for background different sets of Faddeev amplitudes[4-6] we have shown that only a few AN angular momentum waves are able to improve meaningfully the theoretical description of the data. These are only the (isospin one) $^5S_2$ and $^5P_3$ AN waves, with consequent changes in only two *nd* partial waves, respectively $^3P_2$ and $^3D_3$.

Using for background Garcilazo's[7] set of Faddeev amplitudes, we obtain, with contributions coming from the two above mentioned waves, an almost perfect description of all observables, in the whole energy range of the available data. [Considering that the measurements of tensor polarization observables are still in an early stage, we have chosen not to include their values in the fittings, but rather to treat them as predicted quantities. Actually, in general the tensor observables are not strongly sensitive to the influence of the AN interaction, and still carry large error bars, so that the improvement obtained in their values is not dramatic as with $d\sigma/d\Omega$ and iT_{11}]. It is remarkable that, in spite of the strong energy dependence of the vertex functions, the resulting AN parameters present very smooth energy dependences, easily adjustable to a simple effective range formulae[8]. Our results show absorption only at the lowest energies (below 200 MeV for the pion kinetic energy) in the $^5S_2$ wave.

More recently, experimental data of higher accuracy were obtained for the vector analysing power iT_{11} at five energies. The results for $T_\pi = 219$, 256 and 294 MeV show a still more marked than before structure in iT_{11} in the region of intermediate angles. It has not been possible to fit perfectly these details of the data without introducing the contribution of the $^3S_1$ AN wave, with which also the differential cross section at 219 MeV shows distinct improvement. In contrast to the other two cases, the obtained $^3S_1$ phase shows a rapid energy variation[6]. It must be noted that the smaller value of $K^{(S)}$ for $S = 1$ and the Clebsch-Gordan coefficients in eq. (1) cause this wave to have a weaker contribution to the πd amplitudes than that arising from the other L = 0 wave. This implies that its presence can only be inferred from detailled and accurate data, and its determination depends more critically on the background amplitudes. It is regrettable that the more accurate experiments on iT_{11} have only been made at five energies, two of which being below the more sensitive energy range. It is important to remark that this $^3S_1$ wave is not coupled to the NN channel, where a $\mathbf{I} = 1$, $J^P = 1^+$ state is not allowed by the Pauli principle,so that it must be studied directly in a nuclear AN system, such as formed in the πd collision.

The phase shifts obtained for the three above mentioned AN waves are shown in **fig.2,3,4** . Independent analyses of other experiments have shown similar results for the $^5S_2$ and $^5P_3$ phases' and the mainly repulsive character of the $^3S_1$ interaction has also received independent experimental support[2]. We must remark that the information on the AN interaction is obtained from the difference between experiments and theoretical calculations not including the full AN interaction. There are two main sources at uncertainty: a) The ingredients and corresponding input quantities have their uncertainties, which propagate to the values of the extracted AN phases. Our phases are based on Garcilazo's calculations, but we recall that other sets of Faddeev calculations have yielded[4] similar sets of amplitudes for the AN interaction, though the final description of the data is then much less impressive than in the case of Gacilazo's background amplitudes. We conclude from that fact that the AN interaction yields very characteristic contributions to the ãd scattering amplitudes, and its main features can be extracted with some reliability. b) Another source of uncertainty could be more serious. The contribution of the one pion-exchange diagram containing the $\Delta N\pi$ vertex is already taken into account in the Faddeev calculations so that the AN interaction determined by us represents only a part of the full amplitude. It is clear that an addition of the potentials in general does not lead to an addition of the phases so that our evaluation of the residual interaction could be modified by that fact. Our preliminary investigations[g] have shown that the above mentioned one-pion-exchange gives a small contribution to the $^5P_3$ wave but a rather large one to.the $^5S_2$ wave. The signs and magnitudes tend to fill correctly the difference between ours and the newly determined of phases of ref. 1. We shall report on these results elsewhere[g].

The behavior of these AN waves is similar to that presented by several NN phases of low angular momenta. They are interpreted as representing the AN interaction of short range which occurs before the decomposition of the A into N,n. As mentioned above, the $\Delta N\pi$ vertex is included in Faddeev calculations through the πN P_{33} wave, and thus a part of the AN interaction due to uncorrelated multipion exchanges is there taken into account. However, sirnilarly to the NN system, we expect that the AN short-range interaction is dominated by heavier meson exchanges. This idea can be checked looking for the forms of potentials, of

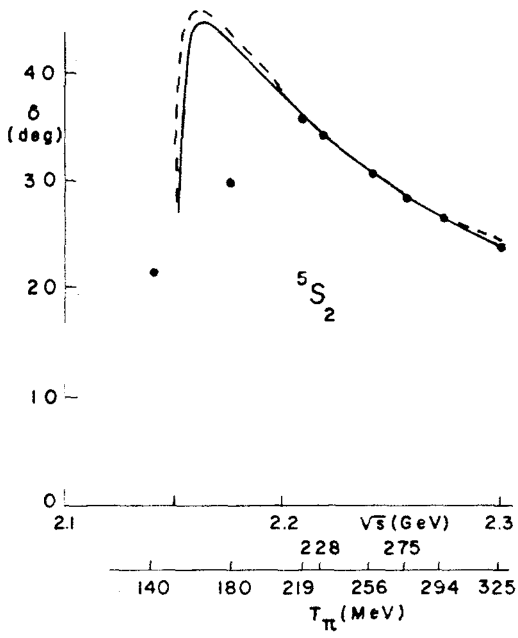

Fig. 2 - Phase shifts for the ΔN interaction in the $^5S_2$ state. The dots indicate the values obtained from the analysis of πd experimental data[5,6]. The full line represents the separable potential values from eq.(17), and the dashed line is obtained from the local Yukawa potential of eq.(22). With a Yukawa term plus a Woods-Saxon repulsive part as in eq.(25), the curve obtained is not distinguishable from the dashed line. The pion kinetic energies of the ãd experiments are shown through the auxiliary scale in the Iower part of the figure. Our previous results[5] show absorption in the $^5S_2$ state below 200 MeV, so that these potential models are not to be applied in the lowest energies.

the conventional types used in the study of the NN system, which produce the AN interaction described above. This is the purpose of the next sections.

2. Separable potentials

Separable potential forms have been developped by several authors[10,11] to describe in all possible detail the NN interaction, in some cases involving several terms and many parameters in each partial wave. Together with the separable potential representations for the pion-nucleon interaction, these representations

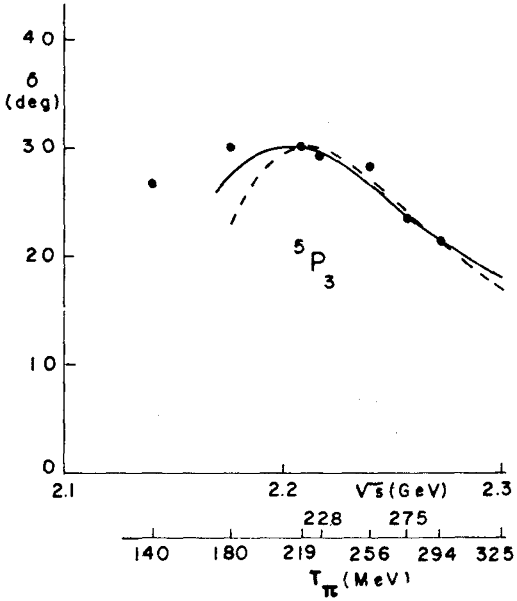

Fig. 3 - Phase shifts for the ΔN interaction in the $^5P_3$ state. The dots indicate the values obtained from the analysis of πd experimental data[5,6]. The full line represents the separable potential values from eq.(18). The dashed line represents the values obtained with an attractive Yukawa potential eq.(22) combined (as in eq.(25)) with a Woods-Saxon repulsive core. See also the caption for fig.2.

of the NN forces constitute important input ingredients for the three-body Faddeev calculations. Separable potentials lead to soluble non-relativistic equations and simplify substantially many-body calculations. They provide models for the off-shell extension of the experimentally determined two-body amplitudes. The motivation for the search of separable potentials for a system like the NN system is mainly a practical one. They should be realistic in the sense of describing faithfully the two-body experimental data, and yet simple and convenient for their use in three- and many- body problems. It is understood that separable potentials are used as a substitute for the (non-existing) knowledge of the true nature of the forces.

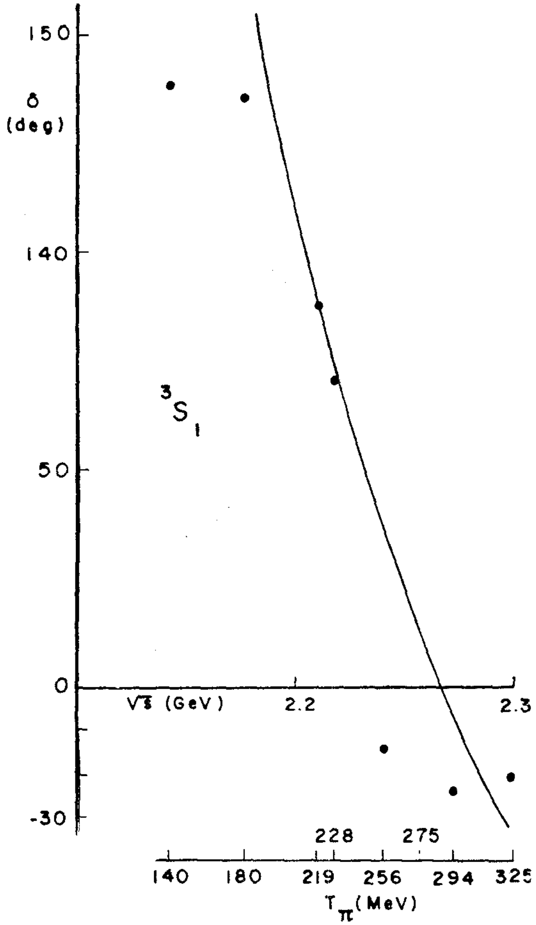

Fig. 4 - Phase shifts for the $^3S_1$ ΔN state. The dots show the values obtained from the analysis of nd experiments. The strong energy variation is only approximately described by the curve, which represents the phase obtained with the potential of **fig.7.**

The inclusion of the direct AN interaction in the three-body calculations will require its representation in terms of separable potentials, which are expected to be similar to those found for the NN system. It will be very interesting to see the similarities and the differences between the NN and the AN interactions, and a common language of separable potentials can be a help in this direction.

In our study of the *nd* system, described in the previous section, we have obtained values for the AN phases only in a limited energy interval, and only representing the short- range part of the interaction. For given values of J and S there may be couplings of different orbitals (L $=$ J, J $\pm$ S for $S = 2$ and L $=$ J $\pm$ S for S $= 1$) but these couplings to higher orbitals have not been observed in our analysis of the πd system , where they are not relevant due to the the decrease in the values of the vertex functions as L increases. Being restricted to the lowest orbitals, couplings to different total spin S values do not occur either. Not having to deal with couplings of different waves substantially simplifies the problem . We thus have to refer only to uncoupled $^5S_2$, $^5P_3$ and $^3S_1$ waves. In the first two cases we have information at six energies, corresponding to πd experiments from 219 to 325 MeV (180 MeV can also be included in the $^5P_3$ case, as no absorption occurs here), and there is a smooth and rather slow energy dependence of the phases. In the $^3S_1$ case , on the contrary, due to the non-existence of the necessary data, there are only three meaningful data points and a rapid energy variation. Besides, in this case the phase passes through zero [cot δ through infinity],and a simple representation with a rank-one potential cannot be found.

In the present work we find very simple representations for the $^5S_2$ and $^5P_3$ phases in terms of separable potentials. We do not include here the $^3S_1$ wave in view of the above mentioned reasons.

In the following, we first present, for convenience, a short review of the properties of separable potentials. Let us assume that the potential operator V can be written as a finite sum of the form

$$V = \sum_J |\chi_J > \lambda_J < \chi_J| \tag{2}$$

where the λ_J's are strength constants and the $|\chi_J >$'s are a convenient set of states. V has the form of a projection operator, and since

$$< \vec{r}'|V|\vec{r} >= \sum_J \chi_J^*(\vec{r}')\ \lambda_J \chi_J(\vec{r})$$

is not of the form $\delta(\vec{r} - \vec{r}')$, it is non-local.

The Lippman-Schwinger equation for the scattering T matrix

$$T(z) = V + VG_0(z)\ T(z) \tag{3}$$

becomes

$$T(z) = \sum_J \lambda_J |\chi_J >< \chi_J| + \sum_J \lambda_J |\chi_J >< \chi_J| G_0(z) T(z) \tag{4}$$

Multiplying from the left by $< \chi_{J'}|G_0(z)$, we obtain

$$\begin{aligned} < \chi_{J'}|G_0(z)\ T(z) &= \sum_J \lambda_J < \chi_{J'}|G_0(z)|\chi_J >< \chi_J| \\ &+ \sum_J \lambda_J < \chi_{J'}|G_0(z)|\chi_J >< \chi_J|G_0(z)\ T(z) \end{aligned} \tag{5}$$

If the states $|\chi_{J'} >$ and $G_0(z)\ |\chi_J >$ with $J \neq J'$ are orthogonal to each other, we obtain a separate equation for each J, with

$$< \chi_J|G_0(z)\ T(z) = \frac{\lambda_J < \chi_J|G_0(z)|\chi_J >}{1 - \lambda_J < \chi_J|G_0(z)|\chi_J >} < \chi_J| \tag{6}$$

Inserting this result into eq (4) we obtain

$$T(z) = \sum_J |\chi_J > \frac{\lambda_J}{1 - \lambda_J < \chi_J|G_0(z)|\chi_J >} < \chi_J| \tag{7}$$

We thus have an exact explicit solution for the T matrix, which is also of separable form.

For our phases we found no mixing of angular momentum states, and it turns out that it is sufficient to use one term for each L value:

$$< \vec{k}|\chi_{LM} >= Y_{LM}(\Theta_{\vec{k}}, \phi_{\vec{k}})\ g_L(k) \tag{8}$$

where $g_L(k)$ is a convenient form factor. Then a rotationally invariant potential is of the form

$$
\begin{aligned}
< \vec{k}'|V|\vec{k} > &= \sum_{L,M} \lambda_L g_L(k')\, g_L(k)\, Y^*_{L,M}(\Theta_{\vec{k}}, \varphi_{\vec{k}})\, Y_{L,M}(\Theta_{\vec{k}'}, \varphi_{\vec{k}}) \\
&= \sum_L \frac{2L+1}{4\pi}\, \lambda_L g_L(k')\, g_L(k)\, P_L(\widehat{k}' \cdot \widehat{k}) \qquad (9)
\end{aligned}
$$

According to eq (7), the transition amplitude is given by

$$
< \vec{k}'|T|\vec{k} >= \sum_L \frac{2L+1}{4\pi}\, \frac{\lambda_L g_L(k')\, g_L(k)}{1-\lambda_L I_L(z)}\, P_L(\widehat{k}' \cdot \widehat{k}) \qquad (10)
$$

where

$$
I_L(z) =< \chi_L|G_0(z)|\chi_L > \qquad (11)
$$

With $z \rightarrow E + ia$, $E = k^2/2\mu$, μ being the reduced mass, the usual partial wave expansion of the T matrix element is

$$
< \vec{k}'|T|\vec{k} >= -\frac{1}{(2\pi)^2\mu} \sum_{L=0}^{\infty} (2L+1)\, P_L(\widehat{k}' \cdot \widehat{k})\, f_L(k) \qquad (12)
$$

so that we identify

$$
f_L(k) = \frac{1}{k(-i+\cot\delta_L)} - -\frac{\mu\pi\lambda_L g_L^2(k)}{1-\lambda_L I_L(k^2)} \qquad (13)
$$

The explicit expression for $I_L(k^2)$ is

$$
I_L(k^2) = \lim_{\varepsilon\rightarrow 0^+} < \chi_L|G_0(E+i\varepsilon)|\chi_L >= \lim_{\varepsilon\rightarrow 0^+} 2\mu \int_{\wedge}^{\infty} \frac{g_L^2(p)\, p^2 dp}{k^2-p^2+i\varepsilon} \qquad (14)
$$

A separable partial wave potential with one term as in eq (9) is said to be of rank-one. The most usual form factors are those of Yamaguchi type[10,11], which for S waves are $g_0(k) = 1/(k^2+\beta^2)$. In the NN case'' the uncoupled waves are described effectively by rank-two potentials, in order to have an attractive part at mid-range and a repulsive shorter-range part. The form of the potentials are chosen so as to produce the expected threshold behavior for the phase-shifts, $\delta_L \sim k^{2L+1}$.

3. Results

The phase-shifts for the $^5S_2$ and $^5P_3$ ΔN waves which we have obtained from the analysis of πd observables are shown in full lines in fig 2 and 3 . Our numerical work has shown that these values are very well represented by rank-one potentials. Then the relevant integrals are only

$$I_0(k^2) = \frac{\mu\pi}{(k^2+\alpha_0^2)^2}\left[\frac{1}{2\alpha_0}\,(k^2-\alpha_0^2) - ik\right] \tag{15}$$

and

$$I_1(k^2) = \frac{\mu\pi}{(k^2+\alpha_1^2)^4}\left[\frac{1}{16\alpha_1^3}\,(k^2-\alpha_1^2)(\alpha_1^4+10\alpha_1^2\,k^2+k^4) - ik^3\right] \tag{16}$$

and the $^5S_2$ and $^5P_3$ phase-shifts are given respectively by

$$k\ \cot\delta_0 = -\frac{(k^2+\alpha_0^2)^2}{\lambda_0\mu\pi} + \frac{1}{2\alpha_0}\,(k^2-\alpha_0^2) \tag{17}$$

and

$$k^3\ \cot\delta_1 = -\frac{(k^2+\alpha_1^2)^4}{\lambda_1\mu\pi} + \frac{(k^2-\alpha_1^2)}{16\alpha_1^3}\,[\alpha_1^4+10\alpha_1^2\,k^2+k^4] \tag{18}$$

which present the expected threshold behavior for these waves (of course we are aware that threshold is a rather vague concept in a collision of a delta resonance).

The values of the momenta used in these expressions are evaluated, in a zero width approximation for the Δ, from

$$k = \frac{1}{2\sqrt{s}}\{[s-(m_\Delta-m_N)^2][s-(m_\Delta+m_N)^2]\}^{1/2} \tag{19}$$

with $m_\Delta = 1.211$ GeV , $m_N = 0.939$ GeV (reduced mass $\mu = 0.529$ GeV). The values of s which are used in our work are those of πd scattering experiments (219, 228, 256, 275, 294 and 325 MeV for the pion kinetic energies). Since we have two parameters for each wave, we have chosen to adjust them to reproduce exactly the phases at two selected energies (219 and 294 MeV). As shown in fig 2 and 3 the phase values obtained for the other energies are then in very good agreement with the ΔN phases to be described[5,6].

Potential representations for the delta-nuclwn interaction

We recall that for the $^5S_2$ wave at 180 MeV our result for the AN amplitude shows absorption, and its description through a potential is not expected to be satisfactory. For the $^5P_3$ wave, we may use eq (18) to include also 180 MeV, since there is no absorption observed in this case, but we see in fig 3 that the predicted value of the phase-shift at this energy is not in very good agreement with the value extracted from the πd analysis.

Below *180* MeV, no description through a potential can be used, because then the momentum value given by eq (19) becomes imaginary, with the AN energy below the threshold as defined by $m_\Delta + m_N$.

The values of the parameters λ_0, λ_1, α_0 and α_1 are given in table 1. We see that in both cases the potentials are attractive, and that the range parameters are similar, corresponding to about three times the pion mass. It is interesting to remark that the potential wells found for the two partial waves have similar depths at their respective ranges, i.e. for p $\sim \alpha_L$.

Table 1

Parameters values for the AN separable potentials

$^5S_2$	$\lambda_0 = -0.0819$ GeV^2	$\alpha_0 = 0.443$ GeV
$^5P_3$	$\lambda_1 = -0.0236$ GeV^4	$\alpha_1 = 0.362$ GeV

The condition for the existence of a bound state is that $f_L(k)$ has a pole for a positive imaginary value of k, k $= i\sqrt{2\mu B}$, where B > 0 is the binding energy . For the L = 0 and L = 1 rank-one potentials described above, the relations between the potential parameters and the binding energies are respectively

$$\lambda_0 = -\frac{2}{\mu\pi}\ \alpha_0(\sqrt{2\mu B_0} + \alpha_0)^2 \tag{20}$$

and

$$\lambda_1 = -\frac{16}{\mu\pi}\ \frac{\alpha_1^3[\alpha_1 + \sqrt{2\mu B_1}\ ]^4}{\alpha_1^2 + 4\alpha_1\sqrt{2\mu B_1} + 2\mu B_1} \tag{21}$$

With the α_0 and α_1 values given in table 1, the thresholds for the potential strengths able to produce bound states (putting B_0 and B_1 equal to zero in eq *(20)* and *(21))* are respectively $\lambda_0^{\min} = -0.105$ GeV^2 and $\lambda_1^{\min} = -0.060$ GeV^4, so that our potentials are too weak to produce bound states. We may observe that the $^5S_2$ potential strength is not very far from that threshold.

4. Configuration space potentials

The Yamaguchi forms of the non-local separable potentials suggest that we are dealing with forces which fall to zero with the distance with a Yukawa-like behavior. We may then look for coordinate space representations $V(r)$ which give the same ΛN phases. We bori-ow from the NN phenomenology[12] the form

$$V_Y(r) = V_0(1 - e^{-cr^2})\; e^{-\mu r}/\mu r \tag{22}$$

where a convergence factor is used to regularize the Yukawa potential near the origin. With parameter values

$$V_0 = -4.524\ GeV, \qquad \mu = 3.80 m_\pi, \qquad c = 0.011\ GeV^2 \tag{23}$$

we obtain a very good representation for the $^5S_2$ phase shift, for all energies of our analysis *(219* to *325* in πd scattering), as shown in **fig 2.** We infer from Levinson's theorem that this potential does not form bound states.

To study the influence of repulsive core about the origin, following the usual NN phenomenology[12] we add to eq *(22)* a Woods-Saxon core

$$W(r) = W_0[1 + e^{\mu_c(r-b)}]^{-1} \tag{24}$$

Then a very good description of the $^5S_2$ phase shift is obtained with

$$V(r) = V_Y(r) + W(r) \tag{25}$$

with parameter values

Potential representations for the *delta-nucleon* interaction

$$V_0 = -1.473 GeV, \quad \mu = \mu_c = 3\ m_\pi, \quad c = 0.08\ GeV^2,$$
$$W_0 = 0.329\ GeV, \quad b = 1.508\ GeV^{-1} \tag{26}$$

The curves representing the potentials in eqs.(22) and (25) are shown in fig.5 [dashed line for eq (25) and full line for eq (22)]. We see that the repulsive core radius is very small(0.2f m) and that the potential is about 65 MeV deep. We must enphasize that the additional Woods-Saxon core does not improve meaningfully the description of the phases, as compared to the pure Yukawa form.

Using the same potentials, with the same parameter values given above, in the L = 1 case, we obtain too low values for the ${}^5P_3$ phases.Other parameter values should then be obtained for this case. We find that a pure Yukawa potential as in eq.(22) is not able to reproduce well the energy dependence of the phases, giving a rather flat energy dependence. The best obtained parameter values in this case are $V_0 = -3.5$ GeV, $\mu = 2.5\ m_\pi$ and c = 0.004GeV^2. We then find that a repulsive core about the origin has an important role in this case, and obtain for parameter values

$$V_0 = -9.669 GeV, \mu = 3.561\ m_{,,}\ \ c = 0.014\ GeV^2,$$
$$W_0 = 2.084\ GeV, b = 3.170\ GeV^{-1}, \mu_c = 3.621\ GeV \tag{27}$$

This potential shape is shown in fig (6), and the corresponding phase-shifts are shown in fig 3 (dashed line). This potential again does not from bound states.

The behavior of the ${}^5S_1$ phase, with rapid decrease and change of sign of the phase-shift occurring at a rather small value of the momentum, indicates a rather large core radius, so that the potentiai is dominantly repulsive. In a fitting using a potential consisting in a sum of two Woods-Saxon forms

$$U(r) = W_1[1 + e^{\mu_1(r - b_1)}]^{-1} + W_2[1 + e^{\mu_2(r - b_2)}]^{-1} \tag{28}$$

we obtain a large core radius of about 1.5 fermis and a weaker attractive part at larger distances. With the parameters values

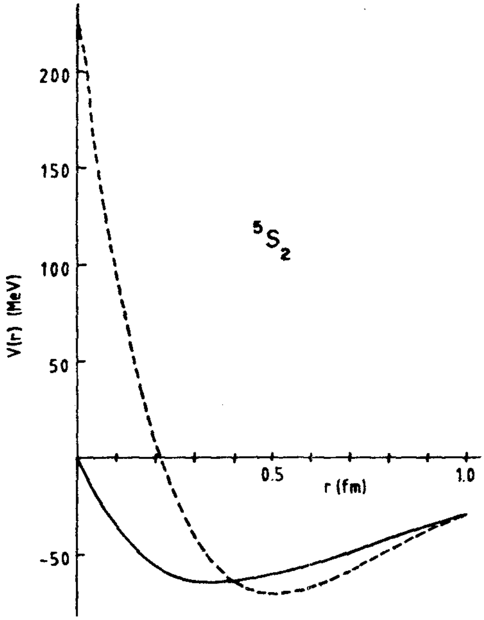

Fig. 5 - Coordinate space potentials, given by eq.(22) (full line) and eq.(25) (dashed line), which produce the $^5S_2$ phase shifts shown in fig.2. The parameter values are given in eqs.(23) and (26) respectively.

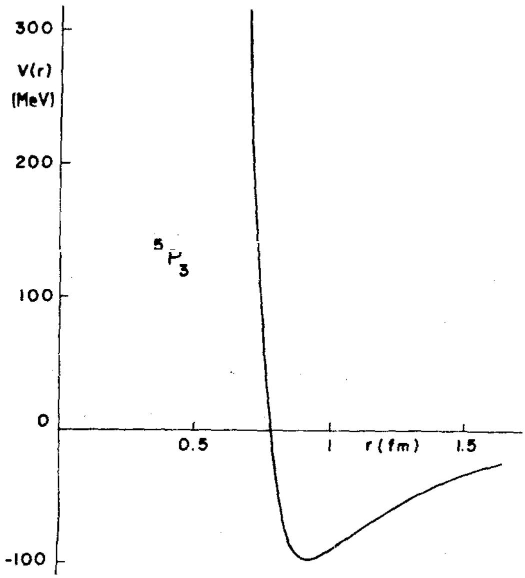

Fig. 6 - Coordinate space potential shape which reproduces the values for the $^5P_3$ ΔN phase-shifts. The potential is given by eq.(25), with parameter values given the text (eq.(27)).

$$W_1 = 20.065\ GeV, b = 3.717\ GeV^{-1}, \mu_1 = 0.269\ GeV$$
$$W_2 = -12.045\ GeV, b = 3.744\ GeV^{-1}, \mu_2 = 0.267\ GeV \tag{29}$$

we obtain the phases represented by the curve in **fig.4** and the potential shape shown in fig.7. The range implied by $\mu_1 \approx \mu_2 \approx 2\ m_\pi$ corresponds to a two-pion-mass exchange. Unfortunately the existing πd experimental data are not enough for a proper study of the AN interaction in this state.

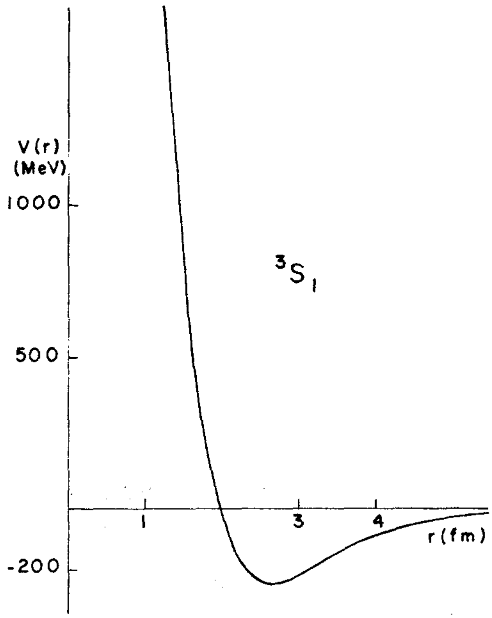

Fig. 7 - The curve shows a potential shape which gives the approximate values for the $^3S_1$ ΔN phase-shifts shown in fig.4. The curve is obtained from a superposition of a repulsive with an attractive Woods-Saxon form, as in eq.(28), with parameter values as given in the text (eq. (29)). We observe a strong repulsive core with large radius, and a much weaker attractive part at larger distances.

5. Comments

The description of the AN interaction through separable potentials is important for its insertion in Faddeev type calculations and in hamiltonian type formulations of the πNN problem. Rank-one potentials of the simplest Yamaguchi forms give very good description of the $^5S_2$ and $^5P_3$ phases obtained in the analysis of the πd scattering observables. It is interesting that similar ranges, corresponding to about 3 pion masses are found in both $^5S_2$ and $^5P_3$ cases. This range is confirmed by the potentials in configuration space which produce these ΔN phases.

In the $^3S_1$ case, it is difficult to describe through simple potential forms the rapid energy dependence of the phase. Our results indicate a strong spin dependence in the potential. We remark that $\vec{S}_\Delta \cdot \vec{S}_N = 3/4$ for $S = 2$ and $\vec{S}_\Delta \cdot \vec{S}_N = -5/4$ for $S = 1$ states, so that a term of this form contributes to the $^3S_1$ state with opposite sign as compared to the two $S = 2$ states here discussed.

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Resumo

Obtemos potenciais separáveis de rank um com fatores de forma de Yamaguchi para descrever as fases $^5S_2$ e $^5P_3$ da interaçáo AN em isospin um, as quais foram obtidas através da análise de observáveis do espalhamento πd. Para ambas ondas, os potenciais separáveis são atrativos, mas não são fortes o bastante para formar estados ligados, e apresentam parâmetros de alcance da ordem de três vezes a massa do píon. Obtemos também potenciais no espaço das configurações de formas simples (Yukawa e Woods-Saxon), sugeridas pela fenomenologia NN, as quais reproduzem as mesmas fases. Novamente obtemos potenciais que são predominantemente atrativos, sem estados ligados, e com alcances correspondentes a cerca de três massas do píon. Para o caso $^5P_3$ temos um caroço repulsivo de cerca de 0,7 fm de alcance. Analisamos também o estado $^3S_1$, cujas fases mostram variação rápida com a energia, implicando na presença de um caroço repulsivo com um grande raio de cerca de 1,5-2 fm.